Width difference of $B_s$ mesons from lattice QCD

Shoji Hashimoto (KEK)
shoji.hashimoto@kek.jp

for the JLQCD collaboration

© BCP4 (International Conference on B physics and CP violation)
February 19–23, 2001
Introduction

Width difference $\Delta \Gamma_s$ in the $B_s - \bar{B}_s$ system:

opens a new possibility to search for new physics beyond the SM, once it is measured and found to be sizable.

Theoretical prediction for $\Delta \Gamma_s$ may be obtained using the Heavy Quark Expansion (HQE) under an assumption of the quark-hadron duality.

Nonperturbative inputs are necessary for $B_B$ and $B_S$.

⇒ *Lattice QCD* calculation may provide them.
Heavy Quark Expansion


$$\Delta \Gamma_{B_s} = \frac{G_F^2 m_b^2}{12\pi M_{B_s}} |V_{cb}^* V_{cs}|^2 \times \left[ c_L(z) \langle O_L \rangle + c_S(z) \langle O_S \rangle + c_{1/m}(z) \delta_{1/m} \right]$$

$c_L(z)$, $c_S(z)$ and $c_{1/m}(z)$ are known functions of $z = m_c^2/m_b^2$. 
Normalizing with the total width $\Gamma_{B_s}$, one arrives at

$$\left( \frac{\Delta \Gamma}{\Gamma} \right)_{B_s} = \frac{16\pi^2 B(B_s \to Xe\nu) f_{B_s}^2 M_{B_s}}{g(z)\bar{n}_{QCD} m_b^2} |V_{cs}|^2$$

$$\times \left[ G(z) \frac{8}{3} B_B(m_b) + G_S(z) \frac{5}{3} B_S(m_b) \right] + \sqrt{1 - 4z \delta_{1/m}}$$

$$= \left( \frac{f_{B_s}}{230 \text{ MeV}} \right)^2 \left[ 0.007 B_B(m_b) + 0.132 \frac{B_S(m_b)}{R(m_b)^2} - 0.078 \right],$$

where

$$R(m_b) \equiv \frac{\bar{m}_b(m_b) + \bar{m}_s(m_b)}{M_{B_s}} = 0.81(3).$$

**Note:** After we published a paper, APE group used the notation $R(m_b)$ for a different quantity. $R$ is always a "ratio", but it can easily introduce confusions.
Nonperturbative inputs

- decay constant $f_{B_s}$

There are many lattice calculations in the quenched approximation. Recently, simulations have also been performed with two-flavors of dynamical quarks:

$$f_{B_s} = \begin{cases} 
215(3)(28)(^{+49}_{-5}) \text{ MeV} & \text{Collins et al., Phys. Rev. D60, 074504 (1999).} \\
217(5)(^{+33}_{-29})(^{+9}_{-0}) \text{ MeV} & \text{MILC, at Lattice 2000, hep-lat/0011029.} \\
250(10)(13)(^{+3}_{-0}) \text{ MeV} & \text{CP-PACS (relativistic), hep-lat/0010009.} \\
242(9)(15)(^{+18}_{-0}) \text{ MeV} & \text{CP-PACS (NRQCD), in preparation.} 
\end{cases}$$

which is 10–20% higher than the quenched results $f_{B_s} \approx 200$ MeV.

We use a “world average” $f_{B_s} = 230(30)$ MeV in the following analysis.

For a new (preliminary) result from the JLQCD collaboration, see a poster by N. Yamada.
**B-parameters** \( B_L \) and \( B_S \)

Matrix elements \( \langle \bar{B}_s | O_L | B_s \rangle \) and \( \langle \bar{B}_s | O_S | B_s \rangle \) of four-quark operators

\[
O_L = \bar{b} \gamma_\mu (1 - \gamma_5) s \bar{b} \gamma_\mu (1 - \gamma_5) s,
\]

\[
O_S = \bar{b} (1 - \gamma_5) s \bar{b} (1 - \gamma_5) s,
\]

are parametrized in terms of \( B \)-parameters:

\[
B_B(\mu) = \frac{\langle \bar{B}_s | O_L(\mu) | B_s \rangle}{\frac{8}{3} f_{B_s}^2 M_{B_s}^2},
\]

\[
B_S(\mu) = \frac{\langle \bar{B}_s | O_S(\mu) | B_s \rangle}{-\frac{8}{3} f_{B_s}^2 M_{B_s}^2} \times \mathcal{R}(\mu)^2.
\]

⇒ We calculate them on the lattice using the lattice NRQCD including the effect of dynamical quarks.
Lattice calculation

See also a poster by N. Yamada for details.

- **NRQCD is used to simulate b quark**
  - including all $1/M$ corrections consistently.
  - no extrapolation in the heavy quark mass is necessary.

