

# Muon $g-2$ : a theoretical review



Tau04  
Nara, September 2004

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# Outline

QED: present and future (T. Kinoshita)

Electroweak loops

Hadronic effects

- \* vacuum polarization

(J. Kühn, S. Eidelman, D. Leone, M. Davier, B. Schwarz, K. Hagiwara)

- \* light-by-light scattering

Summary and outlook

# Muon $g-2$ : Standard Model update

Units:  $10^{-11}$

**QED**                      **116 584 719 (1)**                      **hep-ph/0402206**

**Hadronic**

**LO**                      **6 963 (72)**                      **hep-ph/0308213**

**NLO**                      **- 98 (1)**                      **hep-ph/0312250**

**LBL**                      **120 (40)**                      **tentative, see  
hep-ph/0312226**

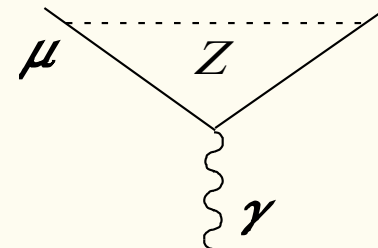
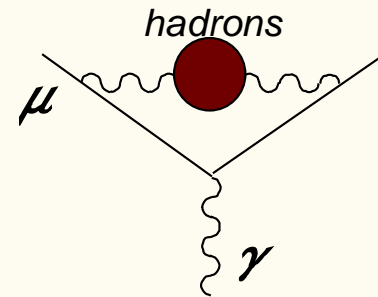
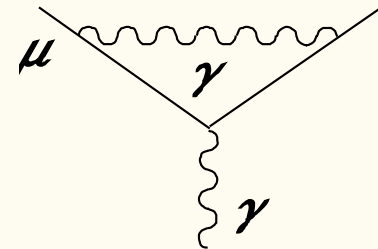
**Electroweak**                      **154 (3)**                      **hep-ph/0212229**

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**Total SM**                      **116 591 858 (82)**

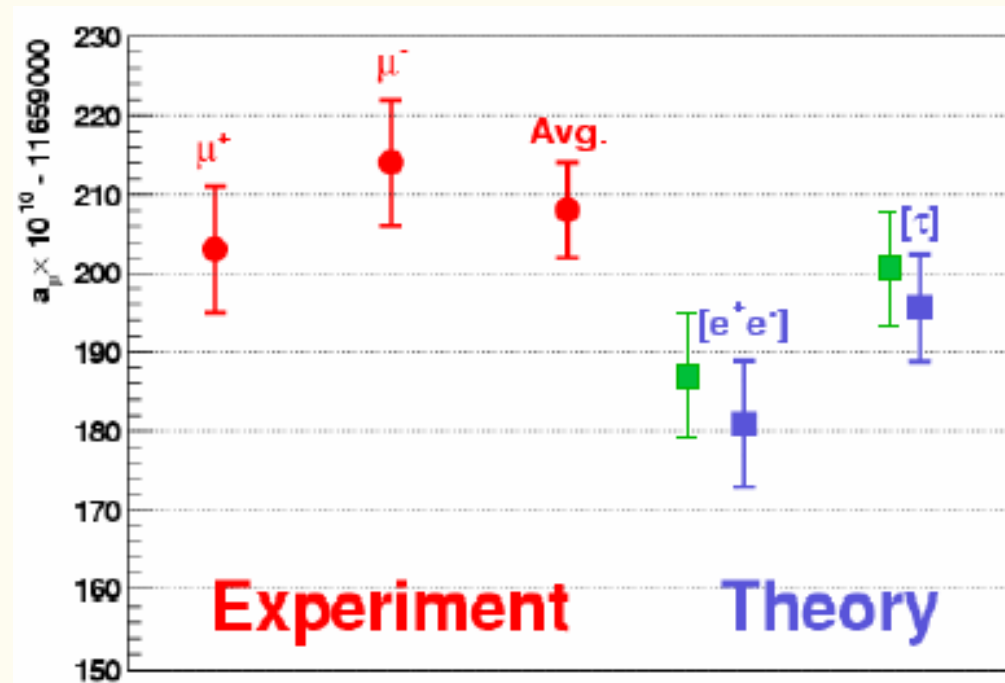
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**Experiment - SM Theory = 222 (102) (2.2 $\sigma$  deviation)**



# Muon $g-2$ : new data

Brookhaven, January 2004:  $\mu$  measurement.



from A. Vainshtein

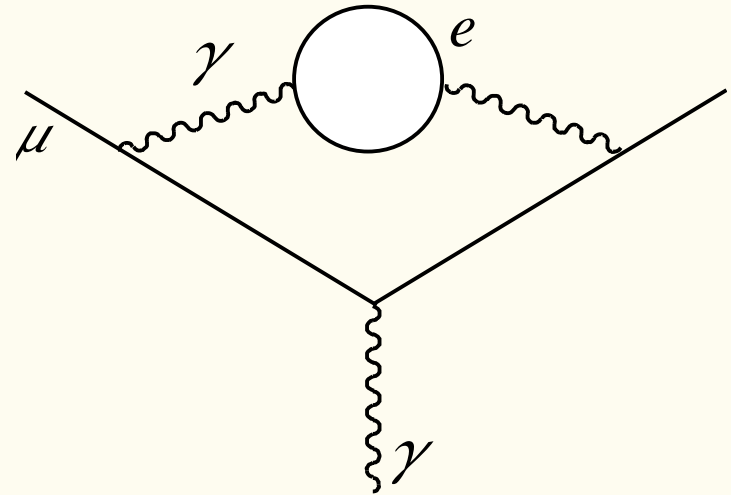
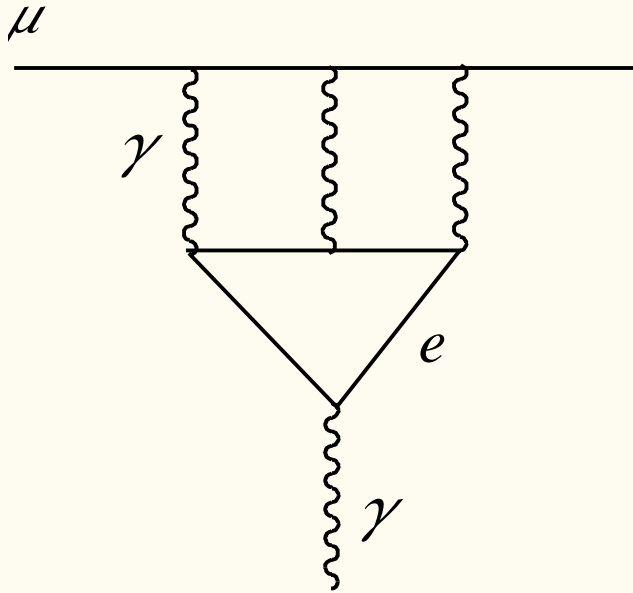
$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 222 \pm 102 \cdot 10^{-11} \\ \rightarrow 2.2\sigma$$

