# Theoretical Aspects of $b \to s\gamma$ Transitions

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## Contents

# Introduction

What is interesting about $B \to X_s \gamma$?

- Loop induced process, test the GIM mechanism of the Standard Model
- May open a window to new Physics
- relevant operator:

$$H_{eff} = a_7 \bar{s}\sigma_{\mu\nu}(1+\gamma_5)b\,F^{\mu\nu} + a_7' \bar{s}\sigma_{\mu\nu}(1-\gamma_5)b\,F^{\mu\nu}$$

- In the standard model

$$a_7 = -\frac{G_F^2 e}{32\sqrt{2}\pi^2} V_{tb} V_{ts}^* C_7 m_b,$$

$$a_7' = -\frac{G_F^2 e}{32\sqrt{2}\pi^2} V_{tb} V_{ts}^* C_7 m_s$$

- Experimentally only the parameters $a_7$ and $a_7'$ can be determined
- To distinguish $a_7$ and $a_7'$: polarization needed: Impossible in $B \to X_s \gamma$!

- Standard Model calculation:
  Inclusive decays are well under control
  Heavy Quark Expansion
  - The leading term is the partonic rate
  - There are no $1/m_b$ corrections
  - $\mathcal{O}(1/m^2)$ terms to the total rate are tiny.
  - Other non-perturbative corrections such as
    $$B \to J/\Psi X_s \to X_s \gamma$$
    have been estimated to be small.
  - Perturbative corrections have been calculated to NLO: $C_7$
- Experimentally: There is always a cut on the Photon Energy
- This induces nonperturbative effects
- Study the photon Spectrum in the inclusive decay $B \to X_s \gamma$!

# Part I: Perturbative Corrections

## Effective Field Theory Framework

- Construct the operators of the EFTH:

$$\begin{aligned} O_{1\cdots 6} &= \text{four fermion operators} \\ O_7 &= m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5)b\,F^{\mu\nu} \\ O_7' &= m_s \bar{s}\sigma_{\mu\nu}(1-\gamma_5)b\,F^{\mu\nu} \\ O_8 &= \bar{s}\sigma^{\mu\nu}T^a(1+\gamma_5)b\,G^a_{\mu\nu} \\ O_8' &= \bar{s}\sigma^{\mu\nu}T^a(1-\gamma_5)b\,G^a_{\mu\nu} \end{aligned}$$

- Calculate the coefficients at $\mu = M_W$:

$$C_i(M_W) = C_i^{(0)}(M_W) + \frac{\alpha_s}{\pi} C_i^{(1)}(M_W) + \cdots$$

Matching

- Calculate anomalous dimensions and use the renormalization group: $\mu = M_W \longrightarrow \mu \sim m_b$: Running

$$
\begin{aligned}
C_i(\mu) &= C_i^{(0)}(M_W) \sum_{n=0} a_n^{(0)} \left( \frac{\alpha_s}{\pi} \ln \left( \frac{M_W^2}{\mu^2} \right) \right)^n \\
&\quad + \frac{\alpha_s}{\pi} C_i^{(1)}(M_W) \sum_{n=0} a_n^{(1)} \left( \frac{\alpha_s}{\pi} \ln \left( \frac{M_W^2}{\mu^2} \right) \right)^n \\
&\quad + \cdots
\end{aligned}
$$

- Compute the matrix elements of the operators
  - Inclusive decays: $1/m_b$ expansion
  - Exclusive decays: models etc.
- Renormalization group has removed the large logarithms from the matrix element.
- With increasing order in $\alpha_s$ the dependence on the renormalization scheme and – scale decreases.

# Status

- The Corrections are known to next-to-leading order:
  - Matching to be computed to $\mathcal{O}(\alpha_s)$:
    Full two-loop Calculation

    Ali, Greub, Hurth, Wyler
  - Running needs the $\mathcal{O}(\alpha_s^2)$ anomalous dimensions:
    Divergent parts of three loop diagrams

