Overview of PQCD approach to B decays

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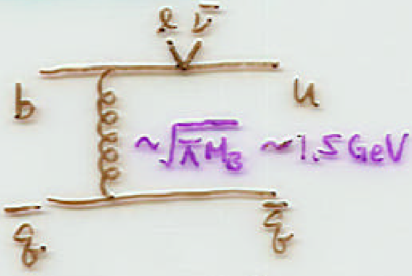
- factorization theorem
- twist expansion
- end-point singularity
- Sudakov suppression
- threshold resummation
- power counting
- predictions
- Summary

Introduction

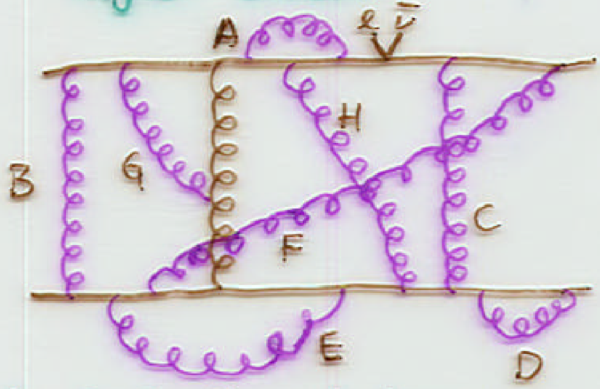
IR divergences

lowest order



Soft $\sim \bar{\Lambda} = M_B - m_b$ fast $\sim M_B$

higher orders



If A, B, C, ... H are all hard, it is calculable.

If not hard, we must deal with IR divergences.

soft $\ell^\mu = (\ell^+, \ell^-, \ell_T) \sim (\bar{\Lambda}, \bar{\Lambda}, \bar{\Lambda})$

collinear $\ell^\mu \sim (\bar{\Lambda}^2/M_B, M_B, \bar{\Lambda})$ $\Rightarrow \ell^2 \sim \bar{\Lambda}^2$

A is IR finite $\in H$ (hard part), B soft $\in \Phi_B$

C, D have soft and coll. div. soft cancel, coll. $\in \Phi_\pi$

E, F, have soft and coll. div. soft cancel, coll. $\in$?

G has soft div. $\in$?

H has coll. div. $\in$?

factorization theorem for $B \to \pi \ell \bar{\nu}$ (Li 0012140)

in the heavy quark limit

Φ_B and Φ_π are universal.

All-order proof of factorization theorem

factorization in momentum space $\Leftrightarrow$ eikonal approximation

spin $\Leftrightarrow$ Fierz identity

color $\Leftrightarrow$ Ward identity for nonabelian gauge theory

BRS transformation

$$\delta A_\mu^a = \omega D_\mu \sigma^a \qquad \delta\psi = i g \omega (T^a \sigma^a)\psi$$

$$\delta \rho^a = -i\omega \partial^\mu A_\mu^a / \xi \qquad \delta\sigma^a = -i g \omega \varepsilon^{abc} \sigma^b \sigma^c / 2$$

A: gauge field ρ, σ : ghost field

ψ : fermion field

ω : small parameter ξ : gauge parameter

choose $\xi = 1$

an amplitude

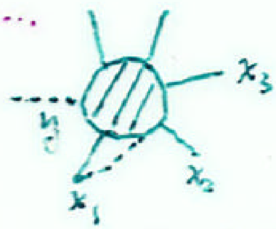
$$G_\mu = \langle \rho(y)\, \psi(x_1)\, \psi(x_2) \cdots \psi(x_n) \rangle$$

is invariant under BRS transformation

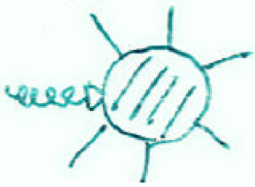
$$\delta G \propto \langle \partial^\mu A_\mu(y) \psi(x_1) \psi(x_2) \cdots \psi(x_n) \rangle \Rightarrow \ell^\mu \langle A_\mu(y) \psi(x_1) \cdots \psi(x_n) \rangle$$

$$+ \langle \rho(y)\, \sigma(x_1) \psi(x_1)\, \psi(x_2) \cdots \psi(x_n) \rangle + \cdots$$