(based on  $e^+e^-$ )

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 123 \pm 89 \cdot 10^{-11} \\ \rightarrow 1.4\sigma$$

(tau)

# QED contributions: muon vs. electron



Enhancement factors:

$$\pi^{2n} \ln \frac{m_\mu}{m_e}$$

$$\ln^n \frac{m_\mu}{m_e}$$

Leading five-loop effects must be included!

# QED contributions: problems at 4-loop order

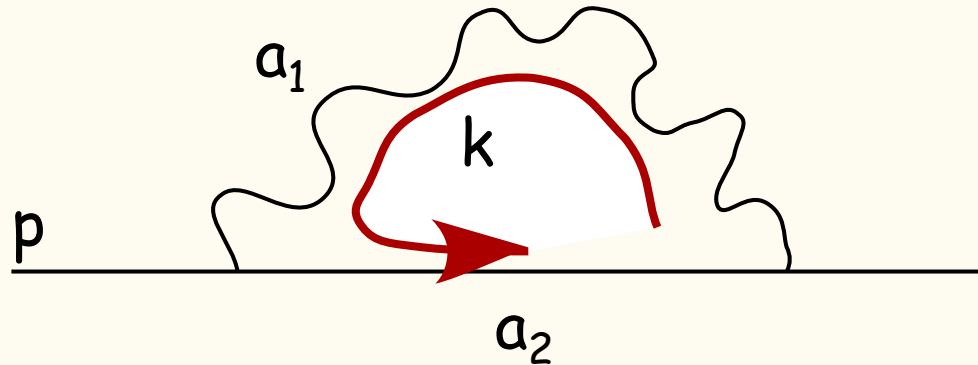
Traditional approach (T. Kinoshita and M. Nio (Nara)):  
numerical problems, digit deficiency

New approach (various groups, in progress):  
Combine numerical and algebraic methods

Reduce all integrals to a smaller basis

Evaluate the primitive integrals numerically,  
with high accuracy.

# Example: integration by parts



$$J(a_1, a_2) = \int d^D k \frac{1}{(k^2)^{a_1} (k^2 + 2kp)^{a_2}}$$

$$0 = p_\mu \int d^D k \frac{\partial}{\partial k_\mu} \frac{1}{(k^2)^{a_1} (k^2 + 2kp)^{a_2}}$$

# New approach to the QED part

Obstacles:

Very large number of integrals

Reduction to primitive integrals

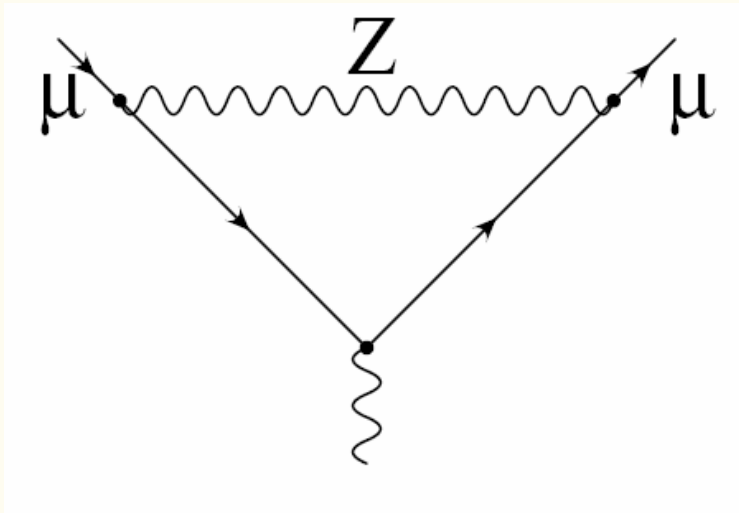
Evaluation of master integrals



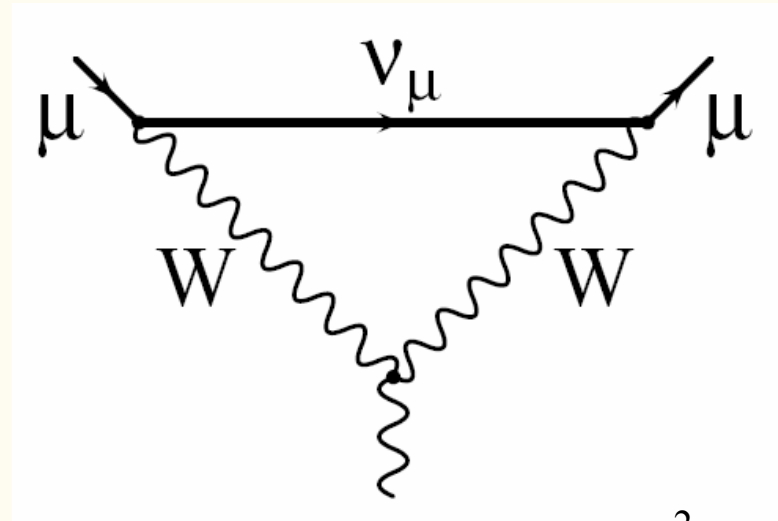
Recent progress: algorithmic reduction (Laporta, 2001)

