Lattice B-Physics

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Introduction

Problem of heavy quark on the lattice

- $a^{-1} = 1 \sim 4$ GeV in currents simulations
- $m_b \simeq 4$ GeV larger than a^{-1} (cut-off)

Current methods for heavy quark

1. Relativistic action at $m_c \oplus$ Static limit ($m_Q = \infty$) $\rightarrow m_b$ (Extrapolation)
2. Non-Relativistic QCD(NRQCD) at m_b, effective theory $\rightarrow$ no continuum limit
3. FNAL method at m_b relativistic action $\oplus$ non-relativistic interpretation

Main systematic errors

finite a, renormalization, quenched approximation

Menu

1. b quark mass (m_b)
2. Leptonic decay constant(f_B)
3. B parameters
4. Form factors of semi-leptonic B decays

Recent reviews:

C.T. Sachrajda, hep-lat/0101003
V. Lubicz, hep-lat/0012003
C. Bernard, hep-lat/0011064
A.S. Kronfeld, hep-lat/0010074
R.D. Kenway, hep-ph/0010219
S. Hashimoto, hep-lat/9909136
S. Aoki, hep-ph/9912288

b quark mass

- a fundamental parameter, important for inclusive decay rates
- renormalon ambiguity in pole mass
  $\rightarrow \overline{\mathrm{MS}}$ quark mass $\overline{m}_b$

Lattice(Static) $\rightarrow$ pole mass

$$m_b^{\mathrm{pole}} = M_B - \xi + \delta m = M_B \quad \underbrace{-\xi + \frac{1}{a}\sum_n C_n \alpha_s(\overline{m}_b)^n}_{\text{power divergence canceled}}$$

M_B: B meson mass from experiment
$\xi \sim 1/a$: binding energy measured on the lattice
δm: 2-loop(Martinelli-Sachrajda), 3-loop(Parma-Milan)

pole mass $\rightarrow \overline{\mathrm{MS}}$ quark mass

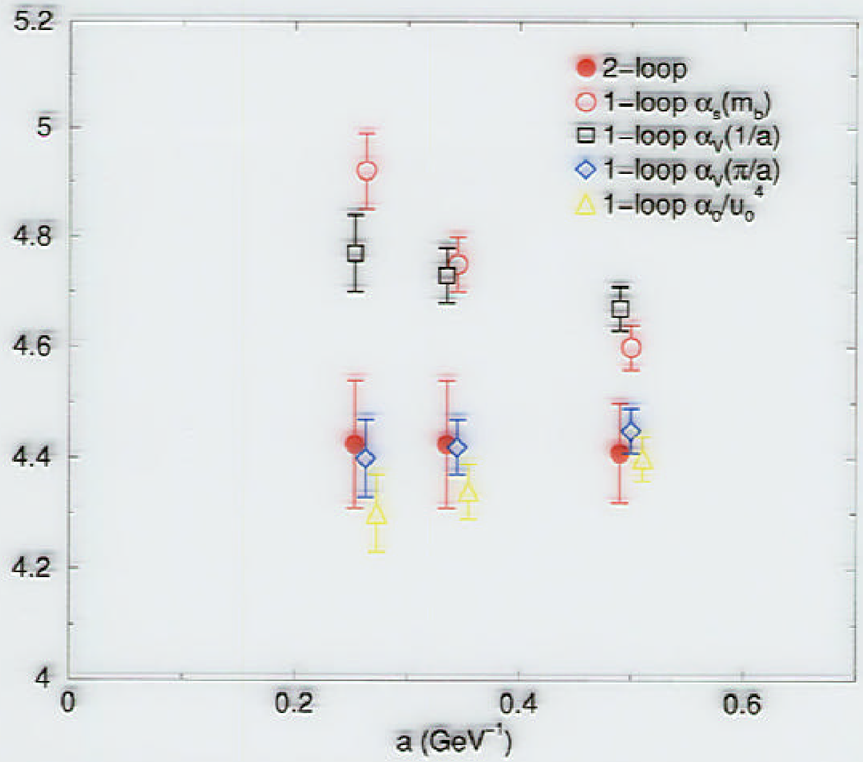
$$\overline{m}_b(\overline{m}_b) = Z(\overline{m}_b) m_b^{\mathrm{pole}} = \quad \underbrace{\left[1 + \sum_n Z_n \alpha_s(\overline{m}_b)^n\right] \times m_b^{\mathrm{pole}}}_{\text{renormalon ambiguity canceled}}$$

Z: 2-loop(Gray-Broadhurst-Grafe-Schilcher), 3-loop(Melnikov-Ritbergen)

Remark

- cancellations in $\overline{m}_b$ are incomplete at finite order
- higher in perturbation theory is better for the result
- the problem may exist in other quantities