- **systematic study on quenched ($N_F=0$) lattices**
  - simulations on three lattice spacings
  - four different methods having different systematic errors
    $\implies$ estimation of the discretization and other systematic errors

- **preliminary result from a unquenched ($N_F=2$) simulation** **NEW**
  - **JLQCD’s new project:**
    two-flavor QCD with a nonperturbatively improved quark.
    at $\beta=5.2$, $\kappa_{SW}=2.02$ ($a \approx 0.1$ fm), on a $20^3 \times 48$ lattice.
Results (preliminary)

- $B$-parameters

\[
B_B(m_b) = \begin{cases} 
0.85(2)(8) \\
0.83(3)(8)
\end{cases}, \quad B_S(m_b) = \begin{cases} 
0.87(1)(9) & (N_f = 0) \\
0.84(6)(8) & (N_f = 2)
\end{cases}
\]

- width difference

\[
\left( \frac{\Delta \Gamma}{\Gamma} \right)_{B_s} = \left( \frac{f_{B_s}}{230 \text{ MeV}} \right)^2 \left[ 0.007 B_B(m_b) + 0.201 B_S(m_b) - 0.078 \right],
\]

\[
= \begin{array}{cccc}
0.007 & 0.014 & 0.025 & 0.020 & 0.016 \\
-0.035 & & & & \\
\end{array}
\]

* smaller than our previous estimate $0.119(29)(32)(17)$.  
⇒ mainly due to the smaller $f_{B_s}$ (previously $245(30)$ MeV). 
⇒ effect of unquenching is less significant ($-6\%$).
Normalizing with $\Delta M_s$

Once $\Delta M_s$ is measured, the uncertainty in $f_{B_s}$ can be avoided by considering

$$
\left( \frac{\Delta \Gamma}{\Delta M} \right)_{B_s} = \frac{\pi}{2} \frac{m_{b}^2}{M_{W}^2} \left| \frac{V_{cb}^* V_{cs}}{V_{tb}^* V_{ts}} \right|^2 \frac{1}{\eta_B(m_b) S_0(x_t)} 
\times \left[ \frac{8}{3} G(z) + \frac{5}{3} G_S(z) \frac{B_S(m_b)}{B_B(m_b)} \frac{1}{R(m_b)^2} + \frac{\sqrt{1 - 4z \delta_{1/m}}}{B_B(m_b)} \right],
$$

$$
= \left( 0.20 + 6.00 \frac{B_S(m_b)}{B_B(m_b)} - 2.85 \right) \times 10^{-3}
$$

$$
= \left[ 3.5 \pm 0.4 \pm 0.6 \right] \times 10^{-3}
$$

The $1/m$ correction is estimated using the factorization approximation as in Beneke-Buchalla-Dunietz(96).

The $1/m$ correction is estimated using the factorization approximation as in Beneke-Buchalla-Dunietz(96).
Comparison with other approaches

1. **HQET** (*b* quark is infinitely heavy.)
   - Giménez-Reyes, hep-lat/0009007, hep-lat/0010048.
   - Smoothly approached from NRQCD by taking a limit $m_Q \to \infty$.
   - An unquenched calculation is available (but with an unimproved action).

2. **relativistic**
   - UKQCD collaboration (Flynn and Lin), hep-ph/0012154.
   - (nonperturbatively improved) relativistic lattice action for heavy quark.
   - Extrapolation is attempted from charm quark mass region.

\[ \text{charm is so heavy. } \implies O((am Q)^2) \text{ error becomes important.} \]
\[ \text{charm is so light. } \implies \text{Is the heavy quark expansion justified?} \]
Comparison with other approaches (cont.)

\[ B_s(m_b) \]

\[ \frac{1}{M_p} \text{ [GeV}^{-1}] \]

- HQET (G.R.), \( N_f=0 \)
- HQET (G.R.), \( N_f=2 \)
- Relativistic (APE)
Normalizing with $\Delta M_{d}$ (APE)

The APE collaboration used $\Delta M_{d}$ to normalize $\Delta \Gamma_{B_s}$.


\[
\left( \frac{\Delta \Gamma}{\Gamma} \right)_{B_s} = K \left( \frac{\tau_{B_s} \Delta M_{d}}{M_{B_s}/M_{B_0}} \right)^{(\text{exp.})} \left( \frac{f_{B_s}^2 \hat{B}_{B_s}}{f_{B_s}^2 \hat{B}_{B_0}} \right)^{(\text{latit.})} \left\| \frac{V_{ts}}{V_{td}} \right\|^2 \\
\times \left[ G(z) + G_S(z) \frac{5}{8} \frac{B_S(m_b)}{B_B(m_b)} \frac{1}{\mathcal{R}(m_b)^2} + \frac{3 \sqrt{1 - 4z}}{8} \delta_{1/m} \right],
\]

= \boxed{0.047 \pm 0.015 \pm 0.016}

\frac{B_s/B_B}{1/m}

* The value of $|V_{td}/V_{ts}|$ is taken from a global fit of the CKM elements.
  ⇒ A smaller value of $f_{B_s}$ is implicitly assumed.

* Stronger cancelation of the leading term and the $1/m$ correction?
Requirements for further improvement

- better determination of $f_{B_s}$
  $\star$ Agreement among different approaches is not yet reached for the unquenched calculations.

- reducing the systematic error in $B_S(m_b)$
  $\star$ The $B$ parameters are numerically more stable than the decay constant, but cross check is necessary.

- calculation of the $1/m$ corrections MOST IMPORTANT
  $\star$ The $1/m$ corrections are written in terms of matrix elements of higher order operators.
  $\star$ The present estimate (as of Beneke-Buchalla-Dunietz) uses the factorization approximation.
  $\Rightarrow$ Lattice QCD may calculate them nonperturbatively (at least, in principle).
Conclusions

- The JLQCD collaboration started a lattice calculation of $B_B$ and $B_S$ including the effect of sea quarks. A preliminary result indicates that the quenching effect is not substantial for these quantities.

- Using the lattice NRQCD, the systematic errors are under control. Simulations with several different methods on three lattice spacings show a reasonable agreement.

- The results for $(\Delta \Gamma/\Gamma)_B$ still have large uncertainty. Better estimation of the $1/m$ corrections will be necessary to improve the accuracy.