CP Violation and Charmless B-meson Decays in the Perturbative QCD

Yong-Yeon Keum
IP/Academia Sinica, Taipei, Taiwan

BCP4-International Workshop, Ise in Japan
20th February, 2001

Outline :

1) Introduction

2) Important Issues
in theoretical point of view.

3) Hard Amplitude in PQCD.

4) Numerical Results
   4-A) $B \to \pi\pi$ decays
   4-B) $B \to K\pi$ decays
   4-C) $B \to \phi K$ decays

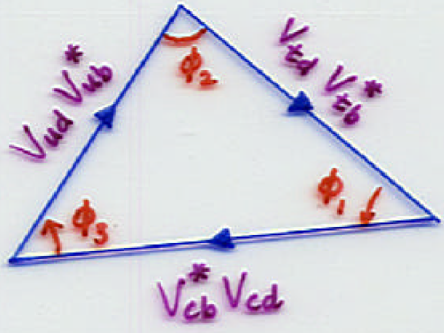
5) Summary

3. Angles of the Unitary triangle :

$$\phi_2(\alpha) \equiv Arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right]$$

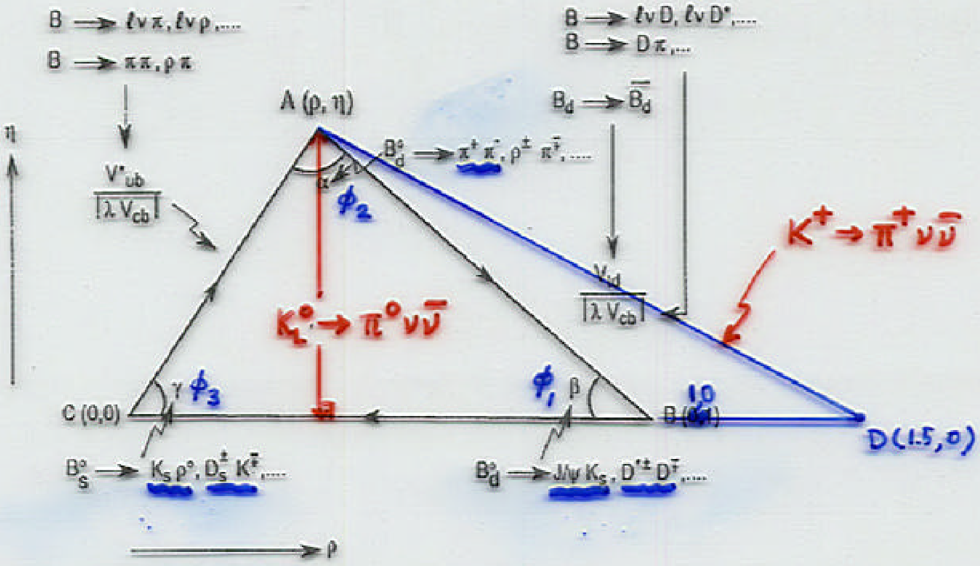
$$\phi_1(\beta) \equiv Arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

$$\phi_3(\gamma) \equiv Arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$



For B meson decays ; (bd)-triangle

$$\boxed{V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0}$$



Unitarity of CKM matrix $\Longrightarrow$ $\alpha+\beta+\gamma=\pi$

If $\alpha+\beta+\gamma\neq\pi$ $\longrightarrow$ Violation of SM.
New physics is coming!

Color Transparancy Argument (Bjorken, S.Brodsky and Lepage)

The quark-antiquark pair produced in the decay does not have enough time to evlove to the real size hadronic entity but remains a small size bound state with a correspondingly small chromomagnetic moment which suppresses the QCD interactions.

The matrix element of O_2 is expressed as

$$\begin{aligned}\langle\pi(P_2)\pi(P_3)|O_2(\mu)|B(P_1)\rangle &\approx \langle\pi(P_2)|(\bar{d}_i q_i)_{V-A}|0\rangle\langle\pi(P_3)|(\bar{q}_j b_j)_{V-A}|B(P_1)\rangle\,, \\ &\propto f_\pi F^{B\pi}(q^2=M_\pi^2)\,, \qquad (1)\end{aligned}$$

where f_π is the pion decay constant and $F^{B\pi}(M_\pi^2)$ the $B\to\pi$ transition form factor evaluated at $q^2=M_\pi^2$.

1. Nonleptonic B-meson Decays :

The aim of the study of weak decay in B-meson is two folds :

(1) To determin the elements of CKM matrix and to explore the origin of CP-Violation in low-energy scale,

(2) To study strong interaction physics related to the confinement of quarks and gluons within hadrons.

Both tasks complement each other : An understanding of the connection between quarks and hadron properties is a necessary prerequiste for a precise determination of CKM matrix elements and CP-Violating phases(KM-phase).

Various Approaches :

A: General Factorization Approach :

B: QCD in Heavy Quark Limit ($m_b \gg \Lambda_{QCD}$):

Beneke et al. (BBNS)

(L. Silverstrini's talk)

K.C. Yang's Poster

C: PQCD approach :

(Li's talk)

2. Important Issues:

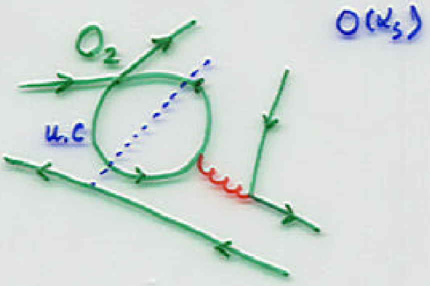
1. Where is the strong phase coming ?