$\overline{m}_b(\overline{m}_b)$ Martinelli-Sachrajda



- results with δm at 1-loop depend on the choice of α_s
- result with δm at 2-loop shows no a dependence

Present best lattice estimate

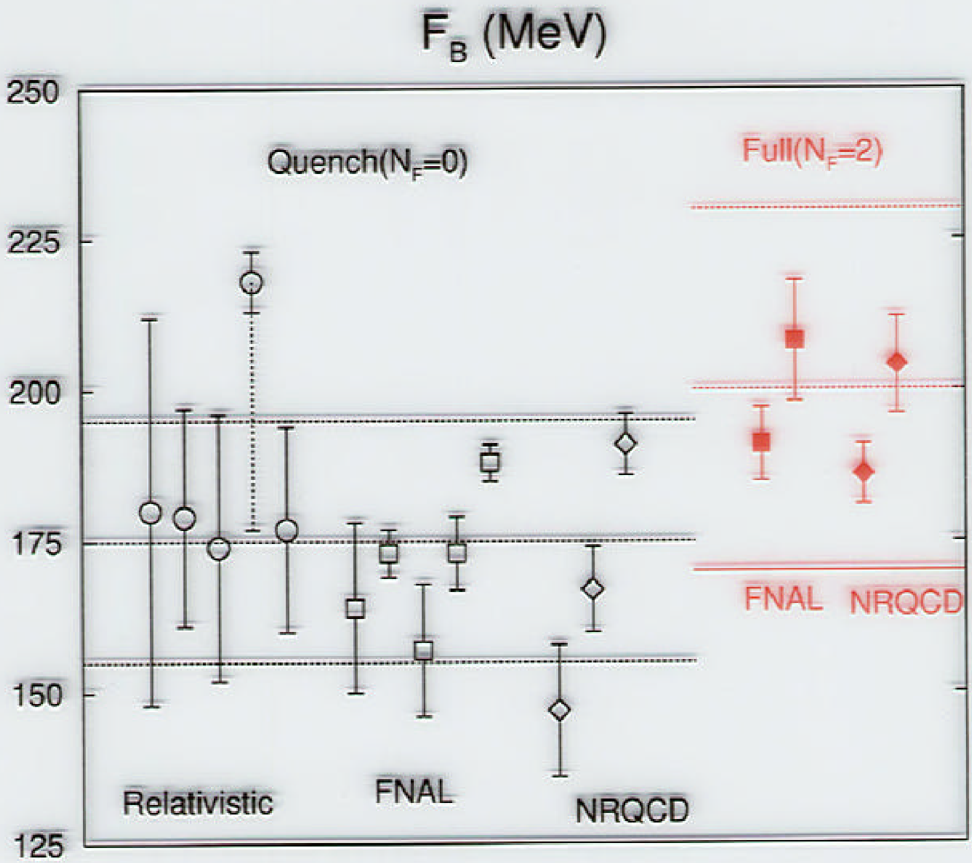
$\overline{m}_b = 4.30(0.05)(0.05)$ GeV ($N_f = 0$, 3-loop)
$\overline{m}_b = 4.26(0.06)(0.07)$ GeV ($N_f = 2$, 2-loop)

Other determination

$\overline{m}_b = 4.20(0.06)$ GeV (Hoang, mass and width of γ mesons)
$\overline{m}_b = 4.25(0.08)$ GeV (Beneke-Singer, $b\bar{b}$ production)
$\overline{m}_b = 4.20(0.10)$ GeV (Melinkov-Yelkhovski, γ sum rule)
$\overline{m}_b = 3.91(0.67)$ GeV (DELPHI, 3 jets at m_Z)

Leptonic decay constant F_B

- important for CKM matrix, most extensively investigated
- results from different methods are consistent in quenched QCD
- $\sim$ 10% increase of F_B in full QCD



APE97/99/00,UKQCD00,Lellouch-Lin00, FNAL97,JLQCD98,MILC98/00,CPPACS00, AliKhan98,JLQCD99,CPPACS00, MILC00,CPPACS00,Collins99,CPPACS00

Summary of lattice results

	$N_f = 0$	$N_f = 2$	Expt.
F_B F_{B_s}/F_B	175(20) MeV 1.15(4)	200(30) MeV 1.16(4)	
F_{D_s}	220(30)MeV	255(30)MeV	280(19)(28)(34)

Present best estimate

$$F_B = 200\ (30)\ \text{MeV}\ (\ N_f = 2\ )$$

See poster by N. Yamada on 22th for details.

