Extraction of $m_s$ and $|V_{us}|$
from Hadronic Tau Decays

Joaquim Prades
CAFPE and Universidad de Granada

with Elvira Gámiz (U. Glasgow), Matthias Jamin (U. Heidelberg),
Antonio Pich (U. València) and Felix Schwab (U. München)

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Plan

Introduction: Theoretical Framework
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Fixed $m_s$: Determination of $|V_{us}|$
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- Combined Fit to Determine $|V_{us}|$ and $m_s$
- Results and Conclusions
ALEPH, OPAL and CLEO → High precision status of

\[ R_\tau \equiv \frac{\Gamma[\tau^- \rightarrow \text{hadrons } \nu_\tau(\gamma)]}{\Gamma[\tau^- \rightarrow e^- \nu_e \nu_\tau(\gamma)]} \]

[and related observables] further increase at B-factories.

Sizeable correction in the semi-inclusive \( \tau \)-decay width into Cabibbo-suppressed modes due SU(3) breaking.

Obtain the strange quark mass and \( |V_{us}| \).

Advantage: The experimental uncertainty can be systematically reduced!
Introduction: Theoretical Framework

A lot of work!

- M. Davier
- S. Chen, A. Höcker, M. Davier
- K. Maltman
- K. Chetyrkin, J. Kühn, A. Pivovarov
- A. Pich, J.P.

Very much improvable with expected B-factories accuracy!
Two-point correlation functions for vector $V^\mu_{ij} = \bar{q}_i \gamma^\mu q_j$ and axial-vector $A^\mu_{ij} = \bar{q}_i \gamma^\mu \gamma^5 q_j$ two-quark currents:

$$\Pi^{\mu \nu}_{V, ij}(q) \equiv i \int d^4 x \, e^{i q \cdot x} \langle 0 | T \left( \left[ V^\mu_{ij} \right]^\dagger (x) V^\nu_{ij} (0) \right) | 0 \rangle$$

$$\Pi^{\mu \nu}_{A, ij}(q) \equiv i \int d^4 x \, e^{i q \cdot x} \langle 0 | T \left( \left[ A^\mu_{ij} \right]^\dagger (x) A^\nu_{ij} (0) \right) | 0 \rangle$$

$i, j = u, d, s$;

Lorentz decomposition:

$$\Pi^{\mu \nu}_{V(A), ij}(q) \equiv \left[ q^\mu q^{\nu} - q^2 g^{\mu \nu} \right] \Pi^{T}_{V(A), ij}(q^2) + q^\mu q^{\nu} \Pi^{L}_{V(A), ij}(q^2)$$

$\text{Im} \, \Pi^{J}_{V(A), ij}(q^2)$ are proportional to the corresponding spectral functions.
Introduction: Theoretical Framework

Using the analytic properties of $\Pi^J(s)$

\[ R_\tau \equiv -i\pi \oint_{|s|=M_\tau^2} \frac{ds}{s} \left[ 1 - \frac{s}{M_\tau^2} \right]^3 \left\{ 3 \left[ 1 + \frac{s}{M_\tau^2} \right] D^{L+T}(s) + 4D^L(s) \right\} ; \]

phase space factors: order three zero in real axis

\[ D^{L+T}(s) \equiv -s \frac{d}{ds} [\Pi^{L+T}(s)] ; \quad D^L(s) \equiv s \frac{d}{ds} \left[ s \Pi^L(s) \right] . \]

Large enough Euclidean $Q^2 \sim \Pi^{L+T}(Q^2)$ and $\Pi^L(Q^2)$ organised in series of dimensional operators using OPE
Moreover, we can decompose $R_{\tau}$ into

$$R_{\tau} \equiv R_{\tau, V} + R_{\tau, A} + R_{\tau, S}$$

according to the quark content

$$\Pi^{J}(s) \equiv |V_{ud}|^2 \{ \Pi^{J}_{V, ud}(s) + \Pi^{J}_{A, ud}(s) \} + |V_{us}|^2 \{ \Pi^{J}_{V, us}(s) + \Pi^{J}_{A, us}(s) \} \bullet$$

Additional information obtained from the moments

$$R^{kl}_{\tau} \equiv \int_{0}^{1} dz \ (1 - z)^{k} z^{l} \frac{dR_{\tau}}{dz} \equiv R^{kl}_{\tau, V + A} + R^{kl}_{\tau, S} \bullet$$
Introduction: Theoretical Framework

\[ R_{\tau}^{kl} \equiv N_c \, S_{\text{EW}} \left( |V_{ud}|^2 + |V_{us}|^2 \right) \left[ 1 + \delta^{kl(0)} \right] + \sum_{D \geq 2} \left[ |V_{ud}|^2 \delta^{kl(D)}_{ud} + |V_{us}|^2 \delta^{kl(D)}_{us} \right] \]

\[ \delta^{kl(D)}_{ud} \text{ and } \delta^{kl(D)}_{us} \sim \text{dimension } D\text{-operators} \]

The most important being \( D = 2 \left[ m_s^2 \right] \) and \( D = 4 \left[ m_s \langle \bar{q}q \rangle \right] \).