2. Can we calculate Form factors ?

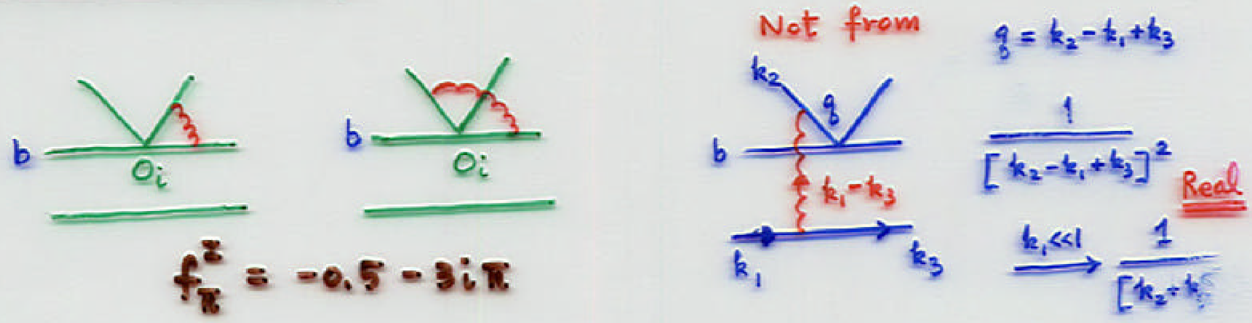
3. How can we treat the Annihilation diagram ?

Q1: Where is the strong phase coming from ??

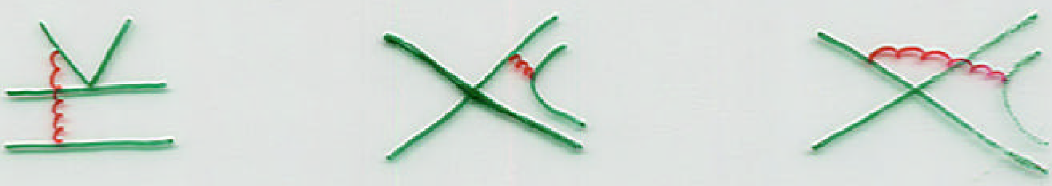
(1) In the General factorization Method, the strong phase comes from the BSS (Bander-Silverman-Soni) mechanism.



(2) In the BBNS Method, it comes from (BSS-Mechanism + Final State Interactions).



(3) In PQCD approach, Strong phases come from the Non-factroized W-emission and Annihilation diagrams.



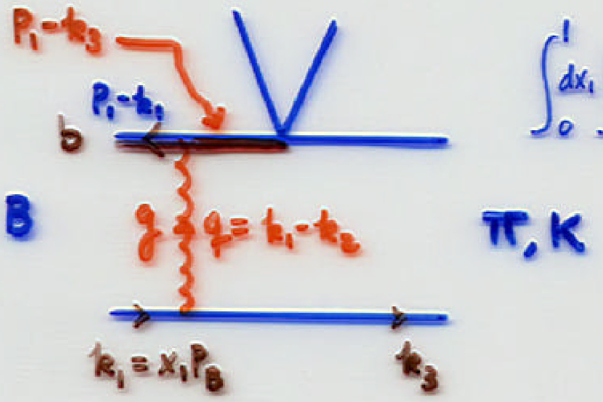
$$\frac{1}{(k_2+k_3-k_1)^2+i\varepsilon} = P\left[\frac{1}{(k_2+k_3-k_1)^2}\right] - i\pi\delta\left[(k_2+k_3-k_1)^2\right], \qquad (2)$$

where P denotes the principle-value prescription.

Q2: Can we calculate Form Factors ??

In the General Factorization Method and BBNS, Form factor is not calculable !

However the PQCD Method including k_T provide us a way to calculate the Form Factor.



$$\int_0^1 dx_1 \int_0^1 dx_3 \frac{1}{x_1 x_3^2 M_B^4} \phi_B(x_1)\, \phi_\pi(x_3) \begin{cases} A: x_3(1-x_3) \rightarrow \text{Log div} \\ P, \sigma: 1+\cdots \rightarrow \text{linear div} \end{cases}$$

* End-point singularity exist !!

For example, the lowest-order hard amplitude in Fig. 1 consists of the internal b quark and gluon propagators proportional to

$$\frac{1}{[(P_1-k_3)^2-m_b^2](k_1-k_3)^2} = \frac{1}{(2P_1\cdot k_3)(2k_1\cdot k_3)} = \frac{1}{x_1 x_3^2 M_B^4}, \qquad (3)$$

Including k_T,

$$\frac{1}{\underbrace{(x_3 M_B^2 + k_{3T}^2)}_{\text{on shell b-quark}}\underbrace{[x_1 x_3 M_B^2 + (\mathbf{k}_{1T}-\mathbf{k}_{3T})^2]}_{\text{gluon propagator}}},$$

which has no singularity at $x_3 \to 0$.

