Width difference of B_s mesons from lattice QCD

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@ BCP4 (International Conference on B physics and CP violation)
February 19–23, 2001

Introduction

Width difference $\Delta\Gamma_s$ in the $B_s - \bar{B}_s$ system:

opens a new possibility to search for new physics beyond the SM, once it is measured and found to be sizable.

Theoretical prediction for $\Delta\Gamma_s$ may be obtained using the Heavy Quark Expansion (HQE) under an assumption of the quark-hadron duality.

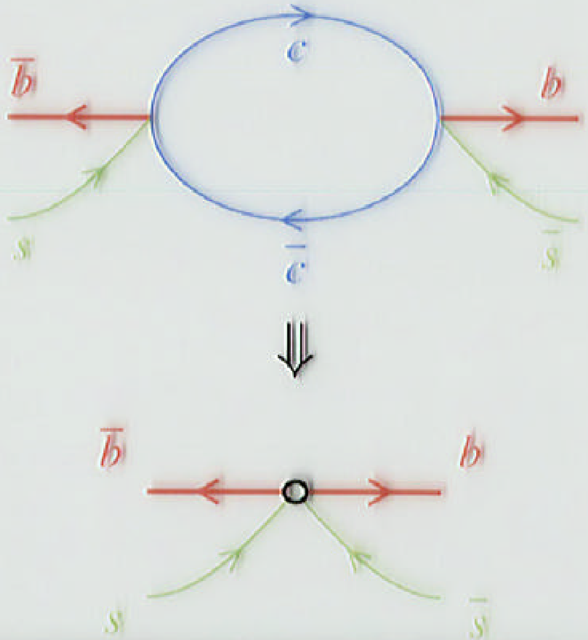
Nonperturbative inputs are necessary for B_B and B_S.

⇒ *Lattice QCD* calculation may provide them.

Heavy Quark Expansion

Beneke, Buchalla, Dunietz, *Phys. Rev.* **D54** (1996) 4419.
Beneke, Buchalla, Greub, Lenz, Nierste, *Phys. Lett.* **B459** (1999) 631.

$$\Delta\Gamma_{B_s} = \frac{G_F^2 m_b^2}{12\pi M_{B_s}} |V_{cb}^* V_{cs}|^2 \times \left[c_L(z)\langle \mathcal{O}_L \rangle + c_S(z)\langle \mathcal{O}_S \rangle + c_{1/m}(z)\delta_{1/m} \right]$$



$c_L(z)$, $c_S(z)$ and $c_{1/m}(z)$ are known functions of $z = m_c^2/m_b^2$.

Normalizing with the total width Γ_{B_s}, one arrives at

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \frac{16\pi^2 B(B_s \to Xe\nu)}{g(z)\tilde{\eta}_{QCD}} \frac{f_{B_s}{}^2 M_{B_s}}{m_b^3} |V_{cs}|^2 \times \left[G(z)\frac{8}{3}B_B(m_b) + G_S(z)\frac{5}{3}\frac{B_S(m_b)}{\mathcal{R}(m_b)^2} + \sqrt{1-4z}\,\delta_{1/m}\right]$$

$$= \left(\frac{f_{B_s}}{230\,\text{MeV}}\right)^2 \left[0.007\, B_B(m_b) + 0.132\,\frac{B_S(m_b)}{\mathcal{R}(m_b)^2} - 0.078\right],$$

where

$$\mathcal{R}(m_b) \equiv \frac{\bar{m}_b(m_b) + \bar{m}_s(m_b)}{M_{B_s}} = 0.81(3).$$

Note: After we published a paper, APE group used the notation $\mathcal{R}(m_b)$ for a different quantity. $\mathcal{R}$ is always a 'ratio', but it can easily introduce confusions.

Nonperturbative inputs

- decay constant f_{B_s}

There are many lattice calculations in the quenched approximation. Recently, simulations have also been performed with two-flavors of dynamical quarks:

$$f_{B_s} = \begin{cases} 215(3)(28)(^{+49}_{-5})\ \text{MeV} & \text{Collins } et\ al.,\ \text{Phys. Rev. } \mathbf{D60},\ 074504\ (1999). \\ 217(5)(^{+33}_{-29})(^{+9}_{-0})\ \text{MeV} & \text{MILC, at Lattice 2000, hep-lat/0011029}. \\ 250(10)(13)(^{+8}_{-0})\ \text{MeV} & \text{CP-PACS (relativistic), hep-lat/0010009}. \\ 242(9)(15)(^{+18}_{-0})\ \text{MeV} & \text{CP-PACS (NRQCD), in preparation}. \end{cases}$$

which is 10–20% higher than the quenched results $f_{B_s} \simeq 200$ MeV.
We use a "world average" $f_{B_s} = 230(30)$ MeV in the following analysis.