B_0-$\bar{B}_0$ mixing parameter B_B

poster by N. Yamada on 22th

$$\langle \bar{B}_q | (\bar{b}q)_{V-A}(\bar{b}q)_{V-A} | B_q \rangle = \frac{8}{3} B_{B_q}(\mu) F_{B_q}^2 m_{B_q}^2$$

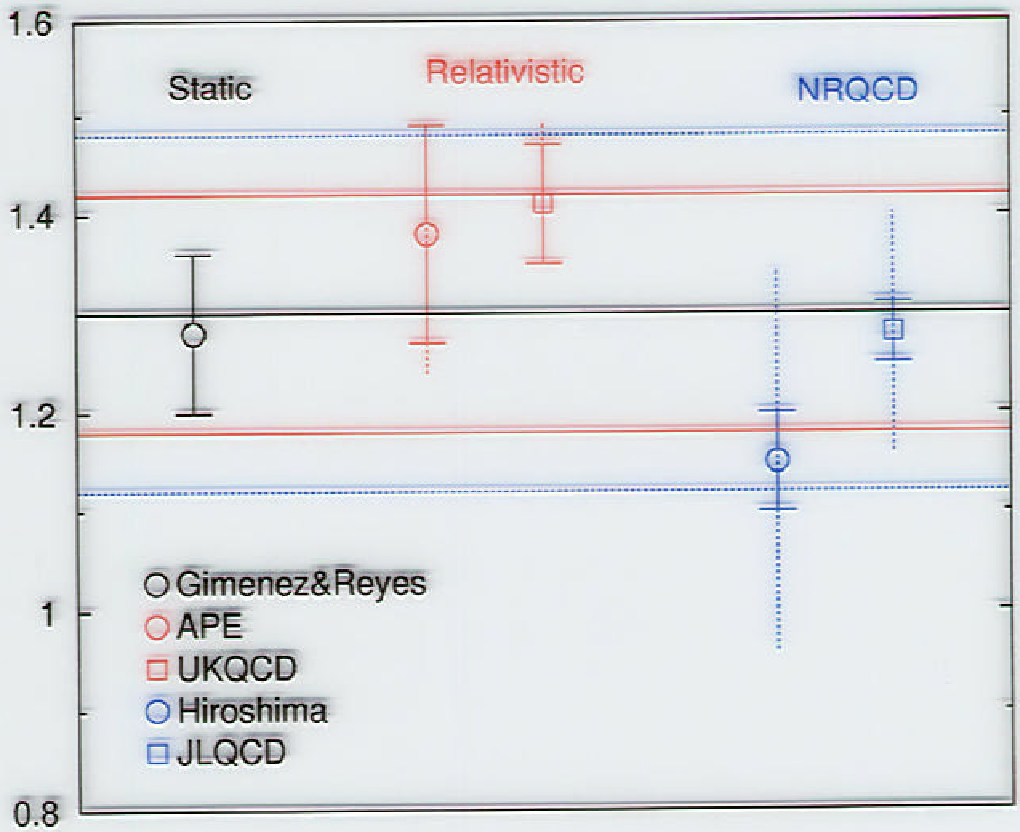
where $q = d,\ s$.

$$\Delta M_q = \frac{G_F^2 M_W^2}{6\pi^2} \eta_{B_q} S(m_t/M_W) F_{B_q}^2 \hat{B}_{B_q} |V_{tq}|^2$$

RG invariant B parameter at NLO

$$\hat{B}_{\bar{B}_q} = \alpha_s(\mu)^{2/\beta_0} \left[1 + \frac{\alpha_s(\mu)}{4\pi} J_{N_f} \right] B_{\bar{B}_q}(\mu)$$

Quenched $\hat{B}_{B_d}$



Present estimate in quenched QCD

$$\hat{B}_{B_d} = 1.30(12)(13)$$

$$F_{B_d}\sqrt{\hat{B}_{B_d}} = 230(40)\ \mathrm{MeV}$$

$$\frac{\hat{B}_{B_s}}{\hat{B}_{B_d}} = 1.00(4)$$

$$\xi \equiv \frac{F_{B_s}\sqrt{\hat{B}_{B_s}}}{F_{B_d}\sqrt{\hat{B}_{B_d}}} = 1.16(5)$$