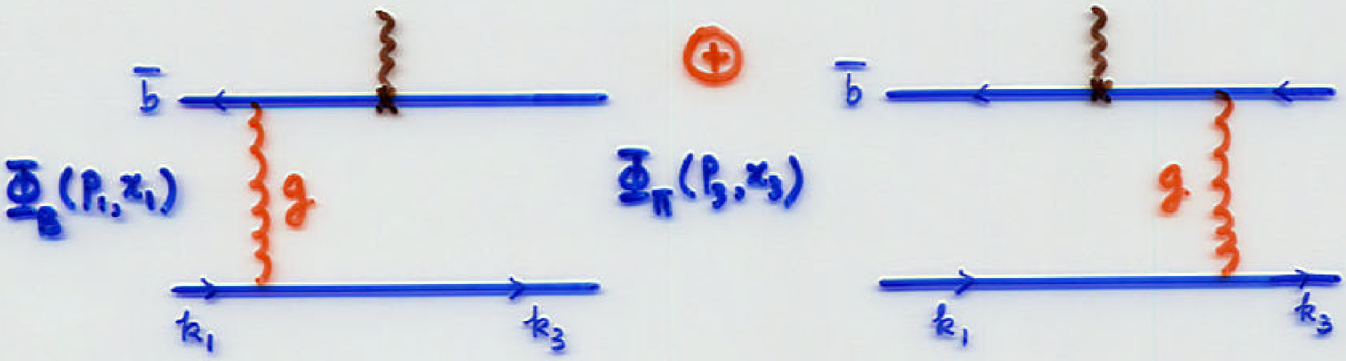
* End-point singularity is smeared !

Table 1: $B \to \pi, K$ transition Form Factor

ω_B	$F^{B\pi}(0)$	$F^{BK}(0)$
0.35	0.356	0.430
0.36	0.343	0.413
0.37	0.330	0.398
0.38	0.318	0.382
0.39	0.306	0.368
0.40	0.295	0.354
0.41	0.285	0.342
0.42	0.275	0.329
0.43	0.266	0.318
0.44	0.257	0.307
0.45	0.248	0.296

with $m_\pi^0 = 1.4,\ GeV$ and $m_0^K = 1.7\ GeV$

$B \to \pi, K$ transition Form Factors:



$$
\begin{aligned}
F_0^{B\pi}(q^2=0) &= -8\pi C_F M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \Phi_B(x_1, b_1) \\
&\times \{[(1+x_3)\Phi_\pi(x_3) + (1-2x_3)\, r_\pi (\Phi_\pi^p(x_3) + \Phi_\pi^\sigma(x_3))]\; Rs(x_3) \\
&\times \alpha_s(t_e^1)\, h_e(x_1, x_3, b_1, b_3)\, exp[-S_B(t_e^1) - S_\pi(t_e^1)] \\
&+ [2 r_\pi \Phi_\pi^p(x_3)]\; Rs(x_1)\; \alpha_s(t_e^2)\, h_e(x_3, x_1, b_3, b_1)\, exp[-S_B(t_e^2) - S_\pi(t_e^2)]\} \, .
\end{aligned}
$$

In our anlysis, we take $0.37 < \omega_B < 0.43$.

$r_\pi \equiv \frac{m_0^\pi}{m_B}$

- $\Phi_B(P, x) = \frac{1}{\sqrt{2N_c}} [\not P + M_B] \gamma_5 \Phi_B(x)$
- $\Phi_\pi(P, x) = \frac{1}{\sqrt{2N_c}} \gamma_5 [\not P \Phi_\pi(x) + m_0^\pi \Phi_\pi^P(x) + m_0^\pi (\not n \not v - 1) \Phi_\pi^\sigma(x)]$

⊙ B-meson wave function ;

$$\Phi_B(x) = N_B\, x^2 (1-x)^2\, Exp\left[-\frac{1}{2}\left(\frac{x M_B}{\omega_B}\right)^2 - \frac{\omega_B^2 b^2}{2}\right]$$

when $\omega_B = 0.4$, $N_B = 91.7835$ GeV

⊙ $\Phi_\pi(x)$, $\Phi_\pi^P(x)$, $\Phi_\pi^\sigma(x)$: LCDAs

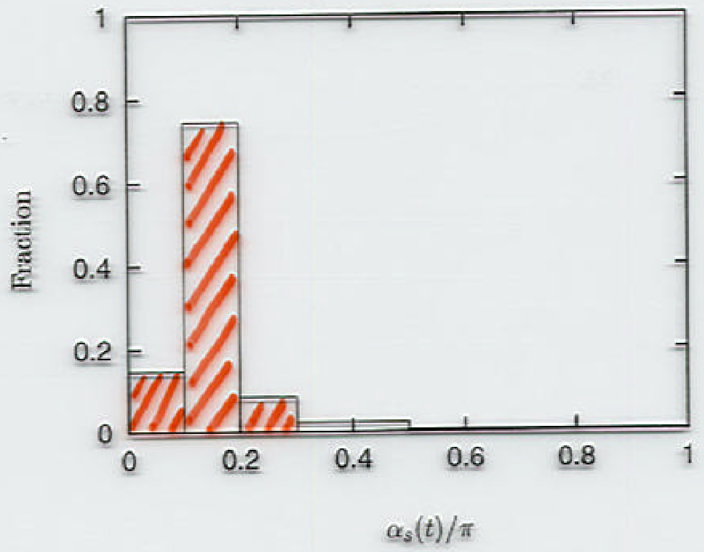


Figure 3:

* 97% of the contribution to $F_0^{B\pi}$ aries from the region with $\alpha_s(t)/\pi < 0.3$!