$$= 0$$



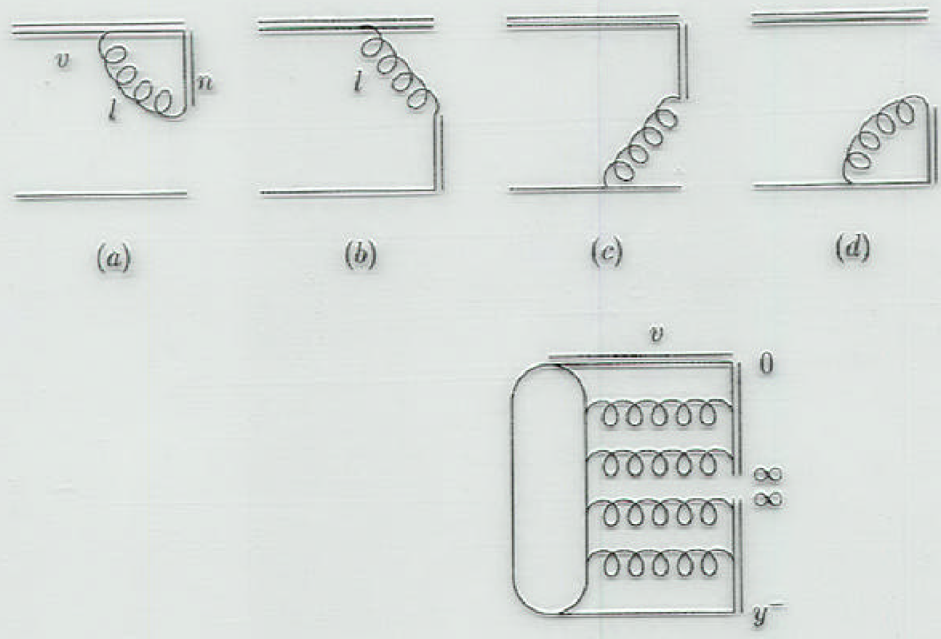
$= 0$



$= 0$ Ward identity

composite field

no physical pole



$$\int \frac{dy^-}{2\pi} e^{-ixP_1^+ y^-} \langle 0 | \bar{q}(y^-) \frac{\gamma_5 \not{n}}{2} P \exp\left[-ig\int_0^{y^-} dz\, n\cdot A(zn)\right] b_v(0) | B(P_1) \rangle$$

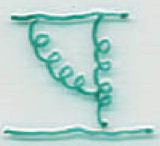
a light-cone B meson wave function

even it absorbs soft dynamics

FIG. 8

gauge invariance $\Leftrightarrow$ hard scale $\bar{\Lambda} M_B$

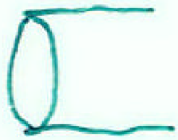
$\Leftrightarrow$ eikonal line along y^-

$\Leftrightarrow$  contributes to ϕ_B

twist expansion

three parton wave functions 

higher order in α_s neglected

two parton wave functions 

from Fierz identity

$$I_{ij} I_{lk} = \frac{1}{4} I_{ik} I_{lj} + \frac{1}{4} (\gamma^\mu)_{ik} (\gamma_\mu)_{lj} + \frac{1}{4} (\gamma_5 \gamma^m)_{ik} (\gamma_m \gamma_5)_{lj} + \frac{1}{4} (\gamma_5)_{ik} (\gamma_5)_{lj} + \frac{1}{4} (\gamma_5 \sigma_{\mu\nu})_{ik} (\gamma_5 \sigma^{\mu\nu})_{lj}$$

pseudoscalar $\pi, K,$

$\langle 0 | \gamma_5 \gamma_\mu | \pi \rangle$, $\langle 0 | \gamma_5 | \pi \rangle$, $\langle 0 | \gamma_5 \sigma_{\mu\nu} | \pi \rangle$

twists $\geq 2 \propto P_\mu \sim O(M_B)$ $\quad \geq 3 \propto m_0 = \frac{M_\pi^2}{m_u + m_d}$ $\quad \geq 3 \propto m_0$

vector ρ, K^*, ϕ

$\langle 0 | \gamma_\mu | \rho \rangle$, $\langle 0 | \sigma_{\mu\nu} | \rho \rangle$, $\langle 0 | I | \rho \rangle$

≥ 2 $\quad \geq 3$ $\quad \geq 3$

$\frac{\gamma_5 \not{P}}{\sqrt{2N}} \phi_\pi$ $\quad \frac{m_0 \gamma_5}{\sqrt{2N}} \phi_{\pi 1}$ $\quad \frac{m_0 \gamma_5 (\not{v}\not{n} - v \cdot n)}{\sqrt{2N}} \phi_{\pi 2}$ $\quad v = (1, 0, 0_T)$, $n = (0, 1, 0_T)$

$\frac{M_\rho \not{\epsilon}}{\sqrt{2N}} \phi_\rho$ $\quad \frac{\not{\epsilon} \not{P}}{\sqrt{2N}} \phi_{\rho 1}$ $\quad \frac{M_\rho}{\sqrt{2N}} \phi_{\rho 2}$

twist-3 wave functions

$i\epsilon_{\mu\nu}$ contracts to $\sigma^{\mu\nu}\gamma_5/2$ in the hard amplitude,

$$\frac{i}{2}\epsilon_{\mu\nu}\sigma^{\mu\nu}\gamma_5 = \frac{i}{2}(\sigma^{-+} - \sigma^{+-})\gamma_5 = \frac{1}{2}(\gamma^+\gamma^- - \gamma^-\gamma^+)\gamma_5 \,. \tag{11}$$

$v = (1, 0, 0_T)$ parallel to P_2 and $n = (0, 1, 0_T)$ parallel to z,

$$\frac{1}{2}(\not n \not v - \not v \not n)\gamma_5 = (\not n \not v - 1)\gamma_5 \,. \tag{12}$$