For a new (preliminary) result from the *JLQCD* collaboration, see a poster by N. Yamada.

- B-parameters B_L and B_S

Matrix elements $\langle \bar{B}_s \| \mathcal{O}_L \| B_s \rangle$ and $\langle \bar{B}_s \| \mathcal{O}_S \| B_s \rangle$ of four-quark operators

$$\mathcal{O}_L = \bar{b}\gamma_\mu(1-\gamma_5)s\,\bar{b}\gamma_\mu(1-\gamma_5)s,$$
$$\mathcal{O}_S = \bar{b}(1-\gamma_5)s\,\bar{b}(1-\gamma_5)s,$$

are parametrized in terms of B-parameters:

$$B_B(\mu) = \frac{\langle \bar{B}_s \| \mathcal{O}_L(\mu) \| B_s \rangle}{\frac{8}{3} f_{B_s}^2 M_{B_s}^2},$$
$$B_S(\mu) = \frac{\langle \bar{B}_s \| \mathcal{O}_S(\mu) \| B_s \rangle}{-\frac{5}{3} f_{B_s}^2 M_{B_s}^2} \times \mathcal{R}(\mu)^2.$$

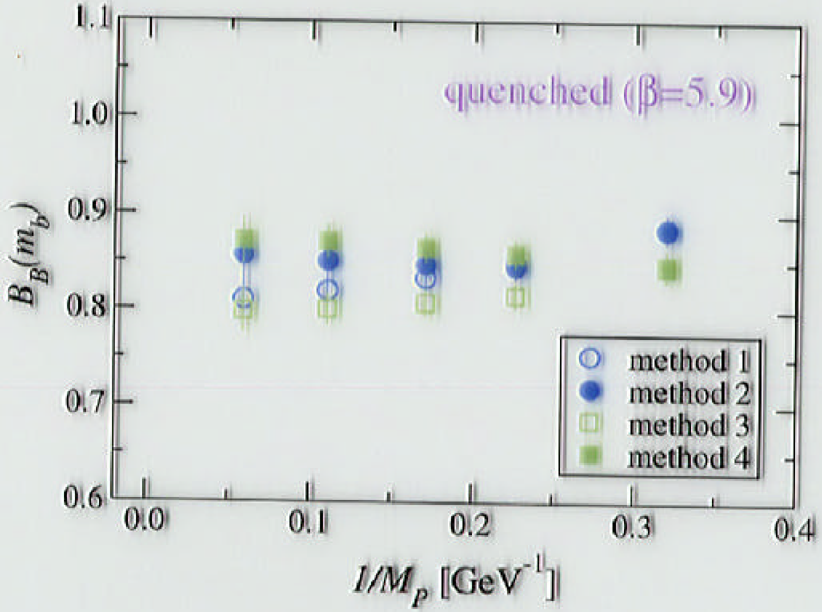
⇒ We calculate them on the lattice using the lattice NRQCD including the effect of dynamical quarks.

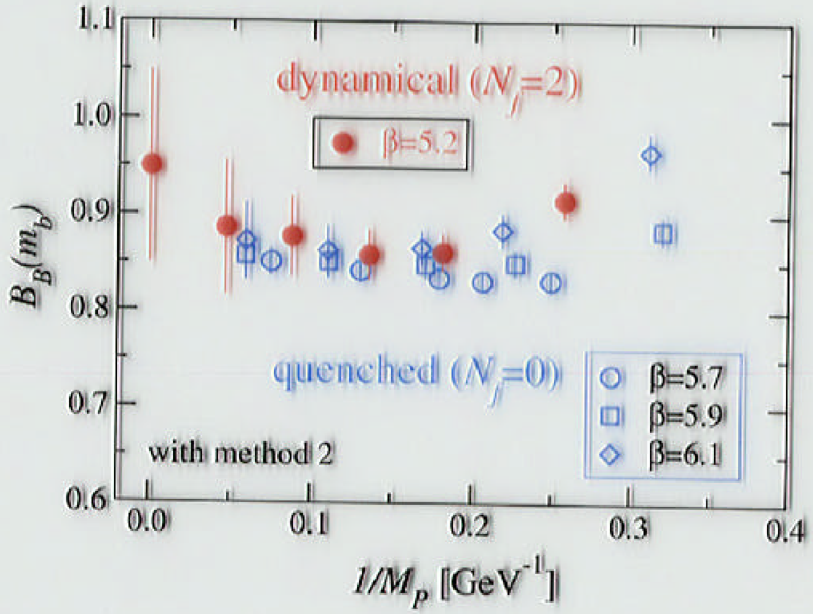
Lattice calculation

See also a poster by N. Yamada for details.