⇒ PQCD approach is quite resonable one!

$F^{B\pi}(0) = 0.3 \pm 0.03$

$F^{BK}(0) = 0.36 \pm 0.04$

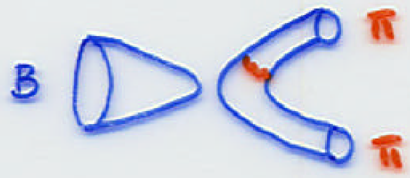
↔ $\omega_B = 0.4 \pm 0.03$

Q3: Can we neglect the annihilation contribution safely ?

Ans: No !

In General Factorization Approach and BBNS Approach, they neglect the annihilation contribution.

$B^0 \to \pi^+\pi^-$



$$\langle \pi\pi | \Gamma | B \rangle_A \simeq \langle \pi\pi | \Gamma | 0 \rangle \langle 0 | \Gamma | B \rangle$$

$$\sim \frac{f_B}{m_B} \underline{F^{\pi\pi}(q^2)}$$

timelike formfactor

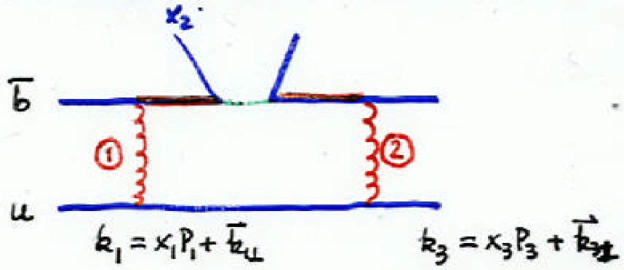
$\ll 1$ (heavy quark limit)

$m_B \to \infty$

However they don't consider the chiral enhanced factor in annihilation term.

In PQCD approach, we include annihilation diagram and the Factorized annihilation diagram provide a large strong phase !

Strong phases in PQCD

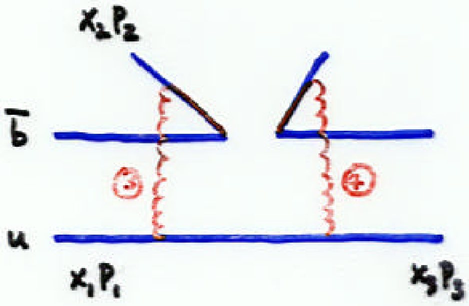


① $$\frac{\phi_B(x_1)\cdot\phi_3(x_3)}{[x_3 M_B^2 + k_{3\perp}^2]\,[x_1 x_3 M_B^2 + |\vec{k}_{1\perp} - \vec{k}_{3\perp}|^2)}$$

$\longrightarrow$ real

② $$\frac{\phi_B(x_1)\cdot\phi_3(x_3)}{[x_1 M_B^2 + |k_{1\perp}|^2]\,[x_1 x_3 M_B^2 + |k_{1\perp} - k_{3\perp}|^2]}$$

$\longrightarrow$ real



③ $$\frac{\phi_B(x_1)\ \phi_2(x_2)\ \phi_3(x_3)}{\left(x_3(1-x_1-x_2)M_B^2 \ominus |k_{1\perp} - k_{2\perp} - k_{3T}|^2\right)\left(x_1 x_3 M_B^2 + |k_{1\perp} - k_{3\perp}|^2\right)}$$

$\exists$ pole

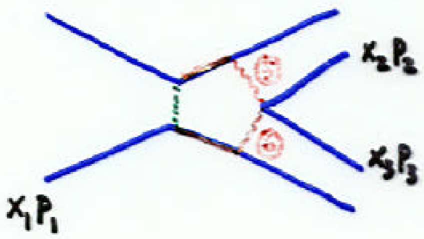
$\longrightarrow$ Complex

④ $$\frac{\phi_B(x_1)\ \phi_2(x_2)\ \phi_3(x_3)}{\left((x_2 - x_1)x_3 M_B^2 \ominus |k_{1\perp} - k_{2\perp} - k_{3\perp}|^2\right)\left(x_1 x_3 M_B^2 + |k_{1\perp} - k_{3\perp}|^2\right)}$$

$\longrightarrow$ complex

Dominant contribution

⊛



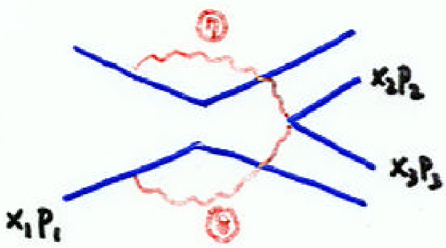
⑤ $$\frac{\phi_2(x_2)\ \phi_3(x_3)}{[x_3 M_B^2 \ominus |k_{3\perp}|^2]\ (x_2 x_3 M_B^2 \ominus |k_{2\perp} - k_{3\perp}|^2)}$$

$\longrightarrow$ complex

⑥ $$\frac{\phi_2(x_2)\ \phi_3(x_3)}{[x_2 M_B^2 \ominus |k_{2\perp}|^2]\ (x_2 x_3 M_B^2 \ominus |k_{2\perp} - k_{3\perp}|^2)}$$

