Precision in V_{ub} as a Function of Luminosity at e^+e^- B Factories (Theoretical Issues)

Adam F. Falk

The Johns Hopkins University

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The Context of the Question

Uncertainties in V_{ub} are almost always dominated by **theory**.

This complicates tremendously the straightforward question:

"What luminosity do you need to measure V_{ub} with method A to precision B?"

Hard numbers are impossible to come by!

(Sorry)

So I've redefined the question. My strategy today:

- Set a goal of measuring V_{ub} to $5-10\%$. This means measuring the "rate" to $10-20\%$.
- The error must include *meaningful* theoretical uncertainties.
- Discuss a variety of methods. Not all can reach our goal!
- Answer the questions...

  1. Which methods could possibly give this precision?
  2. For those methods, what quantities must be measured?
  3. What is the goal for experimental accuracy, for each method?

Methods based on exclusive decays

$B \to H_f \, \ell\nu = (\pi, \eta, \rho \ldots) \, \ell\nu$

Need to know $\langle H_f | \, \bar{u}\gamma^\mu(1-\gamma^5)b \, | B\rangle$ as a function of q^2

Dominant uncertainties are theoretical

• Models? No systematic estimate of errors

• Lattice? Need unquenched calculations. Will do best on π and η final states. (Width of ρ is a problem.)

Experiments need to measure rate and shape of form factor near zero recoil point, as accurately as possible. But theoretical uncertainties will always limit accuracy in V_{ub}.

$$\Downarrow$$

Accuracy of 10% in V_{ub} is *unrealistic* if form factor is modeled (current CLEO measurement?)

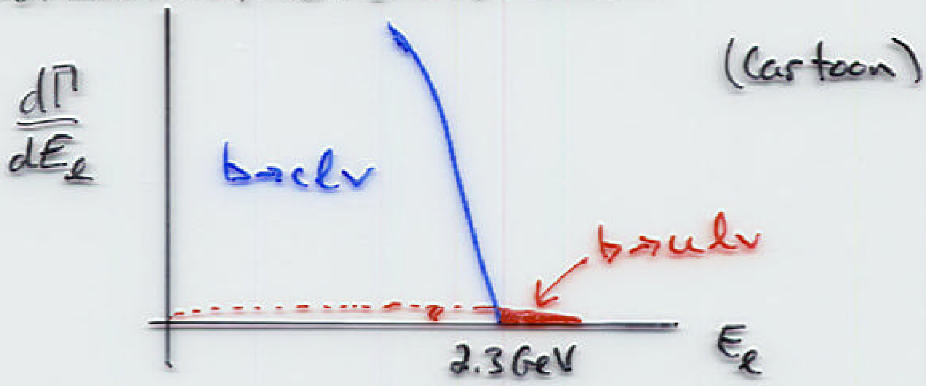
• Constraints based on $SU(3)$ symmetry? Rely on rare decays, such as $B \to K\nu\bar{\nu}$, inaccessible even to high luminosity B Factories at the $\Upsilon(4S)$. ? Need form factors.

Methods based on inclusive decays

$B \to X_u \ell \nu$, where X_u is not reconstructed.

$\sim 100\times$ background from $B \to X_c \ell \nu \Rightarrow$ *kinematic* rejection

e.g., measure $\mathrm{d}\Gamma/\mathrm{d}E_\ell$, require $E_\ell > 2.3\,\mathrm{GeV}$:



Key Message:

Experiments should report measurements *within* restricted kinematic regions.

Experiments should *not* extrapolate to inaccessible regions of phase space, if this can be avoided.

Why? Because the accuracy of theory has extraordinary sensitivity to these kinematic constraints!

Good news: The relation between $|V_{ub}|^2$ and the inclusive rate $\Gamma(B \to X_u \ell \nu)$ is known to approximately 5%

Bad news: Who cares? $\Gamma(B \to X_u \ell \nu)$ cannot be measured!

Theoretical issues

In principle, it's simple:

Large energy release

$\Downarrow$

Operator Product Expansion

$\Downarrow$

Series in $\alpha_s(m_b)$ and $1/m_b$

- Very useful B Factory fact: if initial B rest frame is known, then lepton kinematics determines hadron kinematics.

In practice, there are subtleties:

- The OPE breaks down in the *colinear* region:

Sudakov logarithms, B meson "shape function"

- Parton-hadron duality requires sum over many final states
- Uncalculated corrections, usually $O(\alpha_s^2)$ and $O(1/m_b^3)$

All of these can get worse with kinematic constraints!