B_S

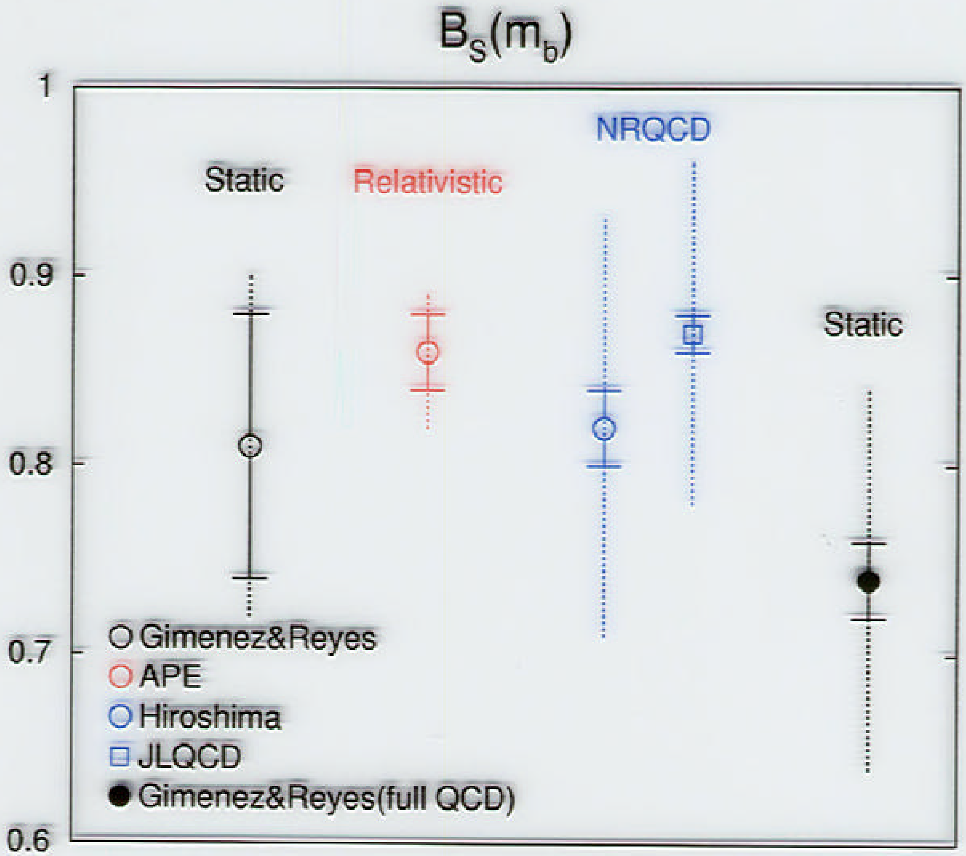
talk by S. Hashimoto on 22th

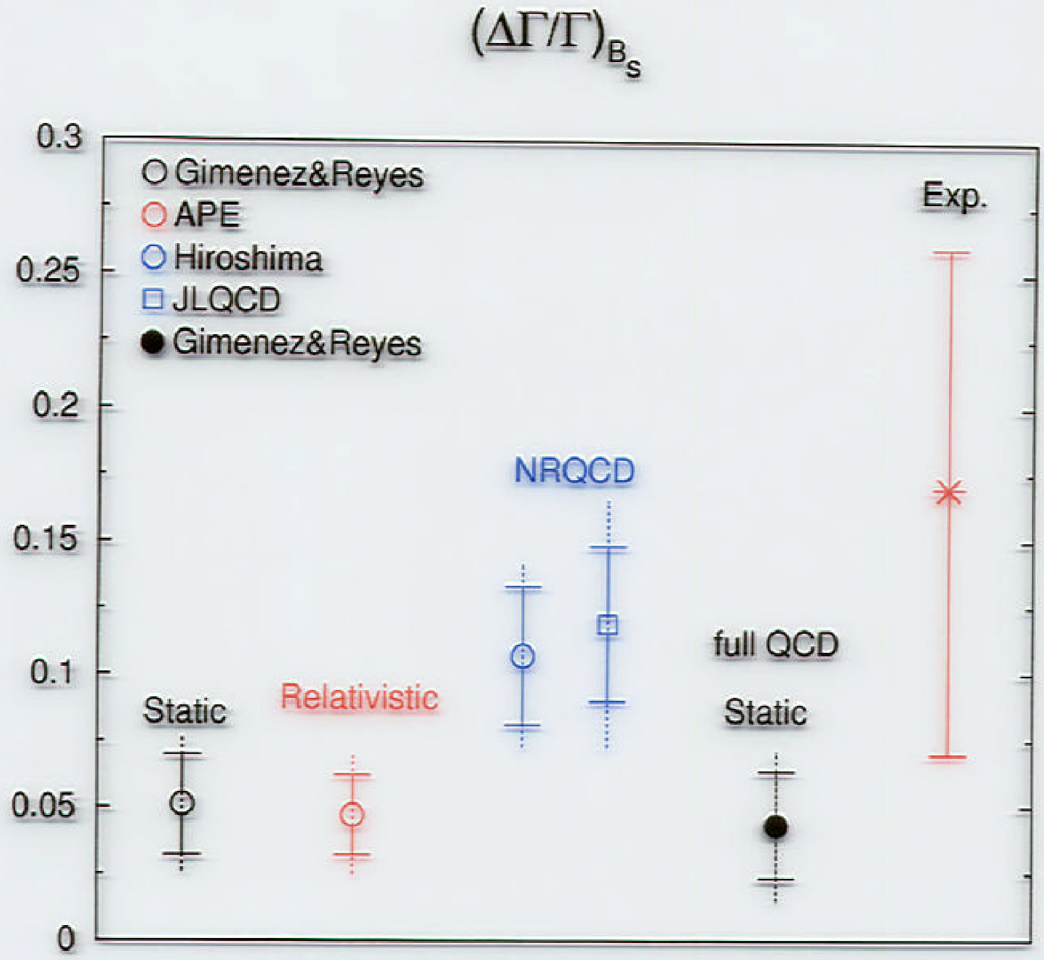
$$B_{S_q}(\mu) = \frac{\langle \bar{B}_q | O_S(\mu) | B_q \rangle}{\frac{5}{3}\langle \bar{B}_q | P(\mu) | 0 \rangle \langle 0 | P(\mu) | B_q \rangle}$$