$\underbrace{x_2 x_3 M_B^2 \ominus |k_{2\perp} - k_{3\perp}|^2}_{D_g}$

$\longrightarrow$ complex



⑦ $$\frac{\phi_B(x_1)\ \phi_2(x_2)\ \phi_3(x_3)}{\left[[(x_1 + x_2 + x_3) \ominus x_3(x_1 + x_2)]M_B^2 + |k_{1\perp} + k_{2\perp} + k_{3\perp}|^2\right]\ D_g}$$

$\longrightarrow$ complex

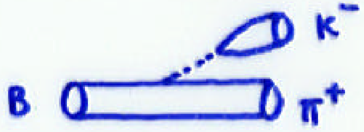
⑧ $$\frac{\phi_B(x_1)\ \phi_2(x_2)\ \phi_3(x_3)}{\left(x_3(x_2 - x_1)M_B^2 \ominus |k_{1\perp} - k_{2\perp} - k_{3\perp}|^2\right)\ D_g}$$

$\longrightarrow$ complex

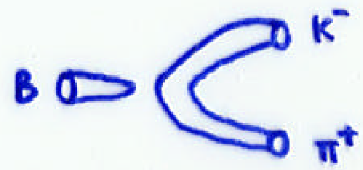
Ex Annihilation contribution : (Fact. Approach)

$$\bar{B}^0 \to K^- \pi^+$$

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* \, a_1 - V_{tb} V_{ts}^* \left[a_4 + a_{10} + 2(a_6 + a_8) \underbrace{\frac{m_K^2}{(m_b - m_u)(m_s + m_u)}}_{= R_6} \right] \right\} \otimes M^{(B\pi^+, K^-)}$$



$$- \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[a_4 - \frac{1}{2} a_{10} + (2a_6 - a_8) \underbrace{\frac{m_B^2}{(m_b + m_d)(m_s - m_d)}}_{= R_A} \right] \otimes M^{(B, \pi^+ K^-)}$$



$B \to K\pi$; $\quad \dfrac{R_A}{R_6} = \left(\dfrac{m_B}{m_K}\right)^2 \sim 10^2$

$B \to \pi\pi \quad \dfrac{R_A}{R_6} = \left(\dfrac{m_B}{m_\pi}\right)^2 \sim 1.5 \times 10^3$

//

3 HARD PART CALCULATION

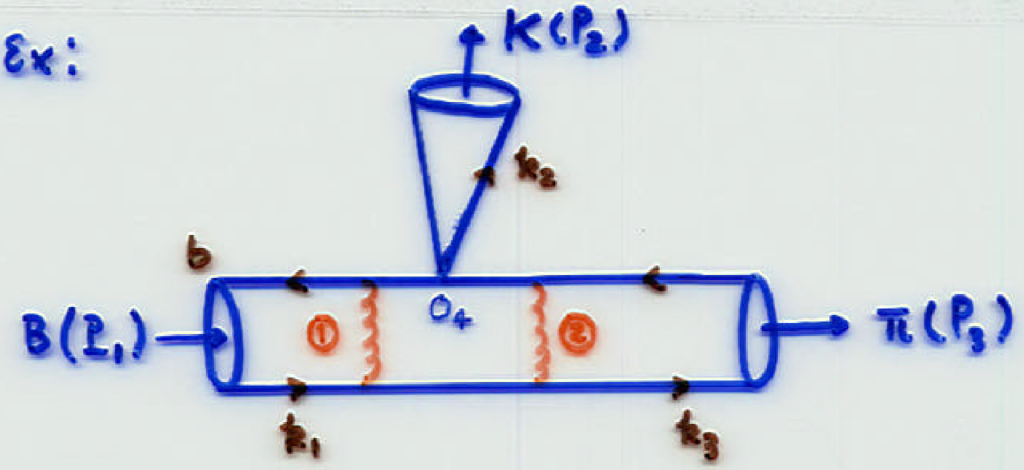
The effective Hamiltonian for the flavor-changing $b \to s$ transition is given by

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_q \left[C_1(\mu) O_1^{(q)}(\mu) + C_2(\mu) O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] , \tag{42}$$

with the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $V_q = V_{qs}^* V_{qb}$ and the operators

$$\begin{aligned}
&O_1^{(q)} = (\bar{s}_i q_j)_{V-A} (\bar{q}_j b_i)_{V-A} , &&O_2^{(q)} = (\bar{s}_i q_i)_{V-A} (\bar{q}_j b_j)_{V-A} , \\
&O_3 = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A} , &&O_4 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A} , \\
&O_5 = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A} , &&O_6 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A} , \\
&O_7 = \frac{3}{2} (\bar{s}_i b_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V+A} , &&O_8 = \frac{3}{2} (\bar{s}_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V+A} , \\
&O_9 = \frac{3}{2} (\bar{s}_i b_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V-A} , &&O_{10} = \frac{3}{2} (\bar{s}_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V-A} ,
\end{aligned} \tag{43}$$

Ex:



$$F_i \sim \int_0^1 d[b_i] \int_0^1 d[x_i] \; \Phi_B(x_1, b_1) \; H(x_i, b_i) \; \Phi_\pi(x_3) \; \Phi_K(x_2)$$