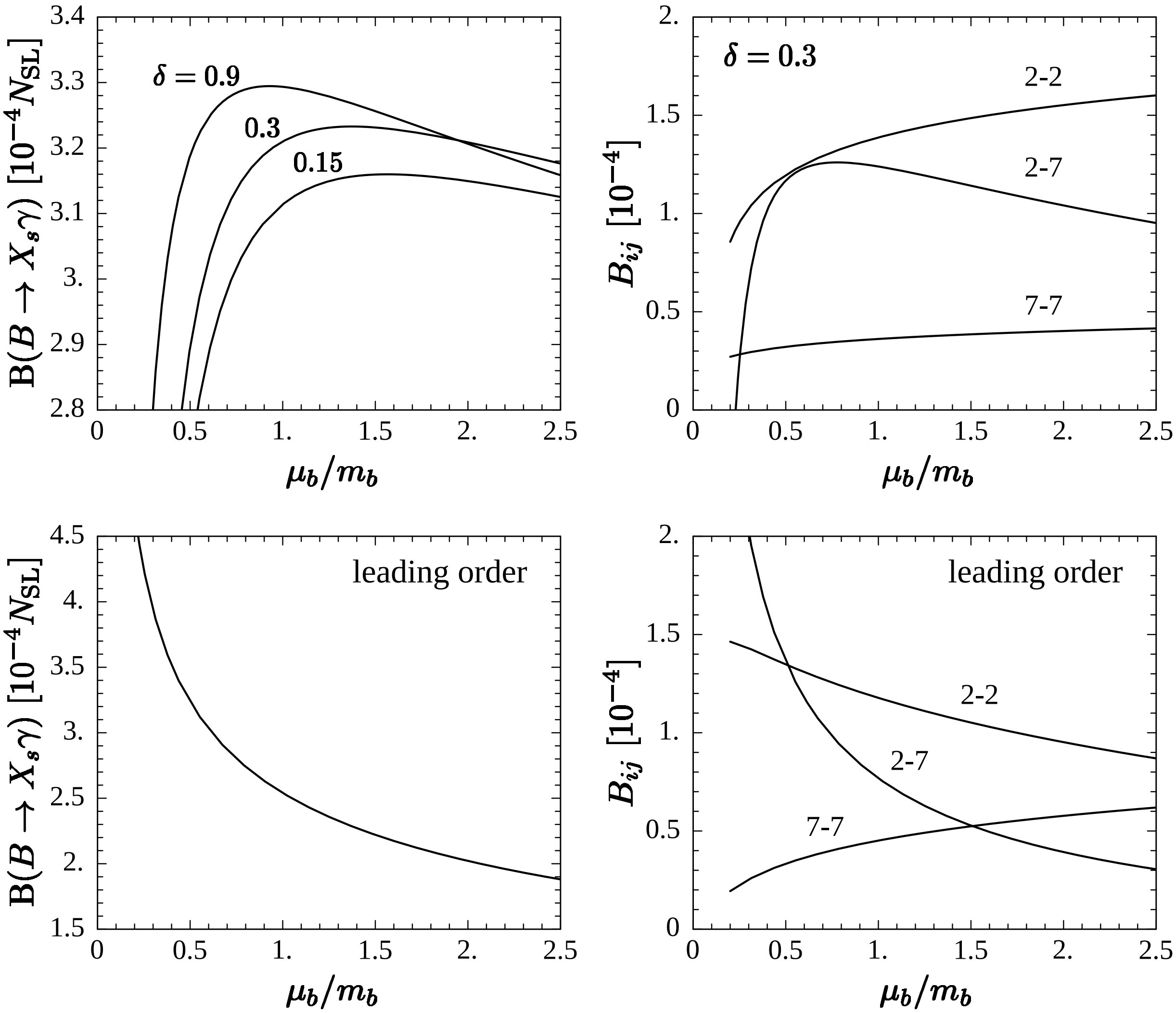
    Chetyrkin, Misiak, Münz
  - Matrix elements to $\mathcal{O}(\alpha_s)$

    Ali, Greub, Hurth, Wyler, Neubert, Kagan
- The QCD corrections are large:

$$C_7(M_W) = -0.19 \longrightarrow C_7(m_b) = -0.31$$

- Dependence on renormalization scale $\mu$

Neubert Kagan




Roughly:

- Leading order: $\delta_\mu = {}^{+27.4\%}_{-20.4\%}$
- Next-to-leading order: $\delta_\mu = {}^{+0.1\%}_{-3.2\%}$
- Small due to (accidential?) cancellations

- The cancellations may not be accidental

  Misiak, Gambino

- Large part of the radiative corrections can be assigned to the running of the bottom quark mass:

$$m_b(\mu \sim m_b)\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b\,F^{\mu\nu}$$