$$O_S(\mu) = P(\mu) \cdot P(\mu), \quad P(\mu) = \bar{b}(1-\gamma_5)q$$

Width difference of B_s–$\bar{B}_s$

$$\left(\frac{\Delta\Gamma}{\Gamma}\right) = \left(\frac{F_{B_s}}{245\mathrm{MeV}}\right)^2 [0.008 B_{B_s}(m_b) + 0.099 B_{S_s}(m_b) - 0.086]$$



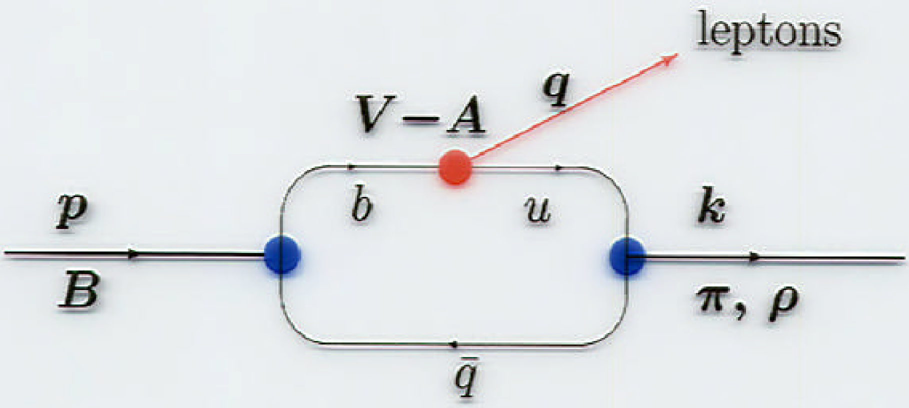


Remark

difference between Static/relativistic and NRQCD is mainly caused by the difference in experimental inputs and formula for $\Delta\Gamma/\Gamma$

Form factors of semi-leptonic decays

rare decays $\Rightarrow |V_{ub}|$



$B \longrightarrow \rho\ell\nu$

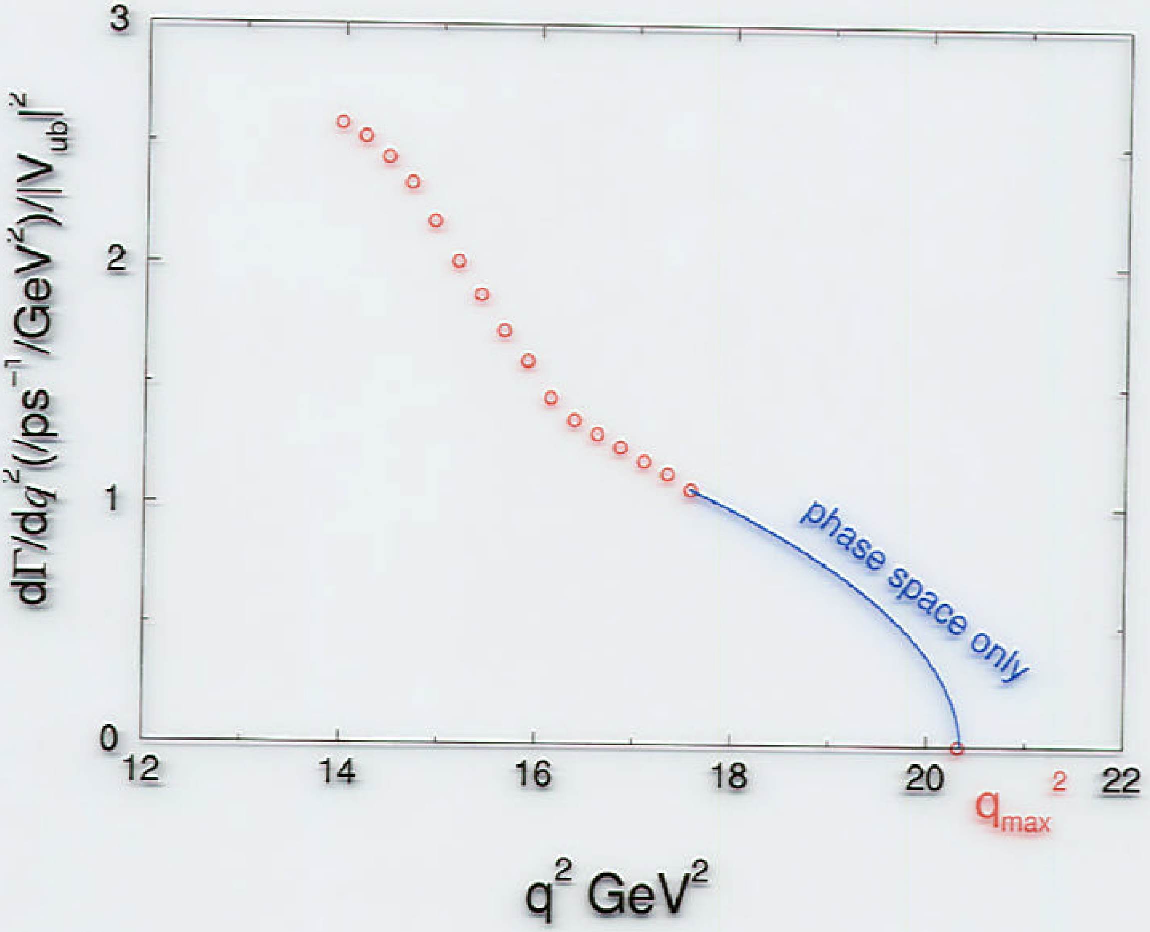
$$\langle \rho(k,\varepsilon)|V^\mu(q)|B(p)\rangle = \frac{2V(q^2)}{m_H+m_V}\epsilon^{\mu\nu\alpha\beta}p_\nu k_\alpha \varepsilon^*_\beta$$