① $\left[(1+x_3) \Phi_\pi(x_3) + r_\pi (1-2x_3) \left(\Phi_\pi^P(x_3) + \Phi_\pi^\sigma(x_3) \right) \right] h_e(x_1, x_3, b_1, b_3) R_S(x_3)$
$\otimes \; \alpha_S(t_1) \, a_4(t_1) \, \mathrm{Exp}\left[-S_B(t_1) - S_\pi(t_1) \right]$

② $2 r_\pi \Phi_\pi^P(x_3) \, \alpha_S(t_2) \, a_4(t_2) \, \mathrm{Exp}\left[-S_B(t_2) - S_\pi(t_2) \right] \; h_e(x_3, x_1, b_3, b_1) R_S(x_1)$

$$R_S(x) = 1.775 \left[x(1-x) \right]^{0.3}$$

$B^0 \to K^+\pi^-$

$O_{1,2}$ contribute to T. 13

O_{3-10} " P.

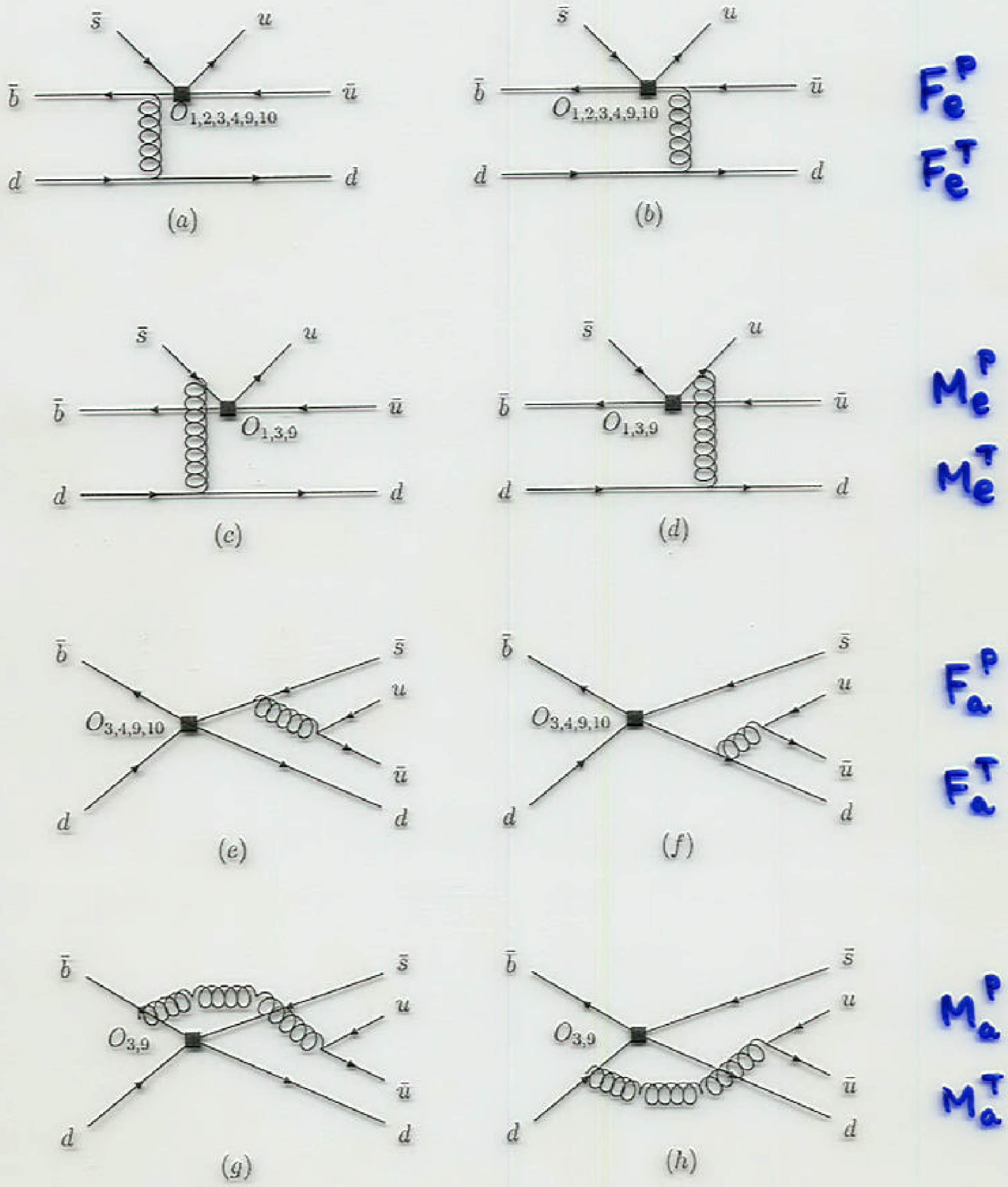


Figure 1: *Feynman diagrams for $B^0 \to K^{\mp}\pi^{\pm}$ decay*

LCDAs (upto twist-3)

