Lattice B-Physics

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BCP4 @ Ise, Japan
2001.2.19
Introduction

Problem of heavy quark on the lattice

- $a^{-1} = 1 \sim 4$ GeV in currents simulations
- $m_b \approx 4$ GeV larger than $a^{-1}$ (cut-off)

Current methods for heavy quark

1. Relativistic action at $m_c \oplus$ Static limit ($m_Q = \infty$) \(\rightarrow m_b\) (Extrapolation)

2. Non-Relativistic QCD(NRQCD) at $m_b$, effective theory \(\rightarrow\) no continuum limit

3. FNAL method at $m_b$
   relativistic action $\oplus$ non-relativistic interpretation

Main systematic errors

finite $a$, renormalization, quenched approximation
Menu

1. $b$ quark mass ($m_b$)

2. Leptonic decay constant ($f_B$)

3. $B$ parameters

4. Form factors of semi-leptonic $B$ decays

Recent reviews:

C.T. Sachrajda, hep-lat/0101003
V. Lubicz, hep-lat/0012003
C. Bernard, hep-lat/0011064
A.S. Kronfeld, hep-lat/0010074
R.D. Kenway, hep-ph/0010219
S. Hashimoto, hep-lat/9909136
S. Aoki, hep-ph/9912288
b quark mass

- a fundamental parameter, important for inclusive decay rates
- renormalon ambiguity in pole mass
  \[ \overline{\text{MS}} \text{ quark mass } m_b \]

Lattice (Static) \( \rightarrow \) pole mass

\[
m_b^{\text{pole}} = M_B - \xi + \delta m = M_B - \xi + \frac{1}{\alpha_s} \sum_n C_n \alpha_s (m_b)^n
\]

Power divergence canceled

\( M_B \): B meson mass from experiment
\( \xi \sim 1/\alpha \): binding energy measured on the lattice
\( \delta m \): 2-loop (Martinelli-Sachrajda), 3-loop (Parma-Milan)

pole mass \( \rightarrow \) \( \overline{\text{MS}} \) quark mass

\[
\overline{m}_b(m_b) = Z(m_b) m_b^{\text{pole}} = \left[ 1 + \sum_n Z_n \alpha_s (m_b)^n \right] \times m_b^{\text{pole}}
\]

Renormalon ambiguity canceled

\( Z \): 2-loop (Gray-Broadhurst-Grafe-Schilcher), 3-loop (Melnikov-Ritbergen)

Remark

- cancellations in \( \overline{m}_b \) are incomplete at finite order
- higher in perturbation theory is better for the result
- the problem may exist in other quantities
results with $\delta m$ at 1-loop depend on the choice of $\alpha_s$
result with $\delta m$ at 2-loop shows no $a$ dependence

Present best lattice estimate

$\overline{m}_b = 4.30(0.05)(0.05)$ GeV ($N_f = 0$, 3-loop)
$\overline{m}_b = 4.26(0.06)(0.07)$ GeV ($N_f = 2$, 2-loop)

Other determination

$\overline{m}_b = 4.20(0.06)$ GeV (Hoang, mass and width of $\gamma$ mesons)
$\overline{m}_b = 4.25(0.08)$ GeV (Beneke-Singer, $b\bar{b}$ production)
$\overline{m}_b = 4.20(0.10)$ GeV (Melinkov-Yelkhovskii, $\gamma$ sum rule)
$\overline{m}_b = 3.91(0.67)$ GeV (DELPHI, 3 jets at $m_Z$)
Leptonic decay constant $F_B$

- important for CKM matrix, most extensively investigated
- results from different methods are consistent in quenched QCD
- $\sim 10\%$ increase of $F_B$ in full QCD

$F_B$ (MeV)

APE97/99/00, UKQCD00, Lellouch-Lin00, FNAL97, JLQCD98, MILC98/00, CPPACS00, AliKhan98, JLQCD99, CPPACS00, MILC00, CPPACS00, Collins99, CPPACS00
Summary of lattice results

<table>
<thead>
<tr>
<th></th>
<th>$N_f = 0$</th>
<th>$N_f = 2$</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_B$</td>
<td>175(20) MeV</td>
<td>200(30) MeV</td>
<td></td>
</tr>
<tr>
<td>$F_{B_s}/F_B$</td>
<td>1.15(4)</td>
<td>1.16(4)</td>
<td></td>
</tr>
</tbody>
</table>

Present best estimate

$$F_B = 200 \ (30) \ MeV \ (\ N_f = 2 )$$

See poster by N. Yamada on 22th for details.
$B_0 - \bar{B}_0$ mixing parameter $B_B$

poster by N. Yamada on 22th

\[ \langle \bar{B}_q | (\bar{b} q)_{V-A} (b q)_{V-A} | B_q \rangle = \frac{8}{3} B_{Bq}(\mu) F_{Bq}^2 m_{Bq}^2 \]

where $q = d, s$.

\[ \Delta M_q = \frac{G_F^2 M_W^2}{6\pi^2} \eta_{Bq} S(m_t/M_W) F_{Bq}^2 \hat{B}_{Bq} |V_{iq}|^2 \]

RG invariant $B$ parameter at NLO

\[ \hat{B}_{Bq} = \alpha_s(\mu)^{2/\beta_0} \left[ 1 + \frac{\alpha_s(\mu)}{4\pi} J_{Nf} \right] B_{Bq}(\mu) \]
Quenched $\hat{B}_{B_d}$

