Perturbative QCD Factorization

Light-Cone Wave Functions

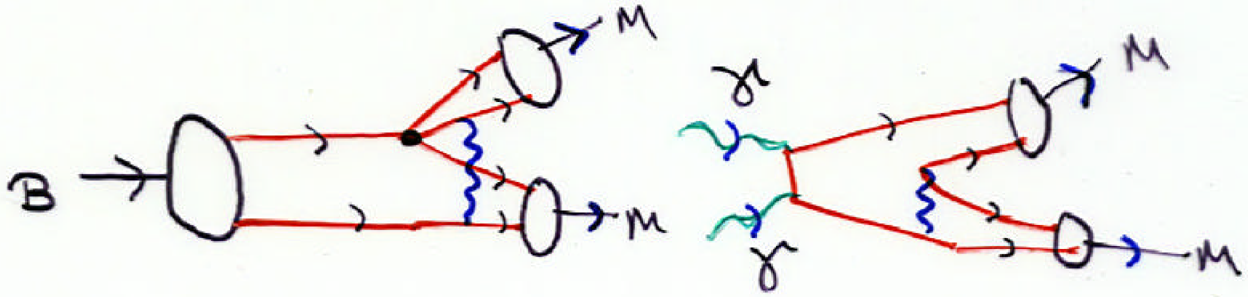
and

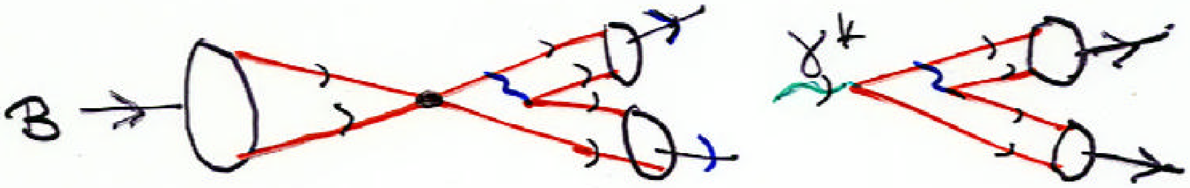
Exclusive B-Decays

S. Brodsky

SLAC

BCP4 Ise, Feb 23, 2001





$$\boxed{\text{Exclusive B-decays}}$$

$B \to M_1 M_2 \,,\ M \ell \bar{\nu} \,,\ M \gamma$

$B \to$ baryon pairs

- Core subject of B-physics, $\cancel{CP}$
- QCD Aspects crucial

Normalization

Scaling

Phases

F.S.I.