# Electroweak effects: pure and hadronic

Small part of the total  $g-2$ :  $154(3) \times 10^{-11}$

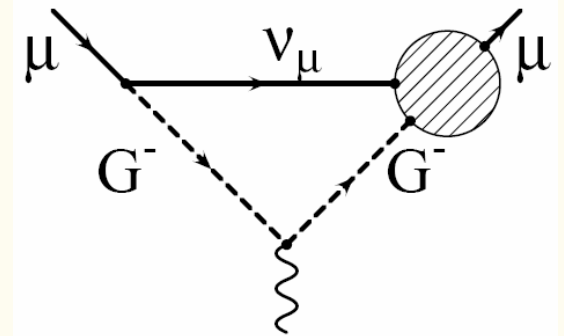
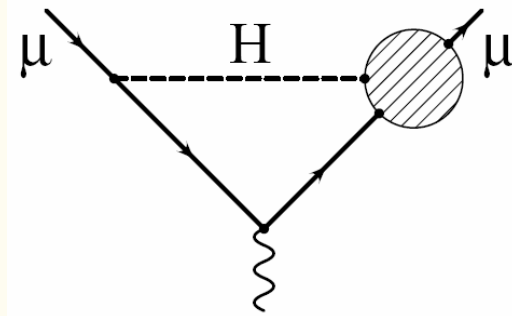
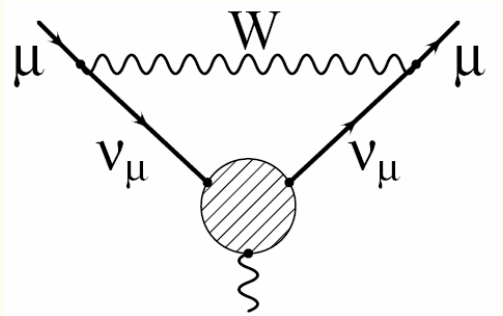
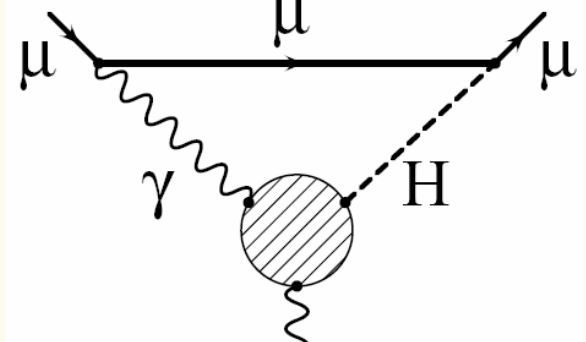
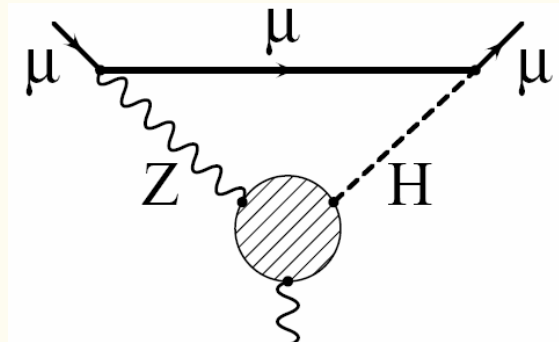
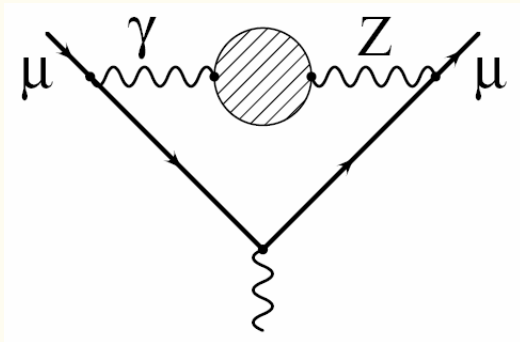


( -1



$$+2) \cdot \frac{5G_{\mu} m_{\mu}^2}{24\sqrt{2}\pi^2} \approx 195 \cdot 10^{-11}$$

# Higher-order electroweak effects



Most important: photonic corrections → large logs

$$\frac{\alpha}{\pi} G_{\mu} m_{\mu}^2 \ln \frac{M_W^2}{m_{\mu}^2} \sim -23\% \text{ of one-loop}$$