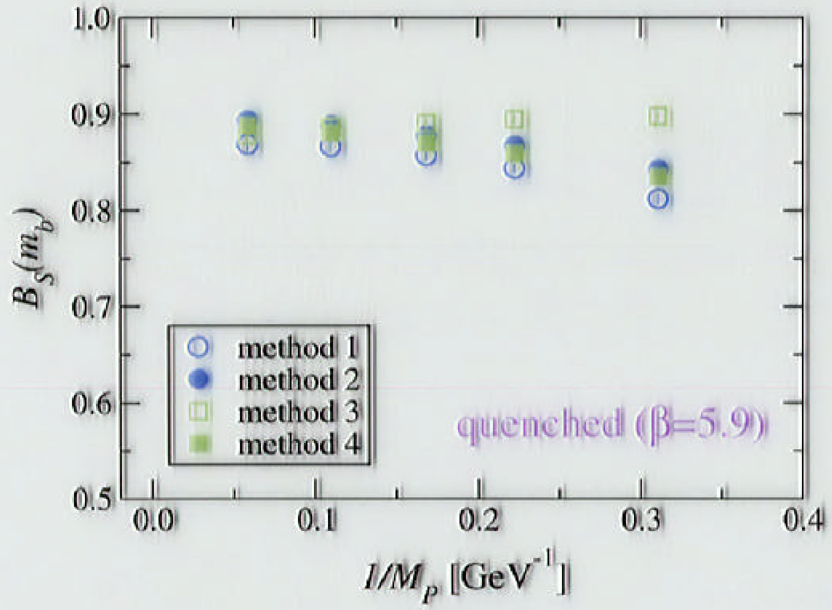
- NRQCD is used to simulate b quark
  - ⋆ including all $1/M$ corrections consistently.
  - ⋆ no extrapolation in the heavy quark mass is necessary.
- systematic study on quenched (N_F=0) lattices
  - ⋆ simulations on three lattice spacings
  - ⋆ four different methods having different systematic errors
    $\Longrightarrow$ estimation of the discretization and other systematic errors
- preliminary result from a unquenched (N_F=2) simulation NEW
  - ⋆ JLQCD's new project:
    two-flavor QCD with a nonperturbatively improved quark.
    at β=5.2, c_{sw}=2.02 ($a \simeq 0.1$ fm), on a $20^3 \times 48$ lattice.

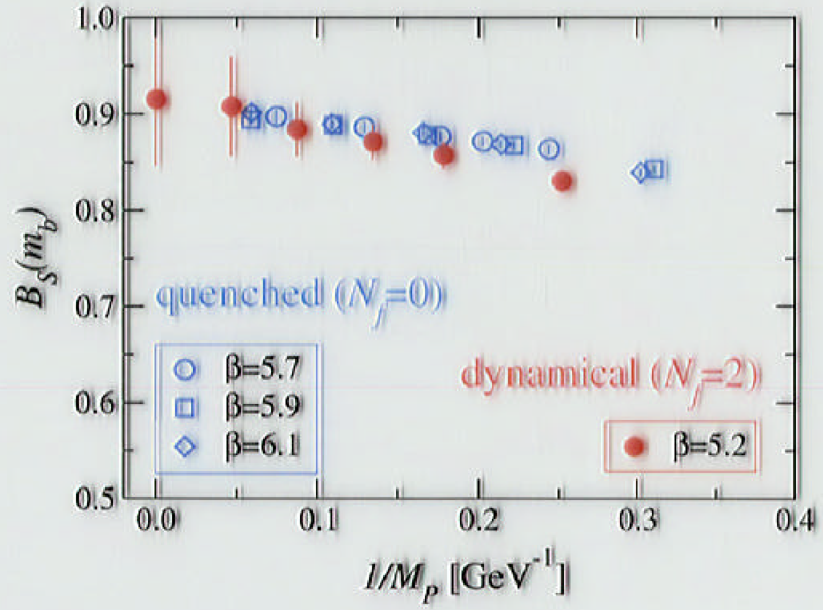
B_B


quenched (β=5.9)
method 1
method 2
method 3
method 4
$B_B(m_b)$
$1/M_P$ [GeV^{-1}]


dynamical (N_f=2)
β=5.2
quenched (N_f=0)
β=5.7
β=5.9
β=6.1
with method 2
$B_B(m_b)$
$1/M_P$ [GeV^{-1}]