Method 1: Require $q^2 > (m_B - m_D)^2$, where $q = p_\ell + p_\nu$

- Automatically cuts out dangerous colinear region
- Full two loop calculation still needed
- Need improved (and consistent) measurement of m_b
- Theory eventually could be accurate to 10% (duality?)

$$\Downarrow$$

Experimental goals are, in order:

- measure rate in region $q^2 > (m_B - m_D)^2 \simeq 11.6\,\mathrm{GeV}$
- measure shape of spectrum in this region
- extend sensitive region lower, down to $10.9\,\mathrm{GeV}$

These measurements should be made to 10%, to match theory

Method 2: Require $S_H \lesssim m_D^2$, where $S_H = (p_B - p_\ell - p_\nu)^2$

Captures more events than q^2 cut, so parton-hadron duality is more secure (good!)

Includes dangerous colinear region (very bad!). Implications:

• Need model of light-cone momentum distribution $f(p^+)$ of b quark inside B. Meaningful estimate of theoretical uncertainty?

• Dependence on $f(p^+)$ becomes more severe if $(S_H)_{\max}$ must be taken to be $(1.5\,\mathrm{GeV})^2$ instead of $m_D^2 \simeq (1.9\,\mathrm{GeV})^2$

$$\Downarrow$$

Accuracy of 10% in V_{ub} is *unrealistic* if $f(p^+)$ is modeled (current LEP measurement?)

Alternative: constrain $f(p^+)$ with E_γ spectrum in $B \to X_s\gamma$

• Relation between $\mathrm{d}\Gamma/\mathrm{d}E_\gamma$ and $\mathrm{d}\Gamma/\mathrm{d}S_H$ in colinear region includes Sudakov logarithms and shape function; known to 10%

• Measurement of the E_γ spectrum to 10% is a worthy experimental goal for a high luminosity B Factory

earlier ?

Method 3: Require $E_\ell > (m_B^2 - m_D^2)/2m_B \approx 2.3\,\text{GeV}$

Historically the first. But theoretically the worst!

• Very restricted region ⇒ duality concerns. Is there already experimental evidence of dominance by ρ and π?

• Region is *dominated* by the colinear configuration! Although $B \to X_s\gamma$ could provide some information, it is fair to conclude that...

...the E_ℓ spectrum will never, by itself, provide a reliable *precision* measurement of V_{ub}, no matter how beautiful the experiment or luminous the accelerator.

But it could be a nice reality check on other methods.

In the end...

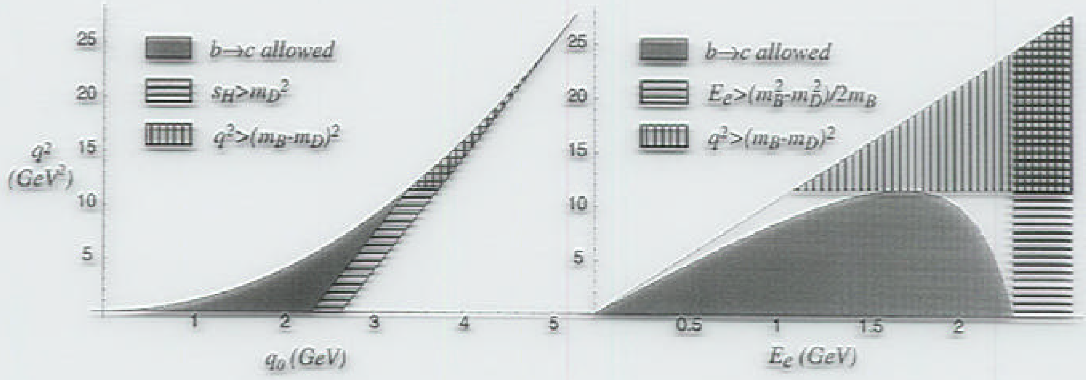
The analysis of inclusive distributions must be *optimized* for the best combination of theoretical predictivity and experimental feasibility.

When this is done, the theory will be good to $\sim 10\%$.

This should be the experimental goal, as well.

An example of work in progress: twice differential distributions,

$$d\Gamma/dq^2 dq_0 \qquad \text{vs.} \qquad d\Gamma/dq^2 dE_\ell$$



(Bauer, Ligeti, Luke)