Up to twist 3, for the initial-state π^- meson,

$$\frac{\not P_3 \gamma_5}{\sqrt{2N_c}}\phi_\pi \,; \quad \frac{m_0\gamma_5}{\sqrt{2N_c}}\phi_\pi^p \,; \quad \frac{m_0(\not n \not v - 1)\gamma_5}{\sqrt{2N_c}}\phi_\pi^t \,, \tag{13}$$

with the wave functions,

$$\phi_\pi(x) = \frac{f_\pi}{2\sqrt{2N_c}}\phi_v(x) \,, \quad \phi_\pi^p(x) = \frac{f_\pi}{2\sqrt{2N_c}}\phi_p(x) \,, \quad \phi_\pi^t(x) = \frac{f_\pi}{12\sqrt{2N_c}}\frac{d}{dx}\phi_\sigma(x) \,. \tag{14}$$

The tensor wave function is normalized to zero,

$$\int_0^1 dx \frac{d}{dx}\phi_\sigma(x) = \phi_\sigma(1) - \phi_\sigma(0) = 0 \,. \tag{15}$$

For the outgoing π^- meson, consider the adjoints, $\gamma^0\gamma_5\gamma^0 = -\gamma_5$

$$\langle \pi^-(P_2^-)|\bar d(z)\gamma_\mu\gamma_5 u(-z)|0\rangle = -if_\pi P_{2\mu}\int_0^1 dx_2 e^{i\xi P_2\cdot z}\phi_v(x_2) \,, \tag{16}$$

$$\langle \pi^-(P_2^-)|\bar d(z)\gamma_5 u(z)|0\rangle = -if_\pi m_0\int_0^1 dx_2 e^{i\xi P_2\cdot z}\phi_p(x_2) \,, \tag{17}$$

$$\langle \pi^-(P_2^-)|\bar d(z)\sigma_{\mu\nu}\gamma_5 u(-z)|0\rangle = -f_\pi m_0\left(1 - \frac{M_\pi^2}{m_0^2}\right)\epsilon_{\mu\nu}\int_0^1 dx_2 e^{i\xi P_2\cdot z}\frac{1}{6}\frac{d}{dx_2}\phi_\sigma(x_2) \,. \tag{18}$$

The tensor structure in Eq. (18) acquires an extra minus sign. For the final-state π^- meson,

$$\frac{\gamma_5 \not P_3}{\sqrt{2N_c}}\phi_\pi \,, \quad \frac{\gamma_5 m_0}{\sqrt{2N_c}}\phi_\pi^p \,, \quad \frac{\gamma_5 m_0(\not v \not n - 1)}{\sqrt{2N_c}}\phi_\pi^t \,. \tag{19}$$

The models

$$\phi_\pi(x) = \frac{3}{\sqrt{2N_c}}f_\pi x(1-x)[1 + 0.44C_2^{3/2}(2x-1)] \,, \quad \text{twist 2} \tag{20}$$

$$\phi_\pi^p(x) = \frac{f_\pi}{2\sqrt{2N_c}}[1 + 0.43C_2^{1/2}(2x-1) + 0.09C_4^{1/2}(2x-1)] \,. \quad \text{twist 3} \tag{21}$$

$$\phi_\pi^t(x) = \frac{f_\pi}{2\sqrt{2N_c}}(1-2x)[1 + 0.55(10x^2 - 10x + 1)] \,, \quad \text{twist 3} \tag{22}$$

with the Gegenbauer polynomials

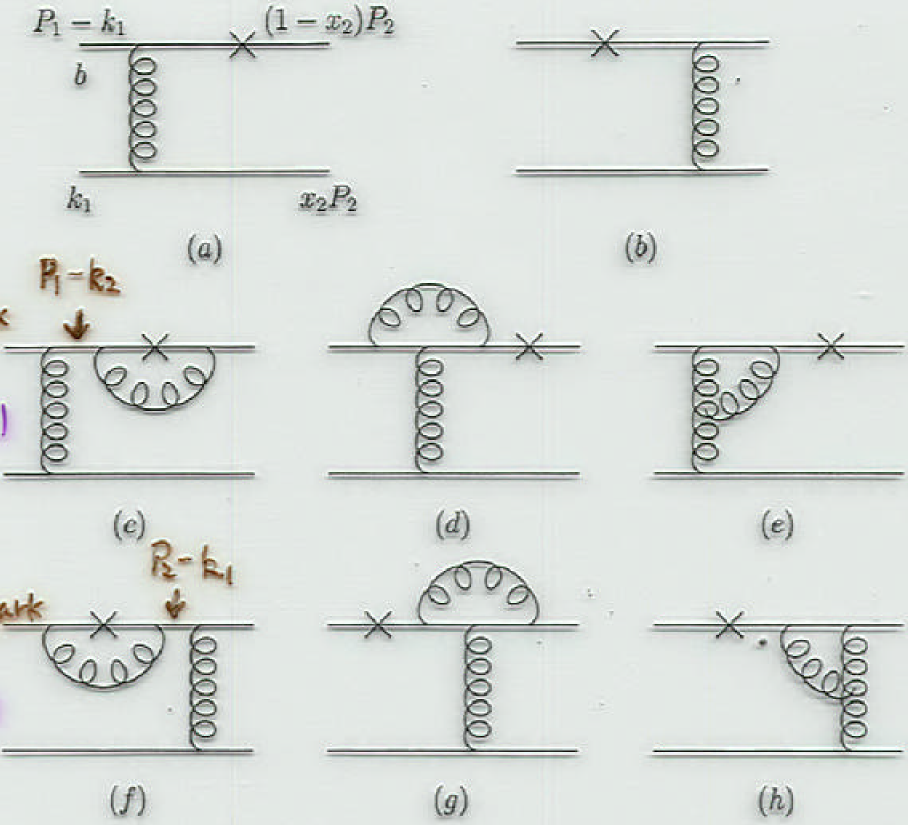
$$C_2^{1/2}(t) = \frac{1}{2}[3t^2 - 1] \,, \quad C_4^{1/2}(t) = \frac{1}{8}[35t^4 - 30t^2 + 3] \,, \quad C_2^{3/2}(t) = \frac{3}{2}[5t^2 - 1] \,. \tag{23}$$