$$\langle \rho(k,\varepsilon)|A^\mu(q)|B(p)\rangle = i(m_H+m_V)A_1(q^2)\,\varepsilon^{*\mu} - i\frac{A_2(q^2)}{m_H+m_V}\varepsilon^*\cdot p\,(p+k)^\mu + i\frac{A(q^2)}{q^2}2m_V\varepsilon^*\cdot p\, q^\mu$$

differential decay rate

$$\frac{d\Gamma}{dq^2}(B\to\rho\ell\nu) = 10^{-12}\frac{G_F^2|V_{ub}|^2}{192\pi^3 m_B^3}c^2 q^2(1+b(q^2-q^2_{\max})) \times \underbrace{\sqrt{(m_B^2+m_\rho^2-q^2)^2-4m_B^2m_\rho^2}}_{\text{phase space factor}}$$

Flynn-Lesk (UKQCD), Quenched QCD



$$\frac{\Delta\Gamma(14 < q^2/\mathrm{GeV}^2 < 20.3)}{\mathrm{ps}^{-1}\mathrm{GeV}^{-2}} = 8.3|V_{ub}|^2$$ UKQCD preliminary

$$= 7.1(2.4) \times 10^{-5}$$ CLEO

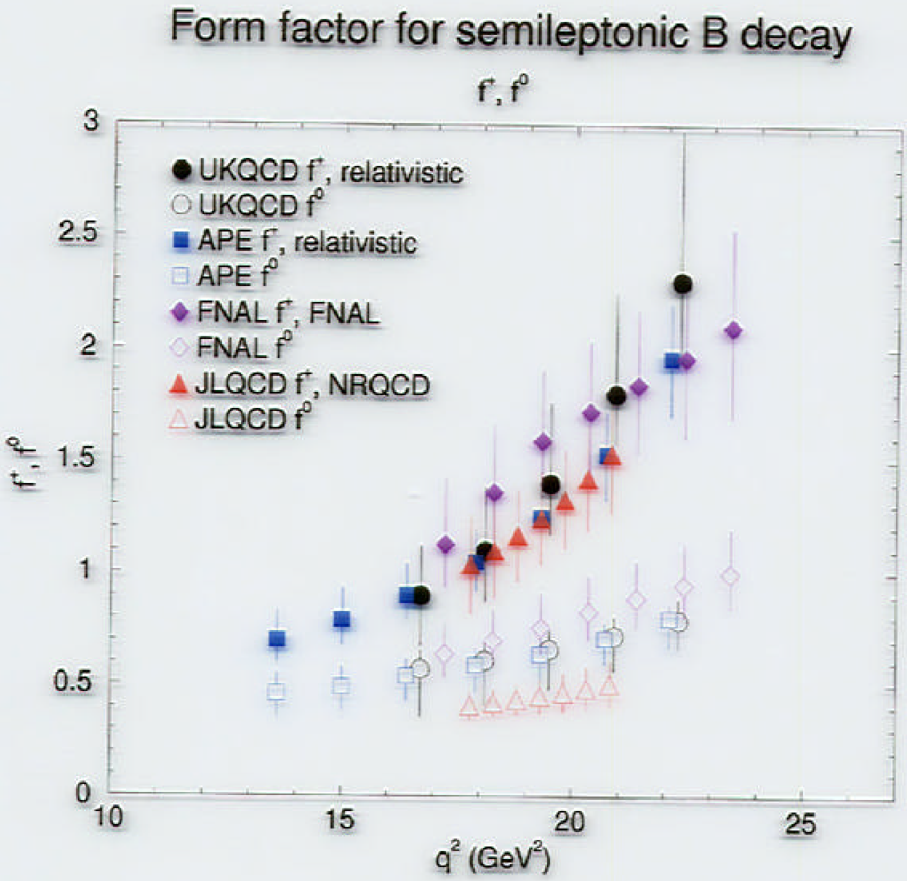
$\Longrightarrow V_{ub} = 2.9(0.5) \times 10^{-3}$ (quenched QCD)