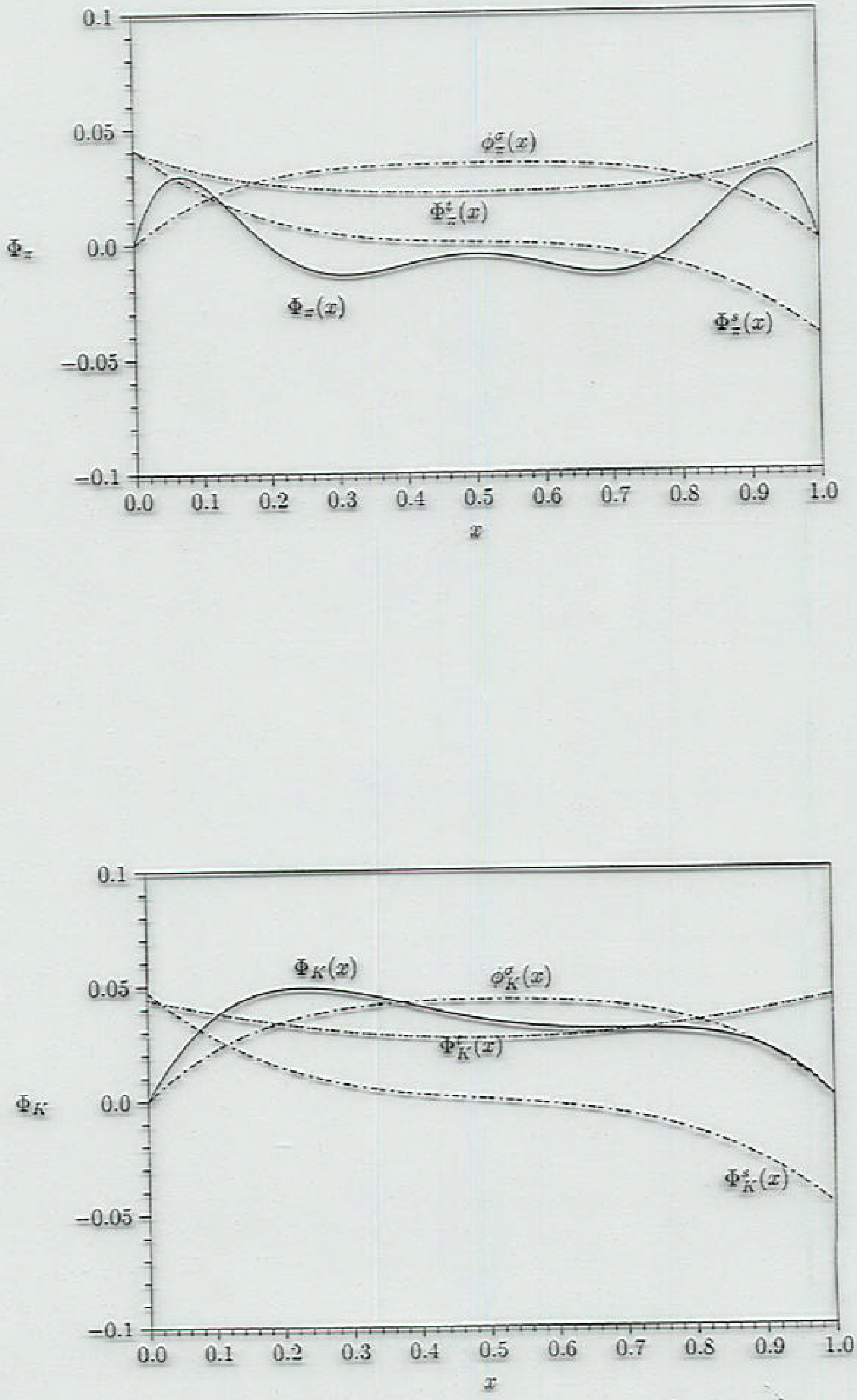
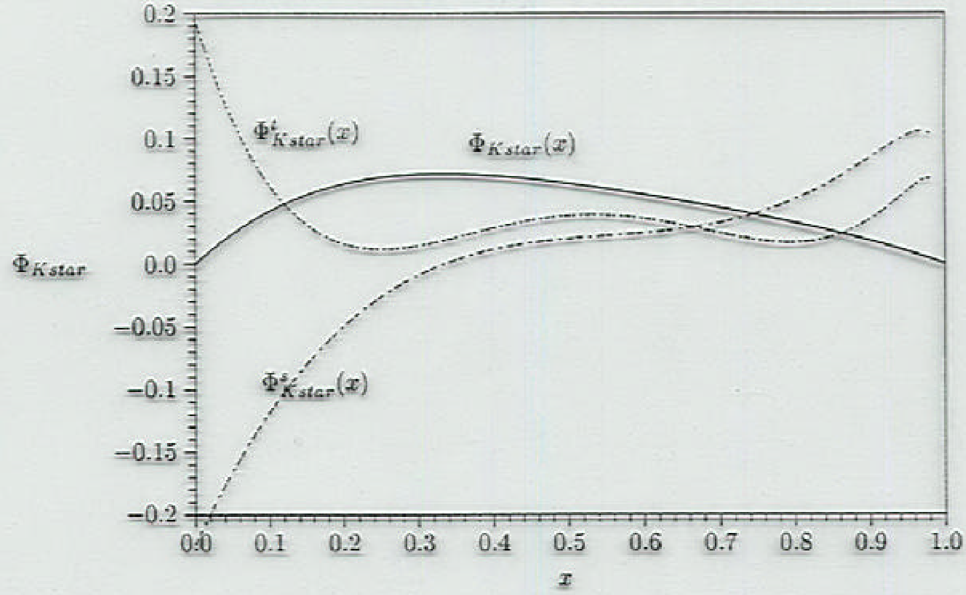