The flavour SU(3)-breaking quantity

\[ \delta R_{\tau}^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = N_c \, S_{\text{EW}} \sum_{D \geq 2} \left[ \delta^{kl(D)}_{ud} - \delta^{kl(D)}_{us} \right] \]

enhances the sensitivity to the strange quark mass.
Introduction: Theoretical Framework

\[ \delta_{ij}^{kl(2)} \text{ known to } \mathcal{O}(a^3) \text{ for } J = L \text{ and } \mathcal{O}(a^2) \text{ for } J = L + T \]

Chetyrkin; Gorishny, Kataev, Larin, Sugurladze; Chetyrkin, Kühn; Becchi, Narison, de Rafael; Bernreuther, Wetzel

Extensive analysis by Pich & J.P.

Perturbative \( L + T \) series converges very well \( \checkmark \)

Perturbative \( L \) series behaves very badly!

In following applications, \[ \delta_{ij}^{kl(4)} \] fully included while \[ \delta_{ij}^{kl(6)} \]
estimated to be of order or smaller than error of \( D = 4 \) \( \bullet \)
Fixed $m_s$: Determination of $|V_{us}|$

QCD Sum Rules, Lattice QCD and Tau Hadronic Data:

$m_s[2\text{GeV}] = 95 \pm 20 \text{ MeV} \sim \delta R_{\tau}^{kl}$ predicted from theory!

Bad QCD behaviour of $J = L$ component in $\delta R_{\tau}^{kl}$

Theory uncertainty much reduced using phenomenology for scalar/pseudoscalar correlators ✓

Dominant pseudoscalar $u_8$ spectral function

$$s^2 \frac{1}{\pi} \text{Im} \Pi_{u_8, A}^L = 2f_K^2 m_K^4 \delta(s - m_K^2) + 2f_{K(1460)}^2 M_{K(1460)}^4 BW(s);$$

Normalized Breit-Wigner: Kambor, Maltman
Scalar spectral functions from M. Jamin, J.A. Oller, A. Pich

Comparison of these spectral functions with QCD

<table>
<thead>
<tr>
<th></th>
<th>$R_{us,A}^{00,L}$</th>
<th>$R_{us,V}^{00,L}$</th>
<th>$R_{ud,A}^{00,L} \times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPE</td>
<td>$-0.144 \pm 0.024$</td>
<td>$-0.028 \pm 0.021$</td>
<td>$-7.79 \pm 0.14$</td>
</tr>
<tr>
<td>Pheno.</td>
<td>$-0.135 \pm 0.003$</td>
<td>$-0.028 \pm 0.004$</td>
<td>$-7.77 \pm 0.08$</td>
</tr>
</tbody>
</table>

★ Perturbative QCD for $J = L + T$ converges very well and OPE included up to $D = 6$ ★

$\delta R_T^{kl,L}$ from phenomenology while $\delta R_T^{kl,L+T}$ from QCD ●
**Fixed $m_s$: Determination of $|V_{us}|$**

Smallest theory uncertainty for $(k, l) = (0, 0)$

$$
\delta R_{\tau,th}^{00} = (0.162 \pm 0.013) + (6.1 \pm 0.6)m_s^2 - (7.8 \pm 0.8)m_s^4 = 0.218 \pm 0.026
$$

(Coefficients are MS-bar at 2 GeV)

$$
|V_{us}|^2 = \frac{R_{\tau,S}^{kl}}{R_{\tau,V+A}^{kl}/|V_{ud}|^2 - \delta R_{\tau,th}^{kl}}
$$

Using OPAL update:

★ OPAL and CLEO: New branching fraction $B(\tau^- \to K^-\pi^+\pi^-\nu)$

★ $R_{\tau,V+A} = 3.469 \pm 0.014$ and $R_{\tau,S} = 0.1677 \pm 0.0050$
Fixed $m_s$: Determination of $|V_{us}|$

$$|V_{us}| = 0.2208 \pm 0.0033_{\text{exp}} \pm 0.0009_{\text{th}} = 0.2208 \pm 0.0034$$

(using PDG value $|V_{ud}| = 0.9738 \pm 0.0005$)

Uncertainty becomes experimental issue!