where the running is

$$m_b(M_W) \to m_b(m_b)$$

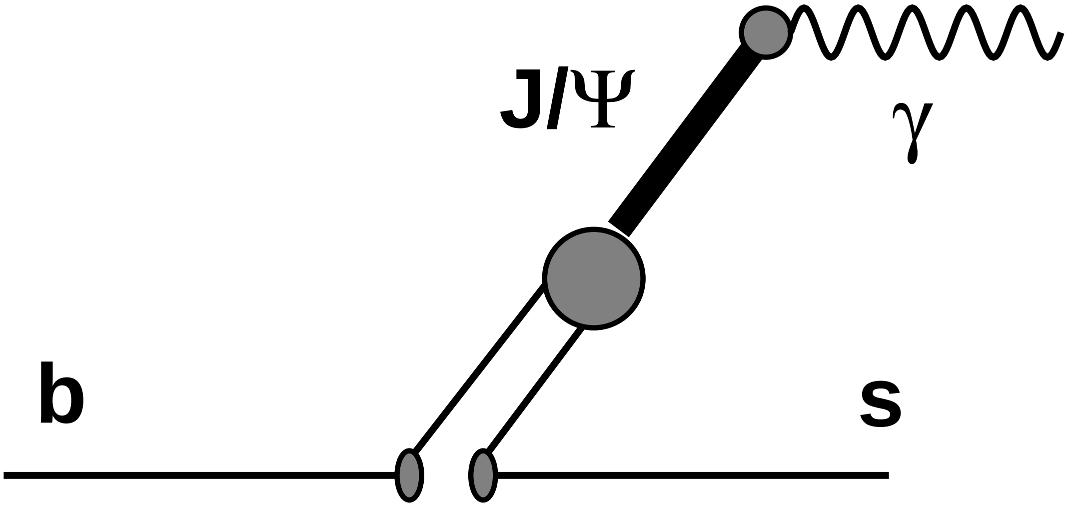
- perturbative corrections are reasonably well under control:

# Part II:
# Non-perturbative Corrections

Various sources of non-perturbative corrections:

- $B \to J/\Psi X_s \to X_s \gamma$
- $1/m_b$ and $1/m_c$ corrections
  in inclusive decays
- Shape functions for the Photon spectrum
- Form factors in exclusive decays

# $B \to J/\Psi X_s \to X_s \gamma$




- $\mathrm{Br}(B \to J/\Psi X) \sim 1 \times 10^{-2}$
- Mediated by the four quark operators $\mathcal{O}_1$ and $\mathcal{O}_2$
- $1/m_c^2$ Suppression by the $J/\Psi$ propagator
- Annihilation of the charm quarks: Suppression by a factor $f_{J/\Psi}^2/m_b^2$
- In total: This contribution is small compared to the short distance contribution mediated by $\mathcal{O}_7$

# $1/m_b$ corrections

Falk Luke Savage, Bigi, Uraltsev, Ali, Hiller ...

- Computed in heavy mass expansion:
  Total rate:
  $$\Gamma = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb^*}|^2 |C_7|^2 \left(1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2} + \cdots\right)$$

- This is integrated over the full photon spectrum

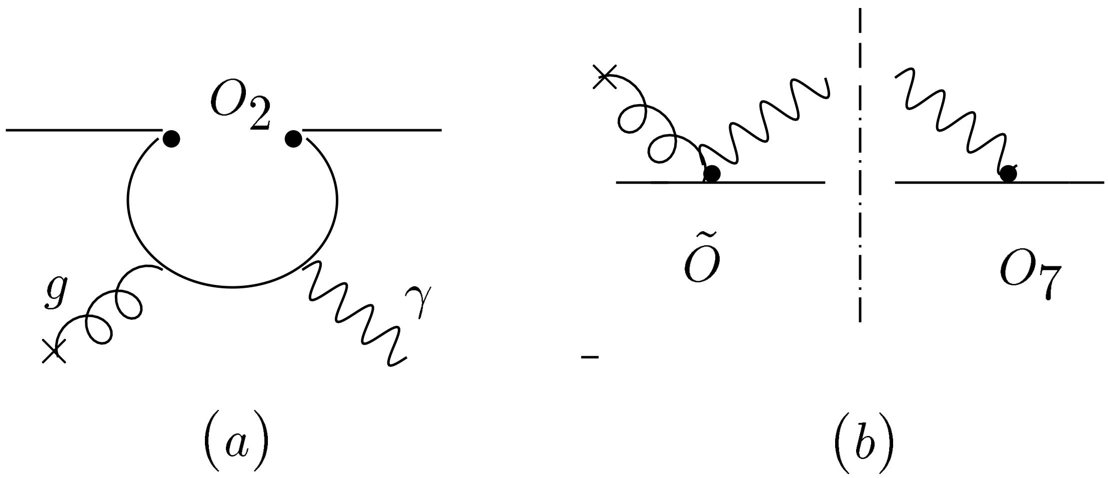
- Compute the photon spectrum in $1/m_b$ expansion:

$$\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb^*}|^2 |C_7|^2$$
$$\left(\delta(1-x) - \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \delta'(1-x) + \frac{\lambda_1}{6m_b^2} \delta''(1-x) + \cdots\right)$$

- Can only be interpreted in terms of spectral moments
  $\longrightarrow$ shape function

# $1/m_c$ corrections

Voloshin, Buchalla, Isidori, Rey, Ligeti, Randall, Wise, ...