We have assumed $m_0 = 1.4$ GeV.

twist-3 contributions are linearly divergent

can be smeared by k_T and threshold resummations

threshold resummation



$k_2 \to 0$, on-shell b quark

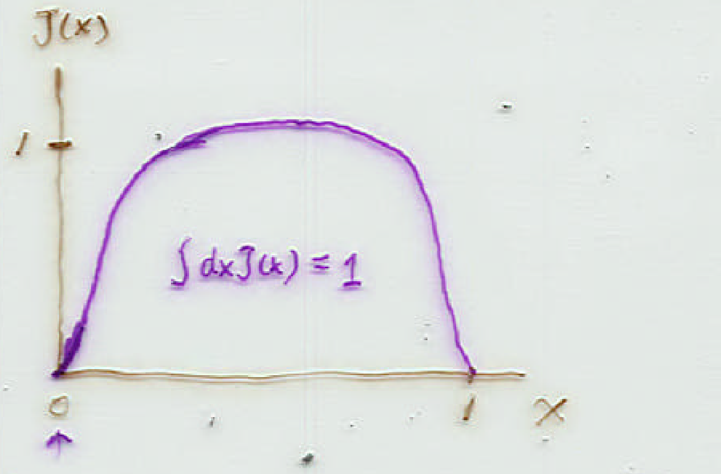
(c) $\Rightarrow -\frac{\alpha_s}{4\pi} C_F \ln^2 x_2 \Rightarrow J(x_2)$

$k_1 \to 0$, on-shell light quark

(f) $\Rightarrow -\frac{\alpha_s}{4\pi} C_F \ln^2 x_1 \Rightarrow J(x_1)$

threshold resummation is universal.

FIG. 2



decrease faster than any power of x

physical picture of Sudakov suppression

consider pion as a color dipole. As a dipole is scattered, it must radiate. An exclusive process without radiation, is then suppressed. A larger b corresponds to a larger dipole $\Rightarrow$ stronger scattering $\Rightarrow$ stronger radiation $\Rightarrow$ stronger suppression.

A larger Q corresponds to a harder scattering $\Rightarrow$ stronger radiation $\Rightarrow$ stronger suppression.

small b is preferred $\Rightarrow$ short distance $\Rightarrow$ more PQCD

A realistic calculation of exclusive processes must consider this mechanism.

Including parton transverse degrees of freedom and Sudakov suppression, PQCD is applicable down to $Q \sim 2\text{-}3$ GeV.

$\langle k_T^2 \rangle \sim \bar{\Lambda} M_B$ in B decays numerically

threshold resummation can be understood in a similar way. just think of a color dipole in the longitudinal direction.

$x \to 0 \Rightarrow$ soft parton $\Rightarrow$ large dipole

Charmless B-meson Nonleptonic decays :

Ⓐ Tree-dominated B-decays ($b \to u$)

$B \to \pi^+\pi^-, \rho^0\pi^\pm, \omega\pi^\pm, \rho^\pm\pi^\mp, \pi^0\pi^0$

Ⓑ Penguin-dominated B-decays ($b \to d, s$)

$B \to K^+\pi^-, K^+\rho^-, K^{*+}\pi^-, K^{*+}\rho^-, \eta' K, \phi K, \phi K^*$

Present Hot issue

① $\phi_3 (\gamma)$? Big contradiction !!

CLEO data with FA approach : $\phi_3(\gamma) > 90^\circ$

Hou et al : $\phi_3 \sim 114^\circ$

H.Y. Cheng : $\phi_3 \sim 110^\circ$

BBNS and Muta et al : $\phi_3 \sim 128^\circ$

However the model independent analysis in ($\rho - \eta$) plan give us (including B_s mixing)

$$43^\circ < \phi_3 < 85^\circ$$

⇒ Factorization Approximation breaks down ?

or

New physics ?