PDG 2004: $|V_{us}| = 0.2200 \pm 0.0026$

KI3 (E865, KTeV, KLOE):
Jamin et al, $|V_{us}| = 0.2229 \pm 0.0026$ to Leutwyler-Roos $0.2259 \pm 0.0022$

$f_K/f_\pi$ Marciano, MILC: $V_{us} = 0.2219 \pm 0.0026$

Unitarity: $|V_{us}| = 0.2265 \pm 0.0022$

**Remark:** If experimental $B(\tau \rightarrow K\nu) = (0.686 \pm 0.023)\%$ is replaced by more precise theoretical value $(0.715 \pm 0.004)\%$

based on $K_{\mu2}$ decay $\Rightarrow |V_{us}| = (0.2219 \pm 0.0034)$
Fixed $|V_{us}|$: Determination of $m_s$

Using OPAL data with $|V_{us}| = 0.2208 \pm 0.0034$ and $\delta R^{kl,L}_{\tau,\text{phen}}$

$$\delta R^{kl,L+T}_{\tau} = \delta R^{kl}_{\tau} - \delta R^{kl,L}_{\tau,\text{phen}}$$

$$m_s^2(M_\tau^2) \approx \frac{M_\tau^2}{1 - \varepsilon_d^2} \frac{1}{\Delta_{kl}^{L+T(2)}(a_{\tau})} \left[ \frac{\delta R^{kl,L+T}_{\tau}}{18 S_{EW}} + \frac{8}{3} \pi^2 \frac{\delta O_4(M_\tau^2)}{M_\tau^4} Q_{kl}^{L+T}(a_{\tau}) \right]$$

known in perturbative QCD: very good convergence

$$\delta O_4(M_\tau^2) \equiv \langle m_s \bar{s}s - m_d \bar{d}d \rangle \approx -[1.5 \pm 0.4]10^{-3} \text{ GeV}^4$$

and $\varepsilon_d \equiv m_d/m_s$
Fixed $|V_{us}|$: Determination of $m_s$

Moments $(0, 0)$ and $(1, 0)$ dominated by experimental uncertainty, we only use

<table>
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<tr>
<th>$(2, 0)$</th>
<th>$(3, 0)$</th>
<th>$(4, 0)$</th>
</tr>
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<td>$m_s(M_\tau)$ MeV</td>
<td>$93.2^{+34}_{-44}$</td>
<td>$86.3^{+25}_{-30}$</td>
</tr>
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</table>

Weighted average

$m_s(M_\tau) = [84 \pm 23]$ MeV
$m_s(2\text{GeV}) = [81 \pm 22]$ MeV

★ Larger OPAL $B(\tau^- \to K^-\pi^+\pi^-\nu)$

~ reduced the strong ALEPH $(k,0)$-moment dependence in $m_s$

~ smaller value for $m_s$ $(115\pm 20 \to 85 \pm 20)$ MeV
Combined Fit to $|V_{us}|$ and $m_s$

**Ultimate procedure**

- **Simultaneous fit to** $|V_{us}|$ and $m_s$ for a set of moments.

First step, **neglect the correlations** and use the five OPAL moments $R_{00}$ to $R_{40}$.

**Fit**

$|V_{us}| = 0.2196$ and $m_s(2\text{GeV}) = 76 \text{ MeV}$

**Compatible with previous results**
Combined Fit to $|V_{us}|$ and $m_s$

- Rather strong correlations $\rightarrow$ expected uncertainties similar to individual ones.

- Moment-dependence of $m_s$ is reduced in the fit.

Full analysis including correlations is under way!
Results and Conclusions

High precision tau hadronic (Cabibbo-suppressed) data from ALEPH, OPAL at LEP and CLEO at CESR provide already competitive results on $|V_{us}|$ and $m_s$.

Using OPAL spectral functions:

$|V_{us}| = 0.2208 \pm 0.0034$

$m_s(2\text{GeV}) = [81 \pm 20] \text{ MeV}$

Combined fit to determine both $|V_{us}|$ and $m_s$ ready soon!
Open questions:

- Moment dependence of $m_s$ very much reduced after OPAL and CLEO new $B(\tau^- \rightarrow K^-\pi^+\pi^-\nu)$
- what happens with $K\pi\pi\pi$?
- Origin of remaining moment dependence?

- ALEPH data, (S. Chen et al)
  $m_s$ determination fulfils quark-hadron duality and OPE,
  what happens with $|V_{us}|$? (see K. Maltman’s talk)

- Low experimental $B(\tau \rightarrow K\nu)$ compared to theoretical prediction based on $K\mu_2$ decays?
Results and Conclusions

Previous issues: theoretical or experimental origin?

~ need more accurate measurements combined with theoretical analyses!

★ With expected B-factories accuracy, \( \tau \) hadronic decays have the potential to provide one of the most accurate measurements for \( |V_{us}| \) and \( m_s \)!