- leads to an operator:

$$\frac{1}{m_c^2}\,\bar{s}\gamma_\mu(1-\gamma_5)T^a b\, G^a_{\nu\lambda}\epsilon^{\mu\nu\rho\sigma}\partial^\lambda F_{\rho\sigma}$$

- The effect is rather small:

$$\frac{\delta\Gamma_{1/m_c^2}}{\Gamma} = -\frac{C_2}{9C_7}\frac{\lambda_2}{m_c^2} \approx 0.03$$

# Shape function and $\gamma$ spectrum

Bigi, Uraltsev, Shifman, Vainshtein, Neubert, M.

- General Structure at tree level
  (no real gluon emmission)

$$\frac{d\Gamma}{dx} = \Gamma_0 \left[ \sum_i a_i \left( \frac{1}{m_b} \right)^i \delta^{(i)}(1-x) + \mathcal{O}((1/m_b)^{i+1} \delta^{(i)}(1-x) \right]$$

- Leading terms can be resummed into a
  shape function:

$$\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb^*}|^2 |C_7|^2 f(1-x)$$

where

$$2M_B f(\omega) = \langle B | \bar{Q}_v \delta(\omega + iD_+) Q_v | B \rangle$$

and $D_+$ is the light-cone component of the heavy quark residual momentum.

- More generally: Convolution of a perturbatively calculable Wilson Coefficient and a matrix element of a non-local Operator

$$d\Gamma = \int d\omega \, C_0(\omega) \langle B | O_0(\omega) | B \rangle$$

with

$$O_0(\omega) = \bar{Q}_v \delta(\omega + iD_+) Q_v$$

- Radiative corrections can be computed by the same procedure as in EFFT: Matching and Running
- Anomalous dimension of the shape function

Aglietti, Ricciardi, Balzereit, Kilian, C. Bauer, Flemming, Stewart, Pirjol, Luke, Ligeti, M ...

# Subleading Shape functions

C. W. Bauer, Luke, M.

- Subleading terms:

$$\frac{d\Gamma_{sub}}{dx} = \Gamma_0 \sum_i b_i \left(\frac{1}{m_b}\right)^{i+1} \delta^{(i)}(1-x)$$

- Can be resummed in terms of nonlocal operators

$$O_1^\mu(\omega) = \bar{Q}_v \left\{iD^\mu, \delta(iD_+ + \omega)\right\} 1 Q_v$$
$$O_2^\mu(\omega) = i\bar{Q}_v \left[iD^\mu, \delta(iD_+ + \omega)\right] 1 Q_v$$
$$O_3^{\mu\nu}(\omega_1, \omega_2) =$$
$$\bar{Q}_v \delta(iD_+ + \omega_2) \left\{iD_\perp^\mu, iD_\perp^\nu\right\} \delta(iD_+ + \omega_1) 1 Q_v$$
$$O_4^{\mu\nu}(\omega_1, \omega_2) =$$
$$g_s \bar{Q}_v \delta(iD_+ + \omega_2) G_\perp^{\mu\nu} \delta(iD_+ + \omega_1) 1 Q_v$$

and the corresponding ones with $1 \longrightarrow \vec{\sigma}$

- Effect of the subleading functions:

  – Introduce for $B$ decays four new functions

  – Need to be modelled: Simple but realistic one parameter model:

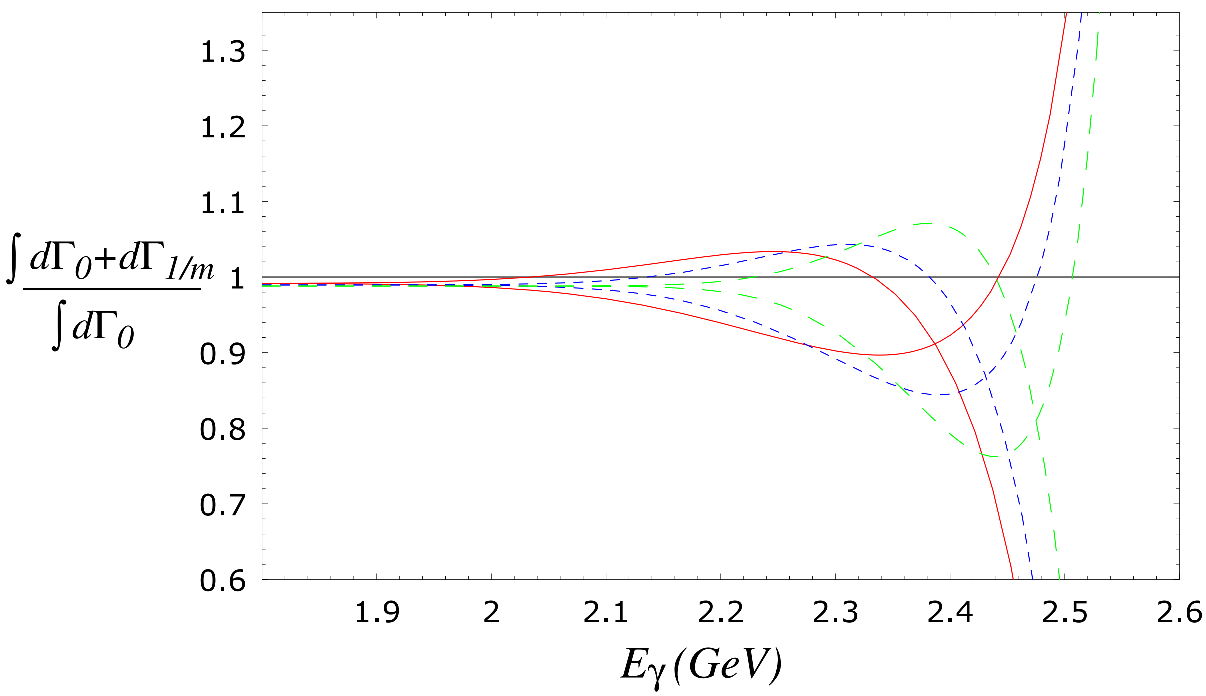



Figure 1: Partially integrated Rate normalized to the leading twist result. The three lines with a peak correspond to $\rho_2 = (500\text{ MeV})^3$, and $\bar{\Lambda} = 570$ MeV (Solid line), $\bar{\Lambda} = 470$ MeV (short dashed line) $\bar{\Lambda} = 370$ MeV (long dashed line). The two lines with a dip have $\rho_2 = -(500\text{ MeV})^3$ and $\bar{\Lambda} = 470$ MeV (dashed line), $\bar{\Lambda} = 370$ MeV (dotted line).

# Form Factors in exclusive decays

- Various methods/models:
  - Form factor models
  - QCD (light cone) sum rules
  - Lattice Calculations
  - ...
- not much can be said independent of "models"
- Typical Prediction

$$\frac{Br(B \to K^* \gamma)}{Br(B \to X_s \gamma)} = (10 - 20)\%$$

Consistent with the measurements

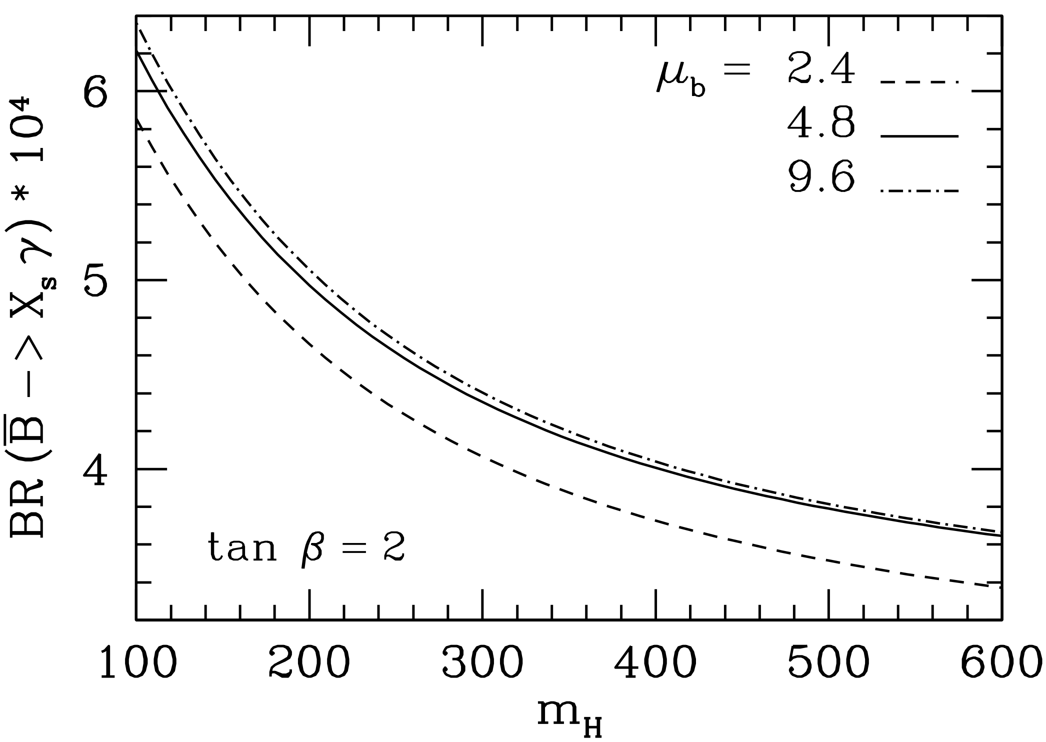
# Part III:
# "New Physics" in $b \to s\gamma$