② Rate $\dfrac{\Gamma(B \to K^+\pi^-)}{\Gamma(B \to \pi^+\pi^-)} \sim 4.0$

③ $Br(B \to K^0\pi^0) = \left(14.6^{+5.9}_{-5.1}{}^{+2.4}_{-3.3}\right) \times 10^{-6}$

Theoretical prediction

in Factorization Approach : 6.0×10^{-6}

$Br(B \to K^{*-}\pi^+) = \left(22^{+8}_{-6}{}^{+4}_{-5}\right) \times 10^{-6}$

Theoretical prediction : 5.5×10^{-6}

④ $B \to \eta' K$ Puzzle : the largest BR in two-body charmless B-decays :

$Br(B^\pm \to \eta' K^\pm) = \left(80^{+10}_{-9} \pm 8\right) \times 10^{-6}$

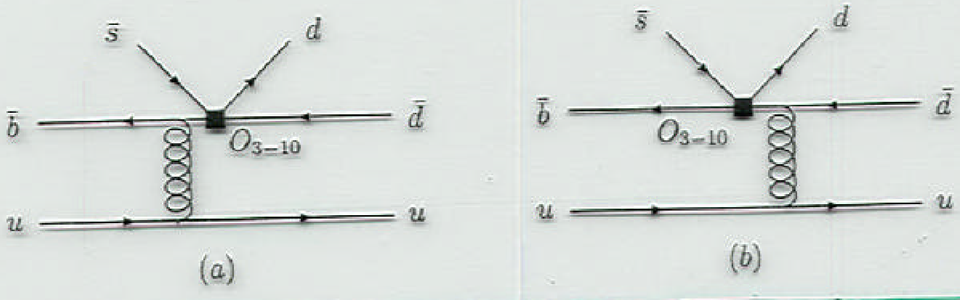
$Br(B^0 \to \eta' K^0) = \left(88^{+18}_{-16} \pm 9\right) \times 10^{-6}$

Without QCD-anomaly

Theoretical prediction : $(1-2) \times 10^{-5}$

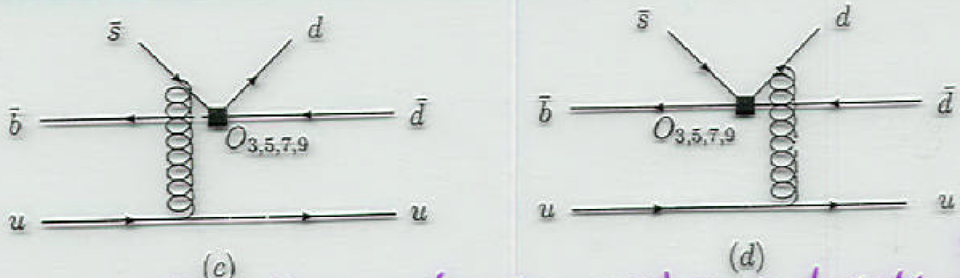
power counting

$B^{\pm} \to K^0\pi^{\pm}$ (hard parts)



factorization approach

$\sim \frac{1}{\bar{\Lambda} M_B}$

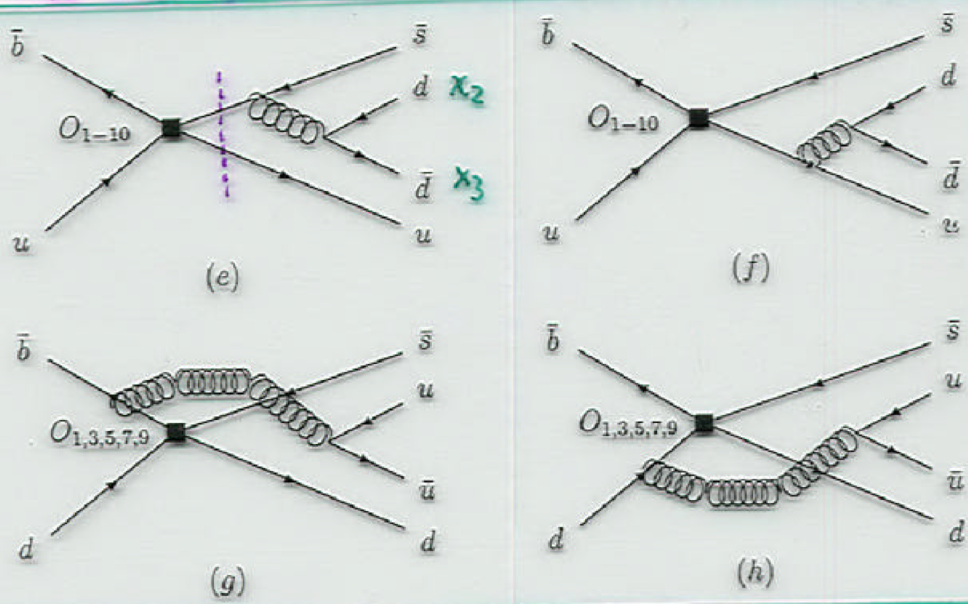


nonfactorizable

$\sim \frac{1}{\bar{\Lambda} M_B}$

sum $\sim 1/M_B^2$

suppressed by 3 wave function overlap and soft cancellation



factorizable annihilation

Re $\sim \frac{1}{M_B^2}$

Im $\sim \frac{1}{\bar{\Lambda} M_B}$

nonfactorizable annihilation

↑ PQCD

Figure 2: *Feynman diagrams for $B^{\pm} \to \bar{K}^0\pi^{\pm}$ decay*

$$\mathrm{Im} \propto \mathrm{Im}\frac{1}{x_3 M_B^2 - k_T^2} \Rightarrow x_3 \sim \frac{\bar{\Lambda}}{M_B} \qquad \frac{1}{x_2 x_3 M_B^2} \sim \frac{1}{\bar{\Lambda} M_B} \gg \mathrm{Re} \sim \frac{1}{M_B^2}$$