$$B \to \pi \ell \nu$$

$$\langle \pi(k)|\bar{u}\gamma_\mu b|B(p)\rangle = f^+(q^2)\left[(p+k)_\mu - \frac{m_B^2 - m_\pi^2}{q^2}q_\mu\right] + f^0(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q_\mu$$

$f^0(q^2)$ is negligible in the decay rate ($q_\mu L_\mu \to m_l$)

$f^+(q^2)$, $f^0(q^2)$(quenched QCD)



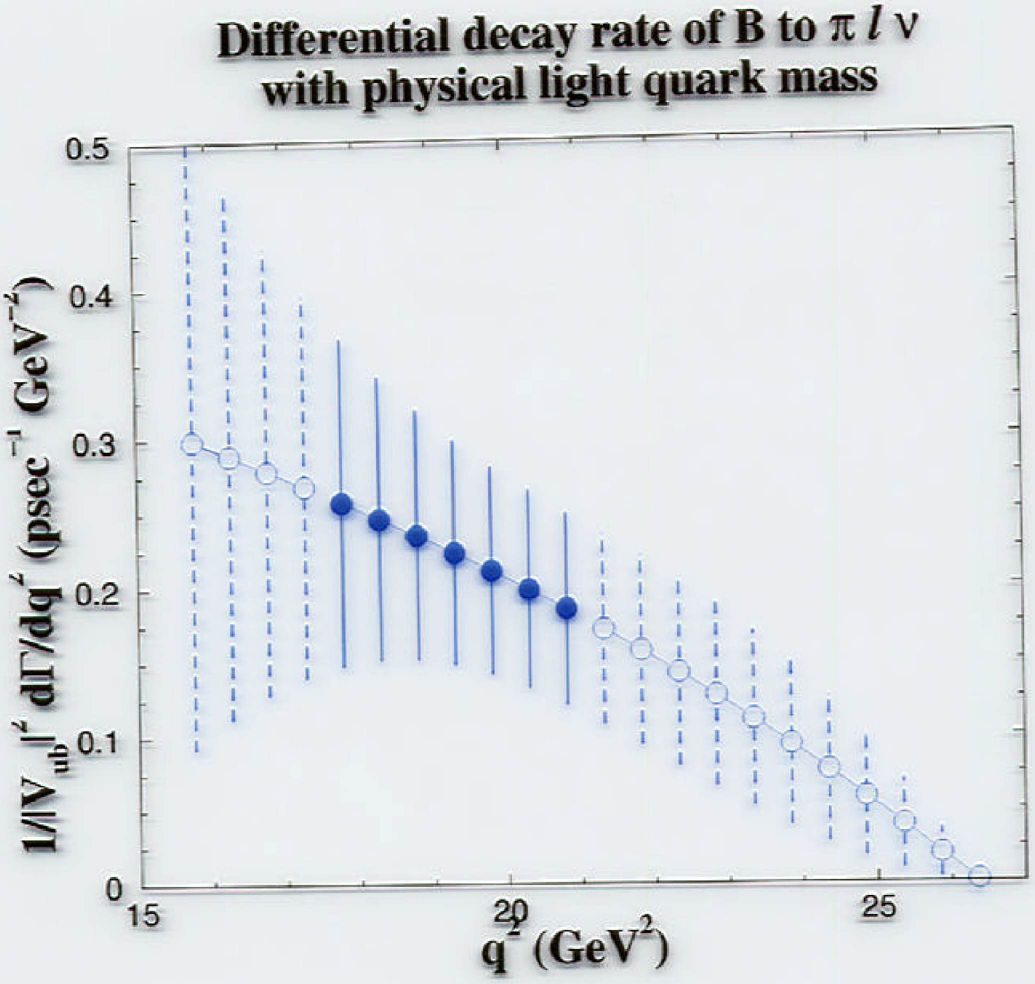
- all groups agree in $f^+(q^2)$
- extrapolation to smaller q^2 is important
- small discrepancy in $f^0(q^2)$ should be understood

  ⇒ Posters by T. Onogi on 19th and by A.S. Kronfeld on22th.