Figure 1a


$\Phi^t_{Kstar}(x)$
$\Phi_{Kstar}(x)$
$\Phi^s_{Kstar}(x)$
Φ_{Kstar}
0.2
0.15
0.1
0.05
0.0
−0.05
−0.1
−0.15
−0.2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x

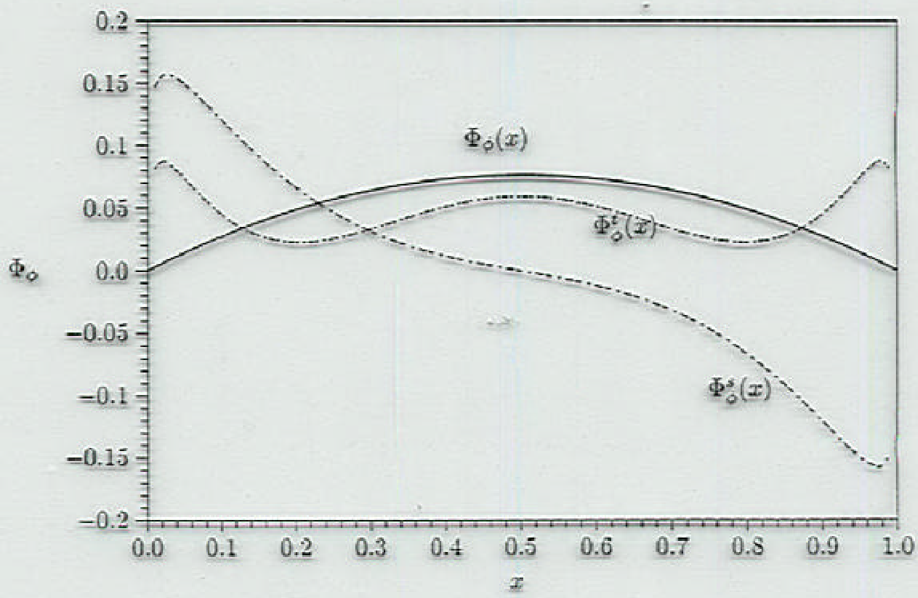

$\Phi_\phi(x)$
$\Phi^t_\phi(x)$
$\Phi^s_\phi(x)$
Φ_ϕ
0.2
0.15
0.1
0.05
0.0
−0.05
−0.1
−0.15
−0.2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x

Figure 1

4. Results:

16

A) B → ππ decays:

Amplitudes	Left-handed column		Right-handed column		PQCD	BBNS
$Re(f_\pi F^T)$	(a)	$4.39 \cdot 10^{-2}$	(b)	$2.95 \cdot 10^{-2}$	$7.34 \cdot 10^{-2}$	$7.57 \cdot 10^{-2}$
$Im(f_\pi F^T)$		–		–	–	$1.13 \cdot 10^{-3}$
$Re(f_\pi F^P)$	(a)	$-3.54 \cdot 10^{-3}$	(b)	$-2.33 \cdot 10^{-3}$	$-5.87 \cdot 10^{-3}$	$-3.04 \cdot 10^{-3}$
$Im(f_\pi F^P)$		–		–	–	$-1.27 \cdot 10^{-3}$
$Re(f_B F_a^P)$	(e)	$5.05 \cdot 10^{-4}$	(f)	$-1.94 \cdot 10^{-3}$	$-1.42 \cdot 10^{-3}$	–
$Im(f_B F_a^P)$	(e)	$2.19 \cdot 10^{-3}$	(f)	$3.72 \cdot 10^{-3}$	$5.91 \cdot 10^{-3}$	–
$Re(M^T)$	(c)	$5.02 \cdot 10^{-3}$	(d)	$-6.55 \cdot 10^{-3}$	$-1.53 \cdot 10^{-3}$	$-7.71 \cdot 10^{-4}$
$Im(M^T)$	(c)	$-3.83 \cdot 10^{-3}$	(d)	$7.03 \cdot 10^{-3}$	$3.20 \cdot 10^{-3}$	–
$Re(M^P)$	(c)	$-2.29 \cdot 10^{-4}$	(d)	$2.75 \cdot 10^{-4}$	$4.66 \cdot 10^{-5}$	$4.17 \cdot 10^{-3}$
$Im(M^P)$	(c)	$1.95 \cdot 10^{-4}$	(d)	$-3.08 \cdot 10^{-4}$	$-1.13 \cdot 10^{-3}$	–
$Re(M_a^P)$	(g)	$1.14 \cdot 10^{-5}$	(h)	$-1.48 \cdot 10^{-4}$	$-1.37 \cdot 10^{-4}$	–
$Im(M_a^P)$	(g)	$-9.12 \cdot 10^{-6}$	(h)	$-1.27 \cdot 10^{-4}$	$-1.36 \cdot 10^{-4}$	–

Table 1: Amplitudes for the $B_d^0 \to \pi^+\pi^-$ decay from Fig. 3, where F (M) denotes factorizable (nonfactorizable) contributions, P (T) denotes the penguin (tree) contributions, and a denotes the annihilation contributions. Here we adopted $\phi_3 = 90^0$, $R_b = 0.38$, and $\alpha_s(m_b) = 0.2552$ in the numerical analysis for the BBNS approach.