Kukhto et al.  
AC, Krause, Marciano  
Heinemaier, Stockinger, Weiglein

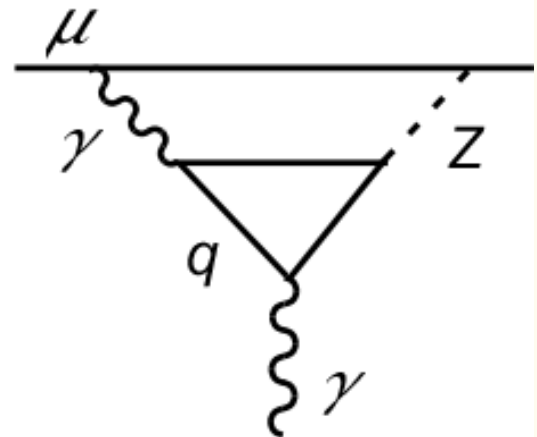
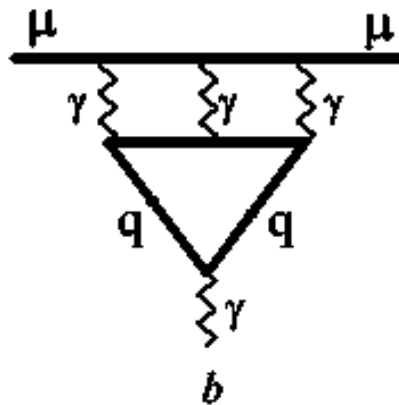
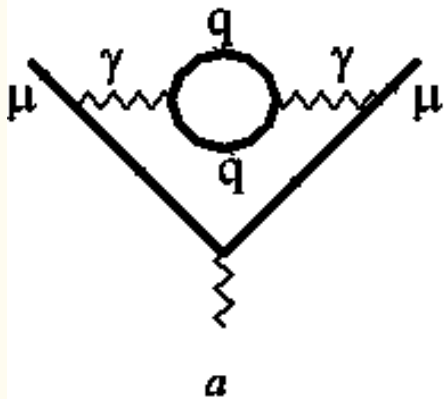
# Muon $g-2$ : hadronic loops

Hadronic effects dominate theoretical uncertainty:

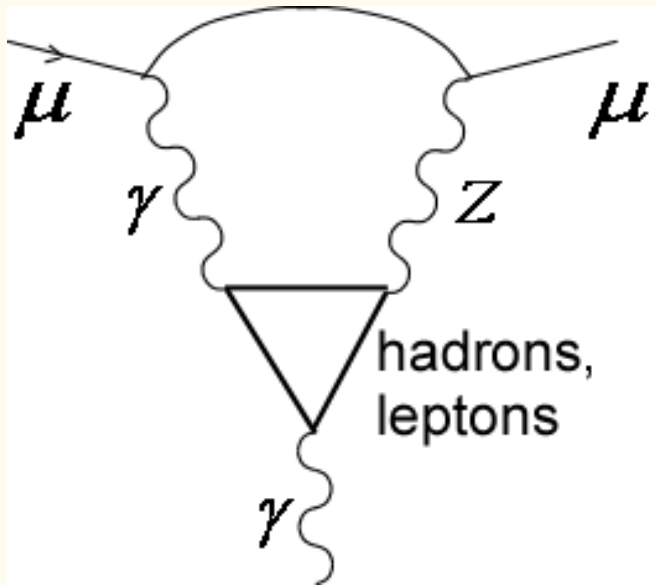
Vacuum polarization

Light-by-light scattering

Electroweak triangle diagrams (numerically small)



# Electroweak-hadronic effects



Large logs  $\ln(m_\mu/M_Z)$  appear in individual fermion contributions;

But cancel in the sum for each generation - like anomalies.

This cancellation between leptons and hadrons was controversial.

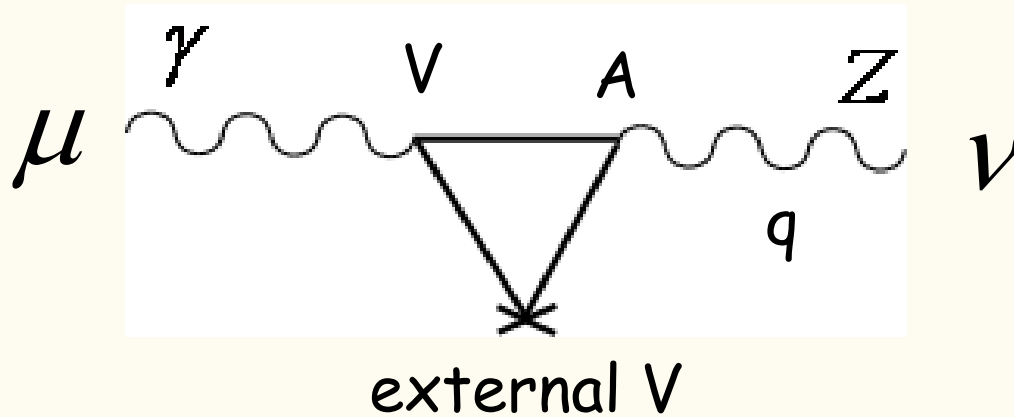
*AC, Marciano, Vainshtein*

*vs.*

*Knecht, Peris, Perrottet, de Rafael*

Useful illustration: similar techniques used in light-by-light

# Structure of the triangle



$$\sim w_T (q^2) \left( -q^2 \tilde{F}_{\mu\nu} + q_\mu q^\sigma \tilde{F}_{\sigma\nu} - q_\nu q^\sigma \tilde{F}_{\sigma\mu} \right) + w_L (q^2) \left( q_\nu q^\sigma \tilde{F}_{\sigma\mu} \right)$$

Perturbative result:

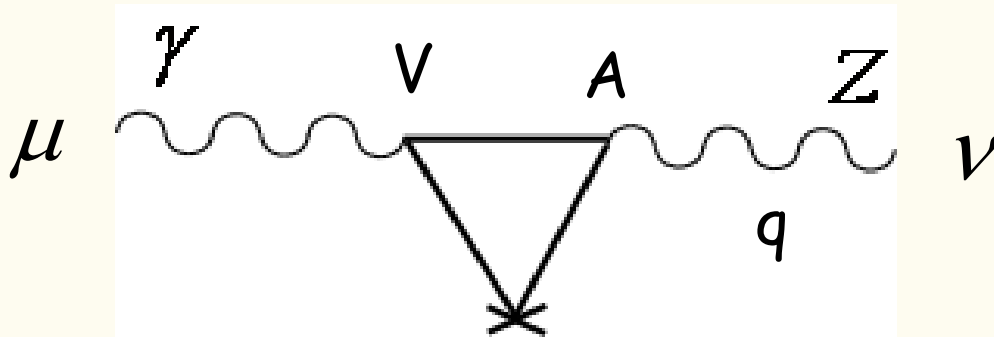
$$w_L = 2w_T \sim \frac{1}{q^2}$$

Anomaly:

$$q_\nu T^{\mu\nu} \sim q^2 w_L \neq 0$$

$$q_\mu T^{\mu\nu} = 0$$

## Vainshtein's non-renormalization theorem for $w_T$



$$T_{\mu\nu} \sim w_T(q^2) \left( -q^2 \tilde{F}_{\mu\nu} + q_\mu q^\sigma \tilde{F}_{\sigma\nu} - q_\nu q^\sigma \tilde{F}_{\sigma\mu} \right) + w_L(q^2) \left( q_\nu q^\sigma \tilde{F}_{\sigma\mu} \right)$$

In the chiral limit,  $w_T$  has no perturbative corrections

Idea of the proof:

$$\text{Im} T_{\mu\nu} \sim q_\mu q^\sigma \tilde{F}_{\sigma\nu} + q_\nu q^\sigma \tilde{F}_{\sigma\mu} \quad (\text{symmetric})$$

$$2w_T(q^2) = w_L(q^2)$$

Guidance from one-loop calculations!

# $w_{T,L}$ in QCD (chiral limit)

Perturbative:  $w_L = 2w_T = \frac{2}{Q^2}$        $Q^2 \equiv -q^2$

Non-perturbative:

Large  $Q^2$

Small  $Q^2$

$$w_L \quad \frac{2}{Q^2}$$

$$\frac{2}{Q^2} \quad (\text{pion pole})$$

$$w_T \quad \frac{1}{Q^2} - \frac{(0.7 \text{ GeV})^4}{Q^6} + O\left(\frac{1}{Q^8}\right)$$

$$\frac{1}{m_{a_1}^2 - m_\rho^2} \left( \frac{m_{a_1}^2 - m_\pi^2}{Q^2 + m_\rho^2} - \frac{m_\rho^2 - m_\pi^2}{Q^2 + m_{a_1}^2} \right)$$

(model for  $w_T$ )

# Contributions to $g-2$

$$\Delta a_\mu \sim \left(\frac{\alpha}{\pi}\right)^2 \frac{m_\mu^2}{M_Z^2} \int_{m_\mu^2}^{\infty} dQ^2 \left( w_L + \frac{M_Z^2}{M_Z^2 + Q^2} w_T \right)$$

Asymptotics:

$$w_{T,L} \xrightarrow{Q^2 \rightarrow \infty} \frac{1}{Q^2} \sum_f I_{3f} N_f Q_f^2$$