differential decay rate

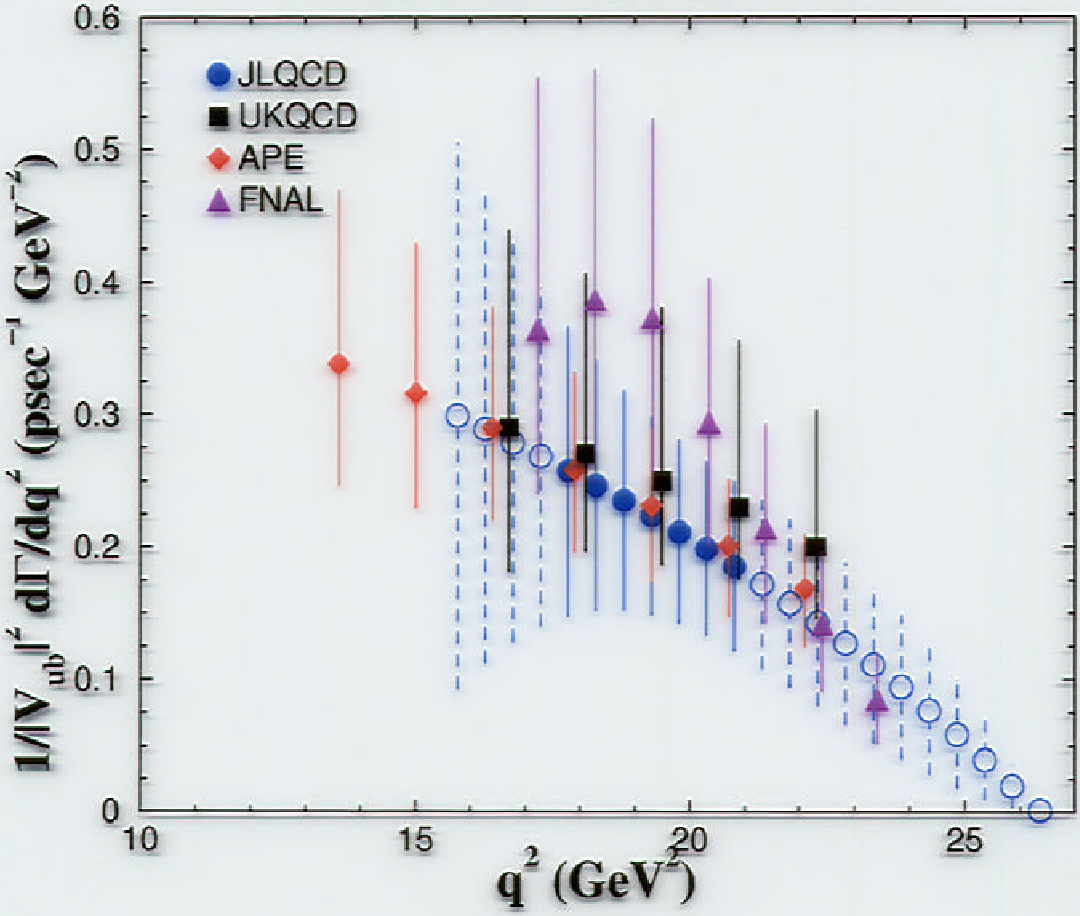
$$\frac{d\Gamma}{dq^2}(B \to \pi \ell \nu) = \frac{G_F^2 |\vec{k}_\pi|^3}{24\pi^2} |V_{ub}|^2 |f^+(q^2)|^2$$

Onogi (JLQCD), quenched QCD



- data at $q^2 = 18 - 22$ GeV^2, obtained by interpolation, are reliable
- differential decay rate is needed but experimentally difficult

Differential decay rate of B to $\pi\, l\, \nu$ with physical light quark mass



- all results on the differential decay rate are consistent within errors
- important to reduce errors

Conclusion

- Lattice is an important players in B-physics, in collaboration with other theoretical methods
- m_b, F_B, B_B, B_S, form factors of $B \to \rho/\pi$, and more
- Lists of some other quantities

quantities	status	N_f	results
$\frac{\tau(B^-)}{\tau(B^0)}$	static	0	1.03(2)(3)
$\frac{\tau(\Lambda_b)}{\tau(B^0)}$	static	0	0.91(1)(1)
$\bar{\Lambda}$	from world data	0	0.68(+2)(−12) GeV
λ_1	from world data	0	-0.45(12) GeV^2
M_{B_c}	NRQCD	0	6.386(9)(98)(15)GeV
	NRQCD	0	6.28(20)
H($\bar{b}bg$)	NRQCD	0	Δm=1.542(8) GeV
H($\bar{c}cg$)	NRQCD	0	Δm=1.323(13)

- impact on determination of CKM matrix

  $\Rightarrow$ Talk by A. Soni on 21th.