Present estimate in quenched QCD

$$\hat{B}_{B_d} = 1.30(12)(13)$$

$$F_{B_d}\sqrt{\hat{B}_{B_d}} = 230(40) \text{ MeV}$$

$$\frac{\hat{B}_{B_s}}{\hat{B}_{B_d}} = 1.00(4)$$

$$\xi = \frac{F_{B_s}\sqrt{\hat{B}_{B_s}}}{F_{B_d}\sqrt{\hat{B}_{B_d}}} = 1.16(5)$$
\[ B_{S_0}(\mu) = \frac{\langle \bar{B}_q | O_S(\mu) | B_q \rangle}{\frac{5}{3} \langle \bar{B}_q | P(\mu) | 0 \rangle \langle 0 | P(\mu) | B_q \rangle} \]

\[ O_S(\mu) = P(\mu) \cdot P(\mu), \quad P(\mu) = \bar{b}(1 - \gamma_5)q \]

Width difference of \( B_s - \bar{B}_s \)

\[ \left( \frac{\Delta \Gamma}{\Gamma} \right) = \left( \frac{F_{B_s}}{245 \text{MeV}} \right)^2 [0.008 B_{B_s}(m_b) + 0.099 B_{\bar{B}_s}(m_b) = 0.086] \]
Remark
difference between Static/relativistic and NRQCD is mainly caused by the difference in experimental inputs and formula for $\Delta \Gamma / \Gamma$
Form factors of semi-leptonic decays

rare decays $\Rightarrow |V_{ub}|$

![Diagram](image)

$B \rightarrow \rho \ell \nu$

$$\langle \rho(k, \varepsilon) | V^\mu(q) | B(p) \rangle = \frac{2V(q^2)}{m_H + m_V} \epsilon^{\mu\nu\alpha\beta} p^\nu k^\alpha \varepsilon^*_\beta$$

$$\langle \rho(k, \varepsilon) | A^\mu(q) | B(p) \rangle = i(m_H + m_V) A_1(q^2) \varepsilon^{*\mu} - i \frac{A_2(q^2)}{m_H + m_V} \varepsilon^* \cdot p (p + k)^\mu$$

$$+ i \frac{A(q^2)}{q^2} 2m_V \varepsilon^* \cdot p \ q^\mu$$

**differential decay rate**

$$\frac{d\Gamma}{dq^2}(B \rightarrow \rho \ell \nu) = 10^{-12} \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m^3_B} c^2 q^2 (1 + b(q^2 - q_{\text{max}}^2))$$

$$\times \sqrt{(m^2_B + m^2_\rho - q^2)^2 - 4m^2_B m^2_\rho}$$

phase space factor
Flynn-Lesk (UKQCD), Quenched QCD

\[ \frac{\Delta \Gamma(14 < q^2/\text{GeV}^2 < 20.3)}{\text{ps}^{-1}\text{GeV}^{-2}} = 8.3|V_{ub}|^2 \]

\[ = 7.1(2.4) \times 10^{-5} \text{ UKQCD preliminary } \]

\[ \text{CLEO} \]

\[ \implies V_{ub} = 2.9(0.5) \times 10^{-3} \text{ (quenched QCD)} \]
\[ B \rightarrow \pi \ell \nu \]
\[ \langle \pi(k)|\bar{u}\gamma_\mu b|B(p)\rangle = f^+(q^2) \left( p + k \right)_\mu - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu + f^0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q_\mu \]

\( f^0(q^2) \) is negligible in the decay rate (\( q_\mu L_\mu \rightarrow m_\ell \))

\( f^+(q^2), f^0(q^2) \) (quenched QCD)

Form factor for semileptonic B decay

- all groups agree in \( f^+(q^2) \)
- extrapolation to smaller \( q^2 \) is important
- small discrepancy in \( f^0(q^2) \) should be understood

⇒ Posters by T. Onogi on 19th and by A.S. Kronfeld on 22th.
Differential decay rate

\[
\frac{d\Gamma}{dq^2}(B \rightarrow \pi \ell \nu) = \frac{G_F^2 |k_\pi|^3}{24\pi^2} |V_{ub}|^2 |f_+^*(q^2)|^2
\]

Onogi (JLQCD), quenched QCD

Differential decay rate of B to π l ν with physical light quark mass

- data at \( q^2 = 18 - 22 \text{ GeV}^2 \), obtained by interpolation, are reliable
- differential decay rate is needed but experimentally difficult
Differential decay rate of $B$ to $\pi \ell \nu$ with physical light quark mass

- all results on the differential decay rate are consistent within errors
- important to reduce errors
Conclusion

- Lattice is an important players in B-physics, in collaboration with other theoretical methods
- \( m_b, F_B, B_B, B_S \), form factors of \( B \to \rho/\pi \), and more
- Lists of some other quantities

<table>
<thead>
<tr>
<th>quantities</th>
<th>status</th>
<th>( N_f )</th>
<th>results</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\tau(B^-)}{\tau(B^0)} )</td>
<td>static</td>
<td>0</td>
<td>1.03(2)(3)</td>
</tr>
<tr>
<td>( \frac{\tau(\Lambda_b)}{\tau(B^0)} )</td>
<td>static</td>
<td>0</td>
<td>0.91(1)(1)</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>from world data</td>
<td>0</td>
<td>( 0.68(2)(-12) ) GeV</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>from world data</td>
<td>0</td>
<td>(-0.45(12) ) GeV^2</td>
</tr>
<tr>
<td>( M_{B_c} )</td>
<td>NRQCD</td>
<td>0</td>
<td>( 6.386(9)(98)(15) ) GeV</td>
</tr>
<tr>
<td>( H(bb) )</td>
<td>NRQCD</td>
<td>0</td>
<td>( \Delta m=1.542(8) ) GeV</td>
</tr>
<tr>
<td>( H(\bar{c}g) )</td>
<td>NRQCD</td>
<td>0</td>
<td>( \Delta m=1.323(13) ) GeV</td>
</tr>
</tbody>
</table>

- impact on determination of CKM matrix
  ⇒ Talk by A. Soni on 21th.