- $b \to s\gamma$ is a loop process in the Standard Model
- can have a large senitivity to "new physics"
- However: This will only show up in the coefficient $|C_7(\mu) V_{ts} V_{tb}^*|^2$.
- This cannot pin down a specific scenario
- Various fashionable scenarios:
  - Two (or Multi) Higgs Doublet Models of various types
  - Various types of Supersymmetry
  - Various types of Technicolor
  - Large Extra Dimensions
  - ...

# $b \to s\gamma$ in Two Higgs Doublet Models

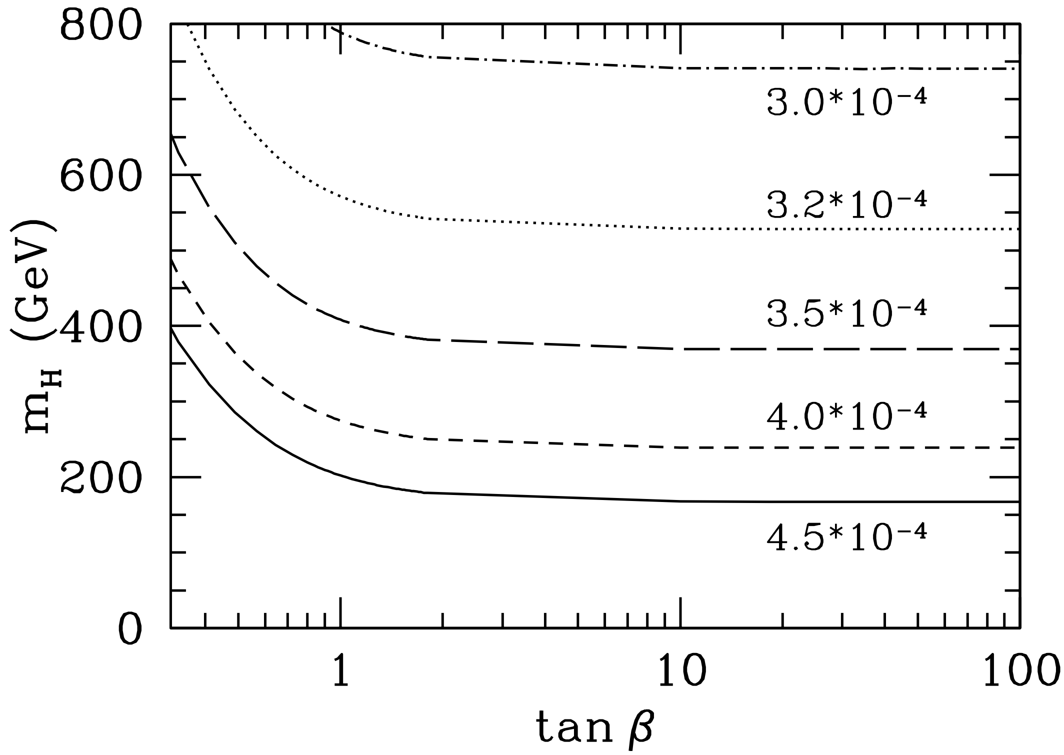
- Charged Higgs boson in the loop
- Parameters:
  $M_{H+}$: Mass of the charged Higgs
  $\tan\beta$: Ratio of the two VEVs
  $\mu_b$: Renormalization scale
- Type II only (related to SUSY)

Borzumati, Greub. Ciuchini et al.



- Contour Plots in the $\tan\beta - M_{H^+}$ plane:

Borzumati, Greub, Ciuchini et al.




- Curves indicate the experimental upper bounds

- Large $\tan\beta$ cancels against (small) $\cot\beta$
  $\longrightarrow$ No effect through a large $\tan\beta$.

# $b \to s\gamma$ in Superymmetry

Okada, Shimizu, Goto, Tanaka ...