B_S


1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.1
0.2
0.3
0.4
$B_S(m_b)$
1/M_P [GeV^{-1}]
method 1
method 2
method 3
method 4
quenched (β=5.9)


1.0
0.9
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0.1
0.2
0.3
0.4
$B_S(m_b)$
1/M_P [GeV^{-1}]
quenched (N_f=0)
β=5.7
β=5.9
β=6.1
dynamical (N_f=2)
β=5.2

Results (preliminary)

- B-parameters

$$B_B(m_b) = \begin{cases} 0.85(2)(8) \\ 0.83(3)(8) \end{cases}, \quad B_S(m_b) = \begin{cases} 0.87(1)(9) & (N_f = 0) \\ 0.84(6)(8) & (N_f = 2) \end{cases}$$

- width difference

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{230\,\text{MeV}}\right)^2 [0.007\, B_B(m_b) + 0.201\, B_S(m_b) - 0.078],$$

$$= \boxed{\underset{\mu}{0.097\,{}^{+0.014}_{-0.035}} \pm \underset{f_{B_s}}{0.025} \pm \underset{B_S}{0.020} \pm \underset{1/m}{0.016}}$$

⋆ smaller than our previous estimate 0.119(29)(32)(17).
⇐ mainly due to the smaller f_{B_s} (previously 245(30) MeV).
⇐ effect of unquenching is less significant (−6%).

Normalizing with ΔM_s

Once ΔM_s is measured, the uncertainty in f_{B_s} can be avoided by considering

$$\begin{aligned}\left(\frac{\Delta\Gamma}{\Delta M}\right)_{B_s} &= \frac{\pi}{2}\frac{m_b^2}{M_W^2}\left|\frac{V_{cb}^*V_{cs}}{V_{tb}^*V_{ts}}\right|^2 \frac{1}{\eta_B(m_b)S_0(x_t)} \\ &\times \left[\frac{8}{3}G(z)+\frac{5}{3}G_S(z)\frac{B_S(m_b)}{B_B(m_b)}\frac{1}{\mathcal{R}(m_b)^2}+\frac{\sqrt{1-4z}\,\delta_{1/m}}{B_B(m_b)}\right], \\ &= \left(0.20+6.00\frac{B_S(m_b)}{B_B(m_b)}-2.85\right)\times 10^{-3} \\ &= \boxed{(3.5\ {}^{+0.4}_{-1.3}\ \pm\ 0.6\ \pm\ 0.6\ )\times 10^{-3}}\end{aligned}$$

$$\qquad\qquad \mu \qquad B_S/B_B \qquad 1/m$$

The $1/m$ correction is estimated using the factorization approximation as in Beneke-Buchalla-Dunietz(96).

Comparison with other approaches

1. HQET (b quark is infinitely heavy.)

   Giménez-Reyes, hep-lat/0009007, hep-lat/0010048.

   ⋆ Smoothly approached from NRQCD by taking a limit $m_Q \to \infty$.

   ⋆ An unquenched calculation is available (but with an unimproved action).

2. relativistic

   APE collaboration (Becirevic *et al.*), *Eur. Phys. J.* **C18** (2000) 157.