$$\int_{m_\mu^2}^{\infty} dQ^2 w_L : \text{diverges}$$

theory inconsistent  
unless anomalies cancel

$$\int_{m_\mu^2}^{\infty} dQ^2 \frac{M_Z^2}{M_Z^2 + Q^2} w_T \sim \ln M_Z^2$$

# "Pure" hadronic contributions

## Recent progress

### Updated studies of $g-2$ using $e^+e^-$ data

Davier, Eidelman, Höcker, Zhang  
Hagiwara, Martin, Nomura, Teubner

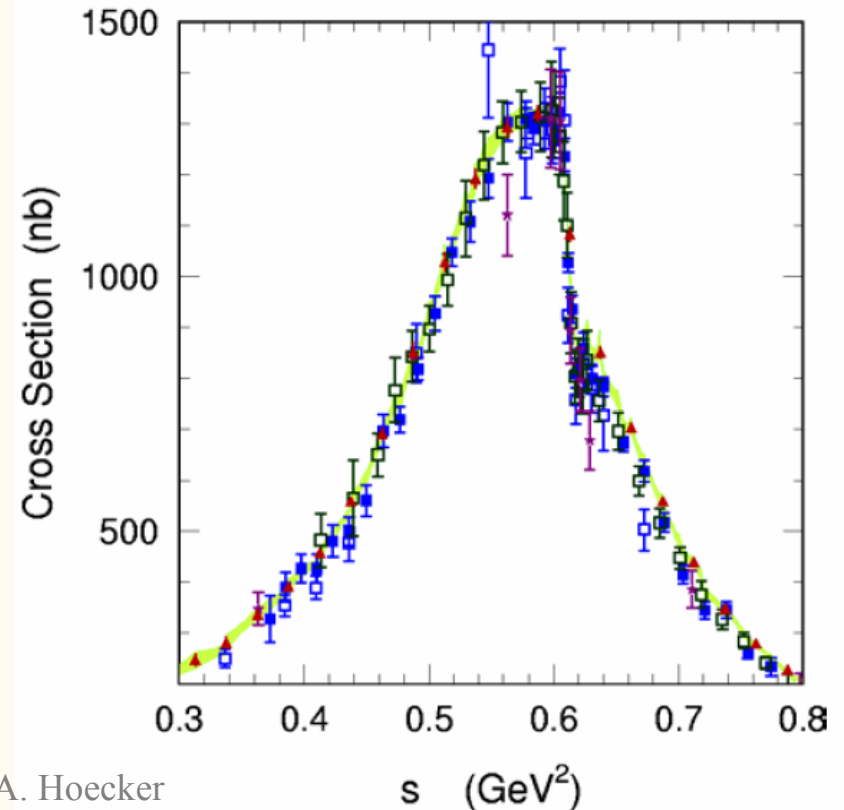
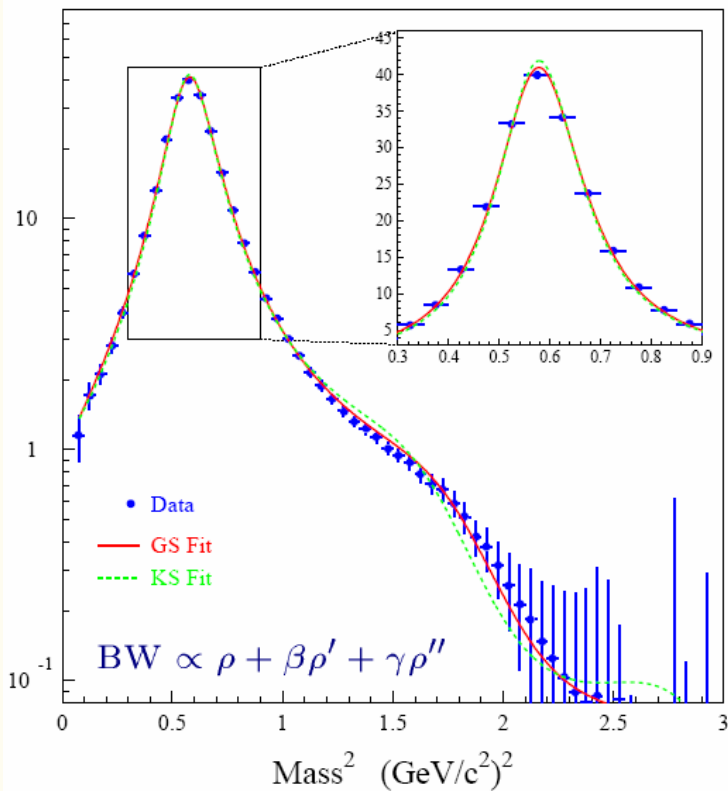
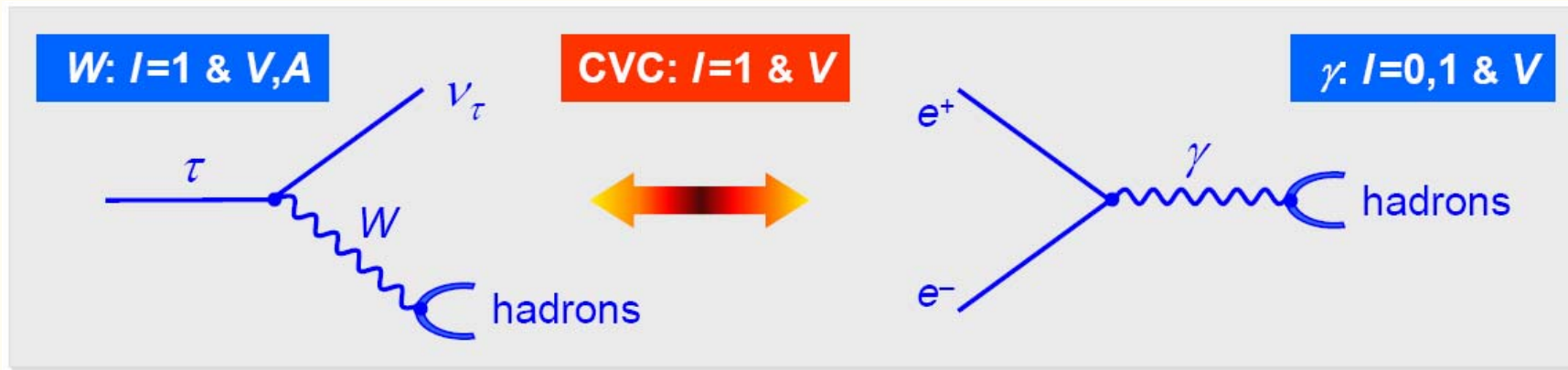
### Novosibirsk results tested by Daphne

See talks by Kühn, Leone, Shwartz

### Shift of the light-by-light prediction

Melnikov and Vainshtein

# Vacuum polarization: $\tau$ decays vs. $e^+e^-$

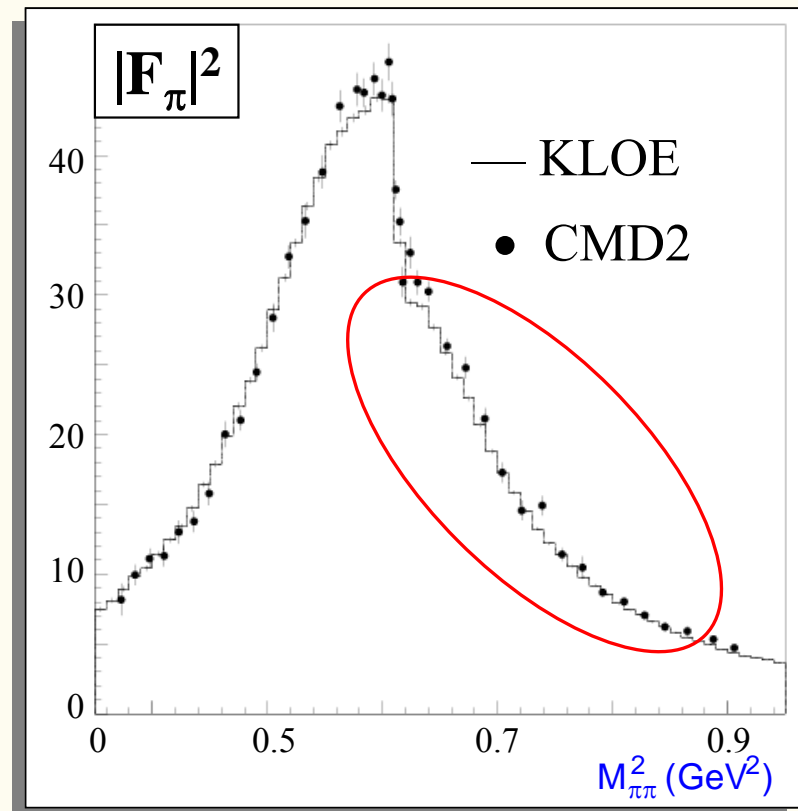


From M. Davier, A. Hoecker

# Vacuum polarization: $e^+e^-$

$e^+e^-$  data have greatly improved

New results from KLOE confirm Novosibirsk CMD2



# Hadronic contributions: outstanding problems

How to reconcile  $e^+e^-$  and  $\tau$  data?