- In exact SuSy $b \to s\gamma$ vanishes
  $\longrightarrow$ $b \to s\gamma$ is sensitiv to SuSy breaking
- In general there are a huge number of SuSy breaking parameters (Talk by A. Masiero)

$\longrightarrow$ SuSy has a Flavour Problem!

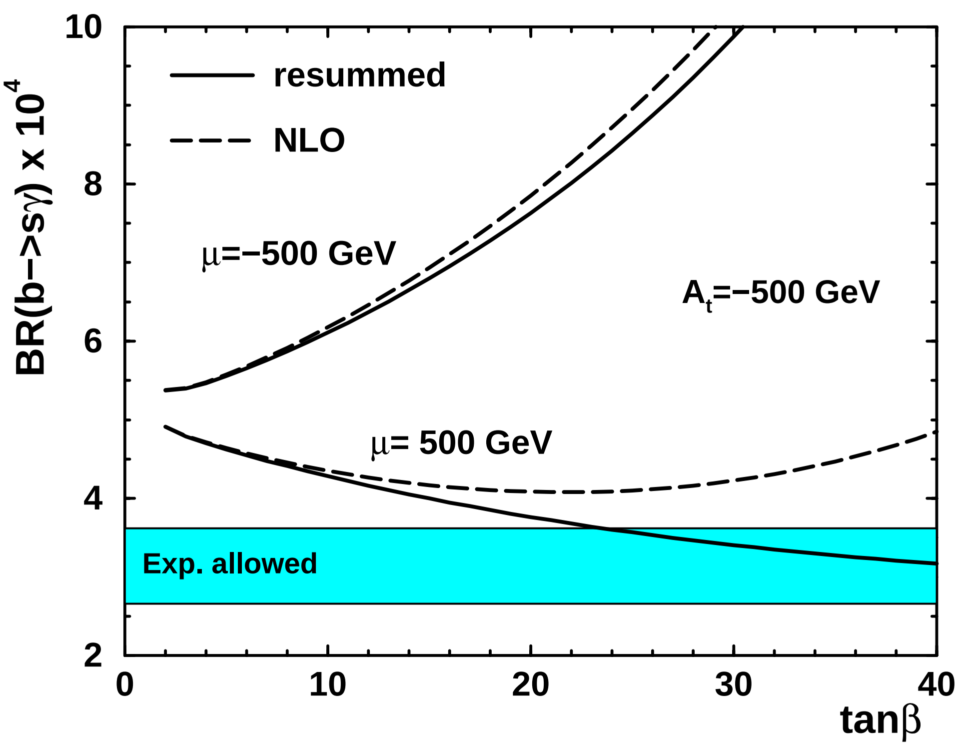
- Strong constraints on the SuSy parameter space.
- Concentrate on recent analysis
  Scenarios with a large $\tan\beta$:

  Carena, Garcia, Nierste, Wagner,
  Degrassi Gambino, Giudice

$\longrightarrow$ In SuSy, large $\tan\beta$ effects can become visible in $b \to s\gamma$

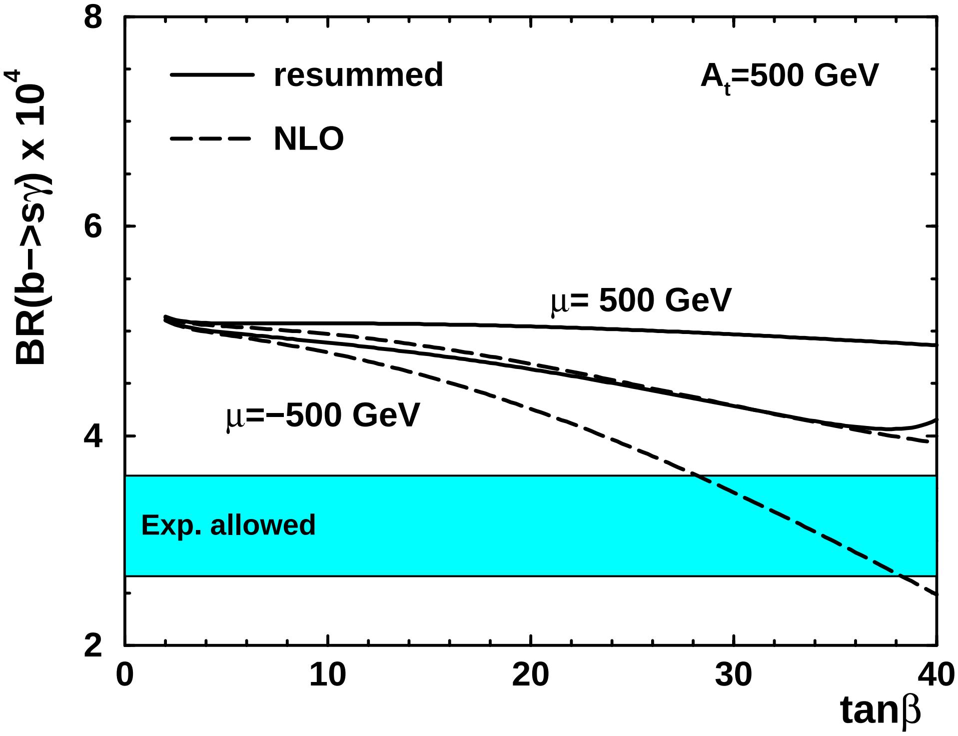
- work in the MSSM: Flavour diagonal
- Parameters:
  $\mu$ : SuSy parameter
  $A_t$ : soft SuSy breaking sector