↑ $\langle k_T^2 \rangle \sim \bar{\Lambda} M_B$

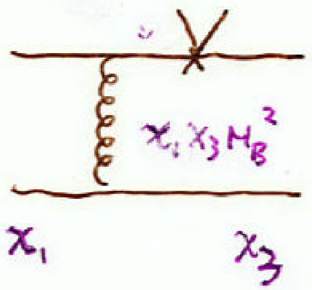
annihilation contributions are mainly imaginary

m_0 contributions

$$\frac{m_0}{M_B}\int_{x_{min}}^{1}\frac{dx}{x^2} \sim \frac{m_0}{\bar{\Lambda}}$$ not suppressed by $\frac{1}{M_B}$

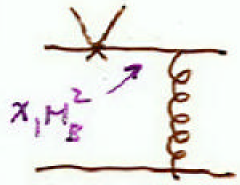
if $x_{min} \sim \frac{\bar{\Lambda}}{M_B}$

penguin enhancement $\Leftrightarrow$ small quark mass in FA



$$x_1 \sim \frac{\bar{\Lambda}}{M_B} = \frac{M_B - m_b}{M_B}$$

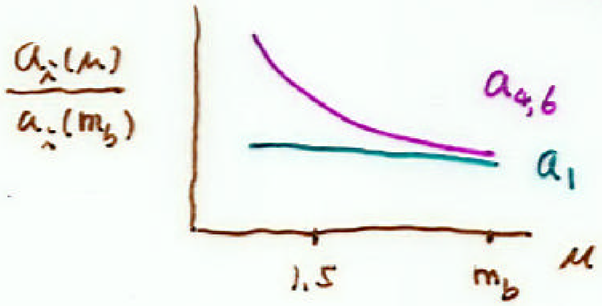
$$x_3 \sim 1$$



hard scale $\langle t \rangle \sim \sqrt{M_B \bar{\Lambda}} = 1.5$ GeV $\Leftrightarrow \langle \frac{\alpha_s}{\pi} \rangle \sim 0.13$

$< m_b$

should be determined by diminishing higher order corr.

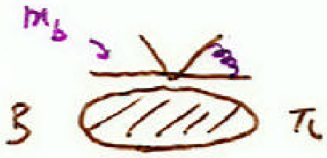


penguin enhanced by 50 %

$$\frac{F_e^{B\pi}|_{t = x_1 x_3 M_B^2}}{F_e^{B\pi}|_{t = m_b}} \sim 1 \qquad \frac{F_P^{B\pi}|_{t = x_1 x_3 M_B^2}}{F_P^{B\pi}|_{t = m_b}} \sim 1.5$$

also give large $B \to \phi K$, ...

in FA (or BBNS)



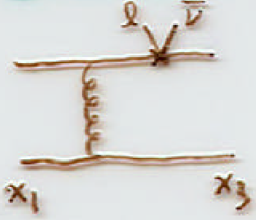
higher order corr. to the vertex

$\Rightarrow \mu = m_b$

form factor is soft

end-point singularity

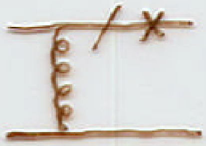
$B \to \pi$ form factor



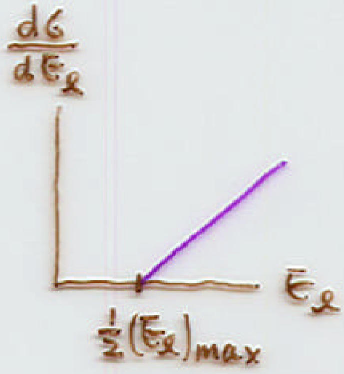
$\propto \dfrac{\phi_B(x_1)\,\phi_\pi(x_3)}{x_1\, x_3^2\, M_B^2} \Rightarrow \begin{cases} \text{log div. if } \phi_\pi \propto x(1-x) \ \text{(twist 2)} \\ \text{linear div if } \phi_\pi \propto 1 \ \text{(twist 3)} \end{cases}$

how to deal with it ?