Can we improve the light-by-light prediction?

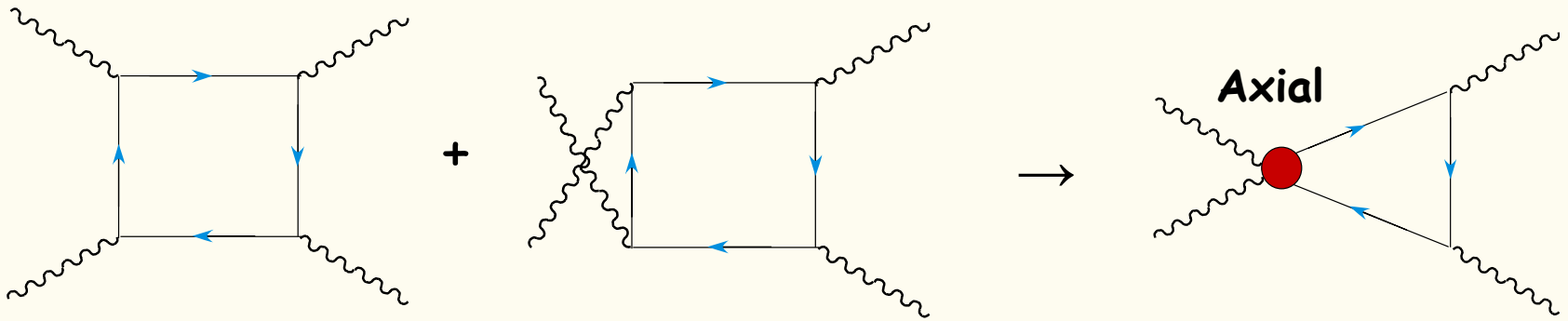
# Light-by-light scattering

Recent evaluations:

Knecht, Nyffeler	80(40)
Hayakawa, Kinoshita	90(15)
Bijnens, Pallante, Prades	83(32)
Melnikov, Vainshtein	136(25)

# Effects enhanced by $N_c$

Quark box: pQCD asymptotics



The same structure as in the EW-hadronic loops.

Dominant contribution:  $\pi^0$  pole

Crucial observation:  $1/Q^2$  asymptotics  $\rightarrow$  no formfactor  
in  $\pi^* \gamma^* \gamma$  if one of the photons soft.

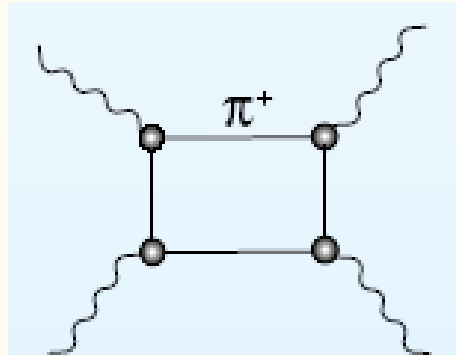
$$\Delta a_{\mu}^{\text{PS}} \simeq (76.5 + 2 \cdot 18) \cdot 10^{-11}$$

$$\Delta a_{\mu}^{\text{PV}} \simeq 22 \cdot 10^{-11}$$

# What about terms subleading in $N_c$ ?

Example: pion box.

It is chirally enhanced,  $m_\mu^2 / m_\pi^2$



Numerical effect: small.

Previous estimates:

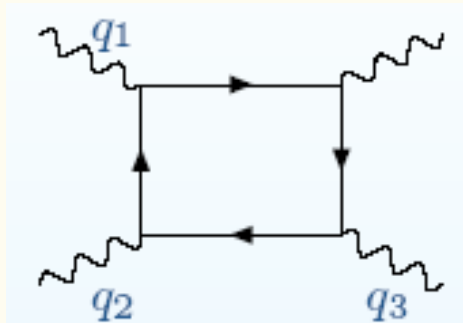
$-4.5(8.5) \times 10^{-11}$	HLS	Hayakawa, Kinoshita, Sanda
$-19(5) \times 10^{-11}$	VMD	Bijnens, Pallante, Prades

Melnikov & Vainshtein:  $0_{\pm 10} \times 10^{-11}$

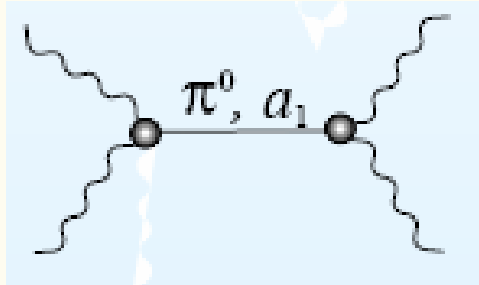
(cancellations with higher orders in the chiral expansion)

# Summary on the light-by-light scattering

From Melnikov and Vainshtein



Matching of hadronic model with perturbative QCD, at asymptotic momentum transfer.  
Large contribution of high virtualities.