As an example:
$M_{H^+} = 200$ GeV, $m_{\tilde{t}_1} = 250$ GeV, all other SuSy particle masses 800 GeV.
Resummation of (large) $\tan\beta$ terms



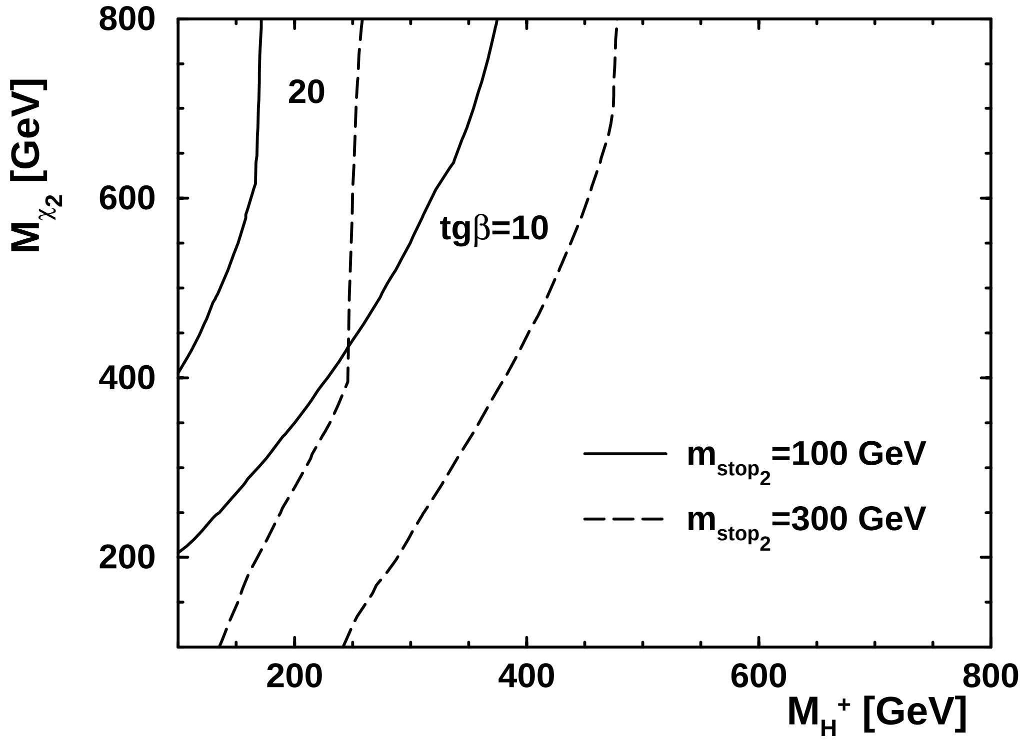

Carena, Garcia, Nierste, Wagner

The same for positive $A_t$




Carena, Garcia, Nierste, Wagner

Mass bounds:




Scanned over $m_{\tilde{t}_2} \leq m_{\tilde{t}_1} \leq 1$ TeV, $m_{\tilde{\chi}_2^+} \leq m_{\tilde{\chi}_1^+} \leq 1$ TeV, $|A_t| \leq 500$ GeV all other SuSy particle masses 1 TeV

Carena, Garcia, Nierste, Wagner

# Conclusions

- Inclusive $b \to s\gamma$ is under reasonable theoretical control

$$\frac{\delta_{th}\Gamma(B \to s\gamma)}{\Gamma(B \to s\gamma)} \approx 10\%$$

including a cut on the photon energy

- Cuts on the photon spectrum can be implemented without getting large theoretical uncertainties
- $b \to s\gamma$ serves as a test of physics beyond the SM
- It may exclude scenarios, if it remains compatible with the SM
- If it is incompatible with the SM, it cannot pin down what is going on ...