1) subtraction of a on-shell b quark propagator



$\Rightarrow$



$F^{B\pi}(0) \ll 0.3$

Akhoury, Sterman, Yao (94)

2) cutoff of x $\qquad \displaystyle\int_{\bar{\Lambda}/M_B}^{1} dx_3 \frac{\phi_\pi(x_3)}{x_3^2} \Rightarrow$ large log correction

sensitivity to cutoff

Brodsky et al. (91) Beneke et al. (00)

3) inclusion of k_T $\qquad \dfrac{1}{x_1 x_3^2 M_B^2} \to \dfrac{1}{x_1 x_3^2 M_B^2 + k_T^2} \sim \dfrac{1}{\bar{\Lambda} M_B}$

a reliable QCD analysis

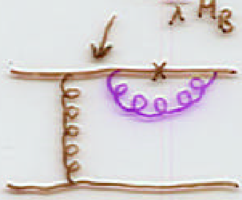
large log corrections $\alpha_s \ln^2 k_T$ are resummed into a Sudakov form factor

Li, Yu (95)

4) threshold resummation

as $x_3 \sim \frac{\bar{\Lambda}}{M_B}$, additional soft div. appear

$\propto \frac{1}{\bar{\Lambda} M_B}$ as $x_3 \sim \frac{\bar{\Lambda}}{M_B}$



$\Rightarrow -\frac{\alpha_s}{4\pi} C_F \ln^2 x_3$

threshold resummation vanishes as $x_3 \to 0$

Li (0102013)

CP Asymmetry of $B^0 \to K^{\pm}\pi^{\mp}$

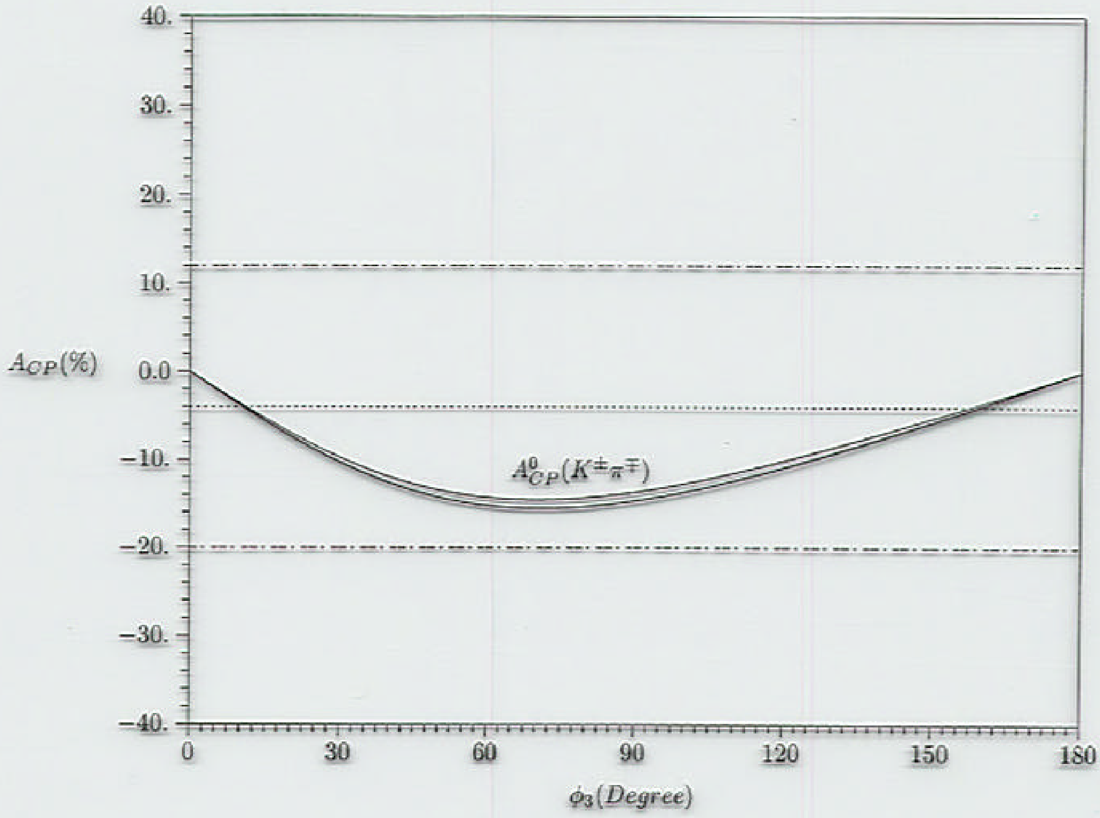


Figure 7

large CP due to large imaginary annihilation

Our predictions for the branching ratio of each mode corresponding to $\phi_3 = 80^o$,

$$
\begin{aligned}
\mathrm{Br}(B^\pm \to K^0\pi^\pm) &= (17.0^{+3.3}_{-2.7}) \times 10^{-6} \ , \\
\mathrm{Br}(B_d^0 \to K^\pm\pi^\mp) &= (16.5^{+3.2}_{-2.5}) \times 10^{-6} \ , \\
\mathrm{Br}(B^\pm \to K^\pm\pi^0) &= (9.0^{+1.9}_{-1.5}) \times 10^{-6} \ , \\
\mathrm{Br}(B_d^0 \to K^0\pi^0) &= (10.6^{+0.3}_{-0.3}) \times 10^{-6} \ . \qquad (71)
\end{aligned}
$$