Dominant in  $N_c \rightarrow \infty$ : pion pole

Still room for improvement: subleading terms

Pomeranchuk and Sakharov on  $g-2 = \frac{\alpha}{\pi}$

If this is true, it's exceptionally important;  
if it isn't true, that, too, is exceptionally important.  
(Pomeranchuk after Sakharov's talk, 1949)



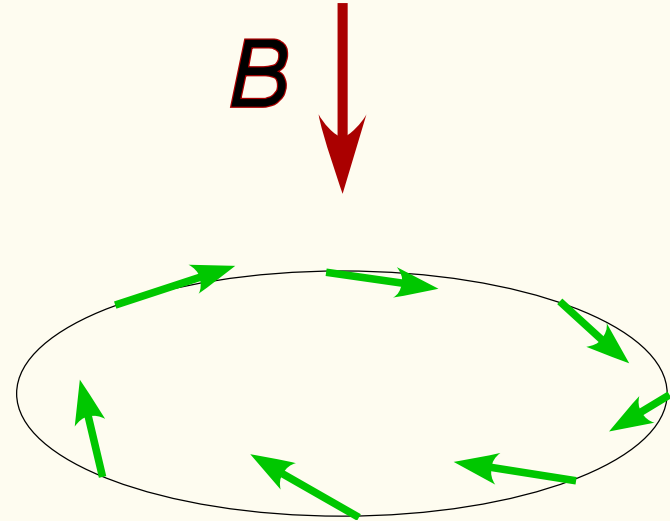
I felt like the messenger of the gods. (Sakharov)

# How do we determine $g-2$ ?

Measure  $\omega_a = \frac{g-2}{2} \frac{e}{m_\mu} B$

$B$  from NMR:  $\omega_p = \frac{2\mu_p B}{\hbar}$

$\frac{e}{m_\mu}$  from  $\mu_\mu \equiv g \frac{e\hbar}{4m_\mu}$



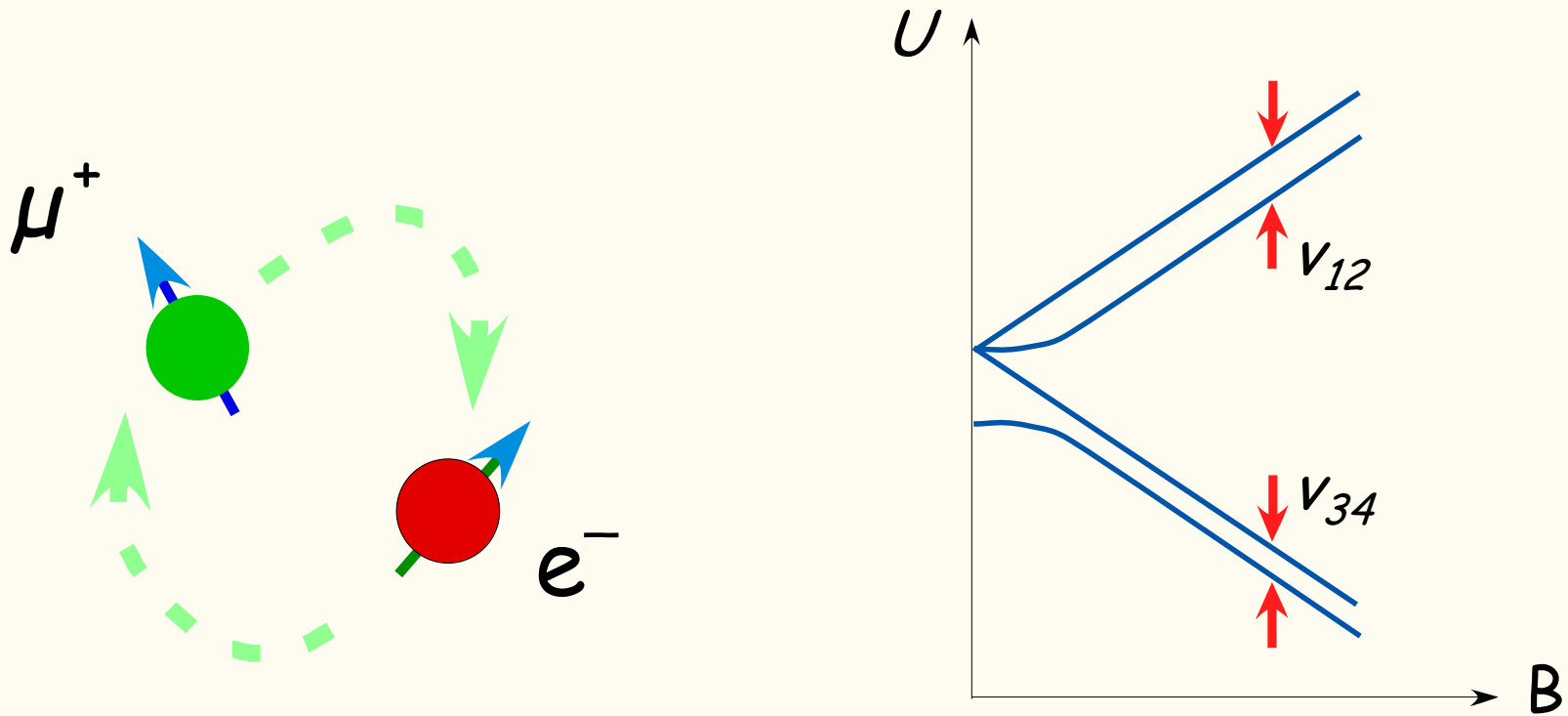
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Master formula:  $\frac{g-2}{2} = \frac{\omega_a / \omega_p}{\mu_\mu / \mu_p - \omega_a / \omega_p}$

Measured by E821

From muonium

# Muonium spectrum determines $\mu_\mu/\mu_p$



$$v_{34} - v_{12} \sim \mu_\mu B \quad \Rightarrow \quad \mu_\mu / \mu_p$$

Measured to relative  $1.2 \cdot 10^{-7}$  (like  $15 \cdot 10^{-11}$  in  $a_\mu$ )

Will need improvement for the "next g-2"

Mu: also  $m_\mu/m_e$  and tests of QED