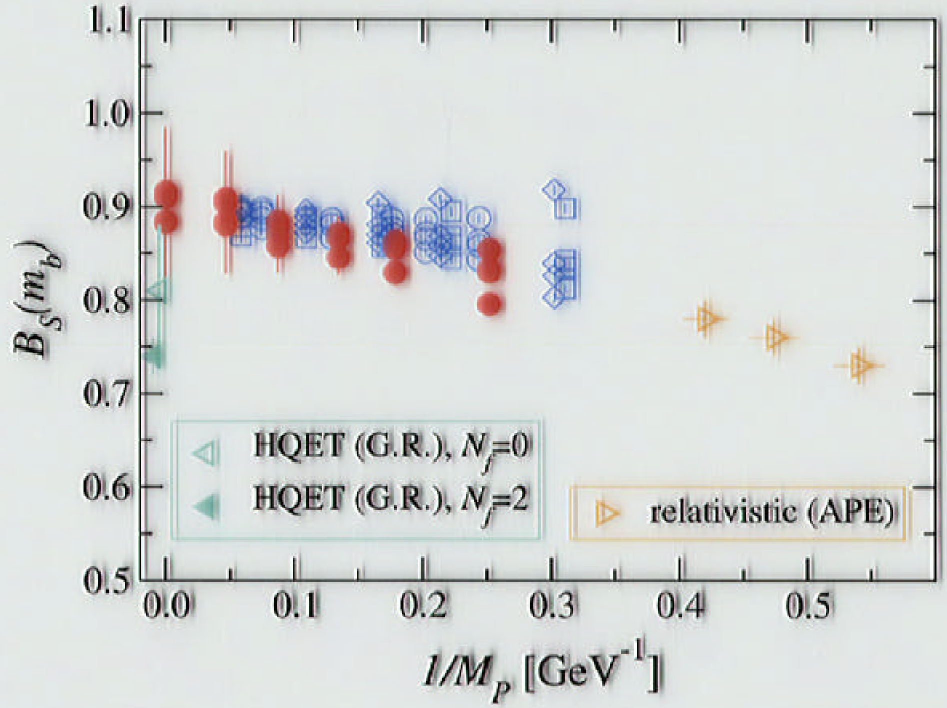
   UKQCD collaboration (Flynn and Lin), hep-ph/0012154.

   ⋆ (nonperturbatively improved) relativistic lattice action for heavy quark.

   ⋆ Extrapolation is attempted from charm quark mass region.

   ⇒ charm is so heavy. → $O((am_Q)^2)$ error becomes important.
   charm is so light. → Is the heavy quark expansion justified?

Comparison with other approaches (cont.)



Normalizing with ΔM_d (APE)

The APE collaboration used ΔM_d to normalize $\Delta\Gamma_{B_s}$,

Becirevic *et al.*, Eur. Phys. J. **C18** (2000) 157.

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = K\left(\tau_{B_s}\Delta M_d \frac{M_{B_s}}{M_{B_d}}\right)^{(\text{exp.})}\left(\frac{f_{B_s}^2 \hat{B}_{B_s}}{f_{B_s}^2 \hat{B}_{B_d}}\right)^{(\text{latt.})}\left|\frac{V_{ts}}{V_{td}}\right|^2$$

$$\times\left[G(z) + G_S(z)\frac{5}{8}\frac{B_S(m_b)}{B_B(m_b)}\frac{1}{\mathcal{R}(m_b)^2} + \frac{3}{8}\frac{\sqrt{1-4z}\,\delta_{1/m}}{B_B(m_b)}\right],$$

$$= \boxed{0.047 \pm \underbrace{0.015}_{B_S/B_B} \pm \underbrace{0.016}_{1/m}}$$

- ⋆ The value of $|V_{td}/V_{ts}|$ is taken from a global fit of the CKM elements.
  - ⇒ A smaller value of f_{B_s} is implicitly assumed.
- ⋆ Stronger cancelation of the leading term and the $1/m$ correction ?

Requirements for further improvement

- better determination of f_{B_s}
  ⋆ Agreement among different approaches is not yet reached for the unquenched calculations.

- reducing the systematic error in $B_S(m_b)$
  ⋆ The B parameters are numerically more stable than the decay constant, but cross check is necessary.

- calculation of the $1/m$ corrections MOST IMPORTANT
  ⋆ The $1/m$ corrections are written in terms of matrix elements of higher order operators.
  ⋆ The present estimate (as of Beneke-Buchalla-Dunietz) uses the factorization approximation.
  ⇒ Lattice QCD may calculate them nonperturbatively (at least, in principle).

Conclusions

- The JLQCD collaboration started a lattice calculation of B_B and B_S including the effect of sea quarks. A preliminary result indicates that the quenching effect is not substantial for these quantities.
- Using the lattice NRQCD, the systematic errors are under control. Simulations with several different methods on three lattice spacings show a reasonable agreement.
- The results for $(\Delta\Gamma/\Gamma)_{B_s}$ still have large uncertainty. Better estimation of the $1/m$ corrections will be necessary to improve the accuracy.