$\mathrm{Br}(B_d^0 \to \pi^\pm\pi^\mp) = (8.2^{+2.0}_{-1.5}) \times 10^{-6}$

are consistent with the CLEO data [16],

$$
\begin{aligned}
\mathrm{Br}(B^\pm \to K^0\pi^\pm) &= (18.2^{+4.6}_{-4.0} \pm 1.6) \times 10^{-6} \ , \\
\mathrm{Br}(B_d^0 \to K^\pm\pi^\mp) &= (17.2^{+2.5}_{-2.4} \pm 1.2) \times 10^{-6} \ , \\
\mathrm{Br}(B^\pm \to K^\pm\pi^0) &= (11.6^{+3.0+1.4}_{-2.7-1.3}) \times 10^{-6} \ , \\
\mathrm{Br}(B_d^0 \to K^0\pi^0) &= (14.6^{+5.9+2.4}_{-5.1-3.3}) \times 10^{-6} \ . \qquad (72)
\end{aligned}
$$

E.W. penguins enhance $K\pi^0$ by $5 \sim 10\%$ at the amplitude level.

Summary

- factorization theorem for B decays is proved to twist-2 (2 parton)
- end-point (log and linear) singularities are smeared by k_T and threshold resummations
- m_0 contributions are not negligible $\propto \frac{m_0}{M_B}$
- hard parts (Im Anni.) scale like $\frac{1}{\bar{\Lambda} M_B}$
  - $\Leftrightarrow$ gauge invariance of factorization
  - $\Leftrightarrow$ penguin enhancement
  - $\Leftrightarrow$ large $\not{CP}$
- predictions are consistent with data

  see Keum's talk

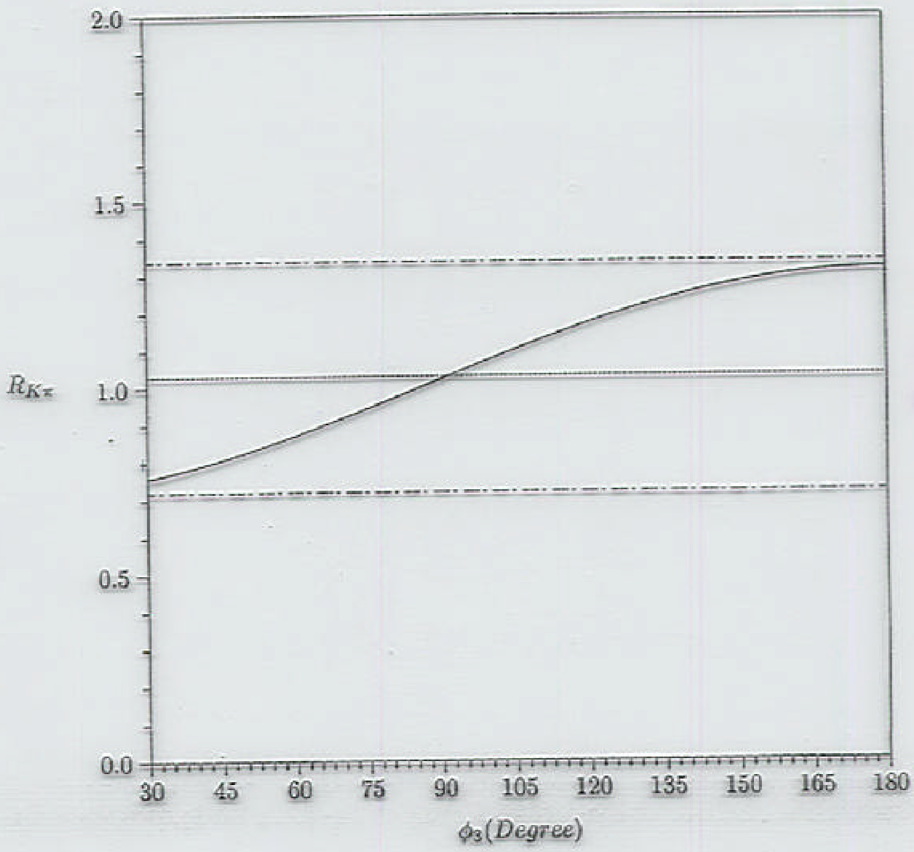
posters	Sanda	$B \to \pi \ell \nu$
	Kurimoto	$B \to D_s \pi$
	Chen	$B \to (\omega, \rho) K$
	Ukai	$B \to D_s K$
	Keum	$B \to \phi K$
	Kuo	$B \to \eta' K$
	Morozumi	$B \to D^* \gamma$; $K^* \ell^+ \ell^-$
	Sinha	$B \to VV$

Keum, Li, Sanda

$\phi_3 \sim 80^\circ$ from data $R \sim 0.95$

$R = \frac{K\pi}{K^0\pi} \leq 1$

insensitive to all parameters



↑ data uncertainty ↓

Figure 6

need more precise data

CP asymmetry in $B^0 \to \pi^+\pi^-$.