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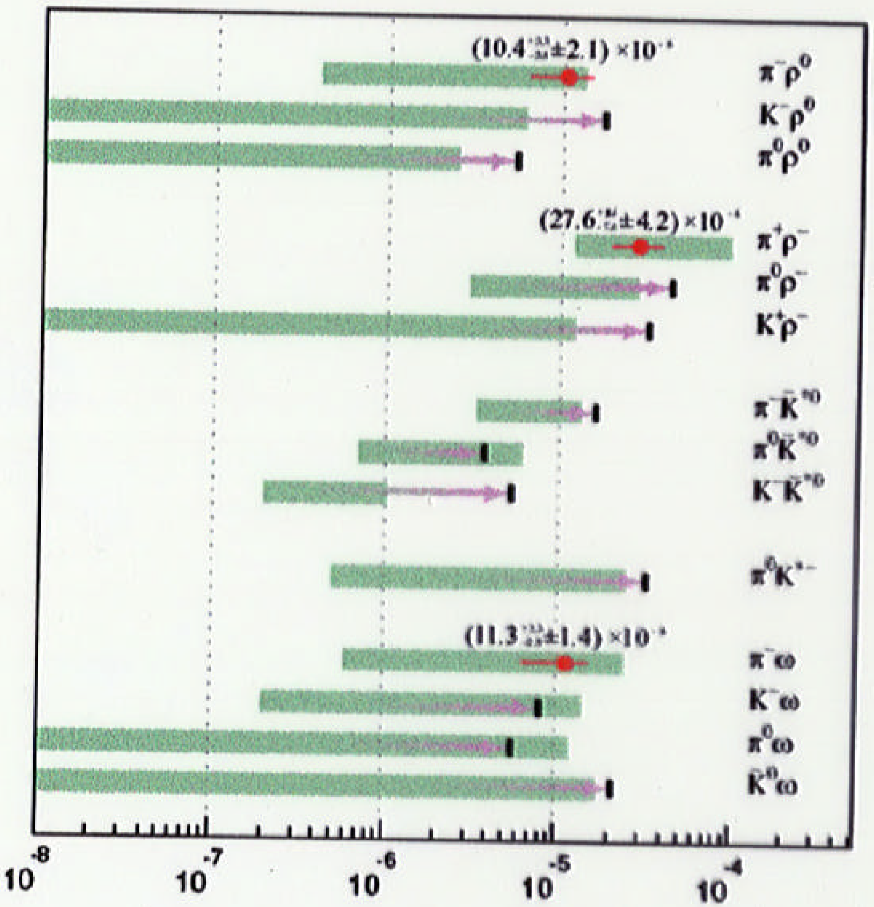
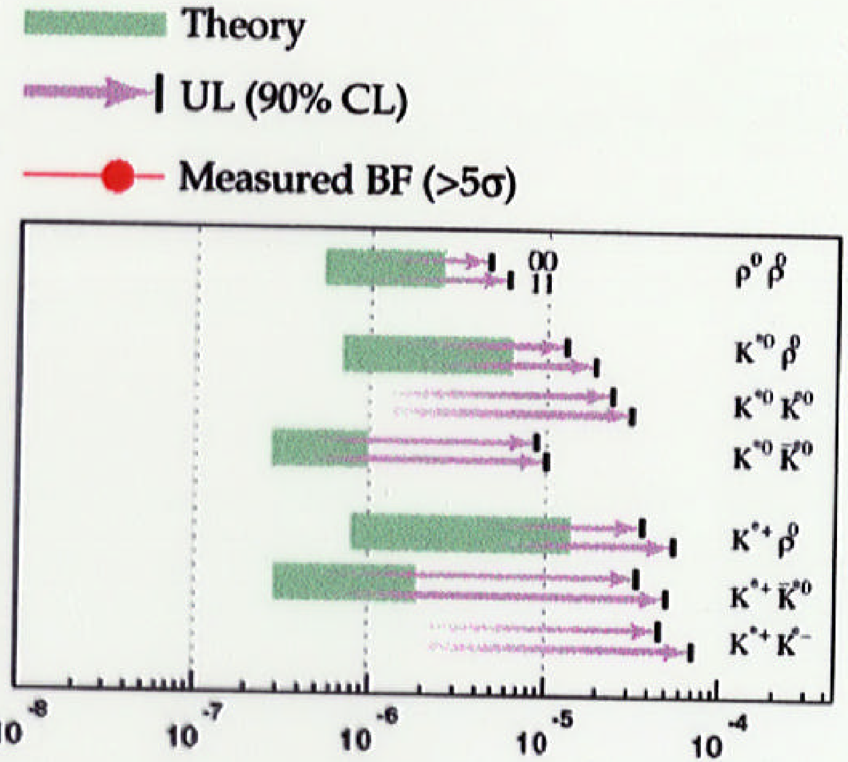
F.S.I.

$B \to PV$, $B \to VV$ modes

PV: PRL **85**, 2881 (2000)
VV: Submitted to PRL (hep-ex/0101029)





A. Lyon (CLEO) – 2001

Many poster contributions on PQCD approa

T. Morozumi $B \to D^* \gamma$, $B \to K^* l^+ l^-$

A.-I. Sanda PQCD

K.-C. Yang $B \to \phi K$

C.-H. Chen $B \to \omega K$, ρK

K. Ukai $B \to D_s K$

T.-W. Yeh Twist -3

C.-D. Lu $B \to \pi \rho$, $\pi \omega$

E. Kou $B \to K \eta'$ *

K.-C. Yang $B \to J/\psi\, K$

R. Sinha + K. Ukai $B \to VV$

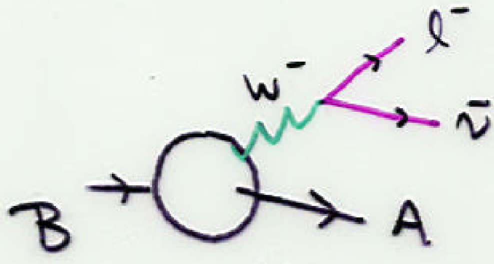
C.-W. Hwang F_π, $F_{\gamma\pi}$

Talks by

H.-N. Li, Y.-Y Keum, L. Silverstrini

SJB
+ Dae Sung Hw

Exact Formulae for Exclusive Semi-Leptonic B-decay



e.g. $B^- \to l^- \bar{\nu} D^0$

$B^- \to l^- \bar{\nu} \pi^0$

$$\mathcal{M} = \mathcal{M}^\mu L_\mu$$

Example of Off-diagonal matrix element

$$\mathcal{M}^\mu = \langle B | J^\mu(0) | A \rangle$$

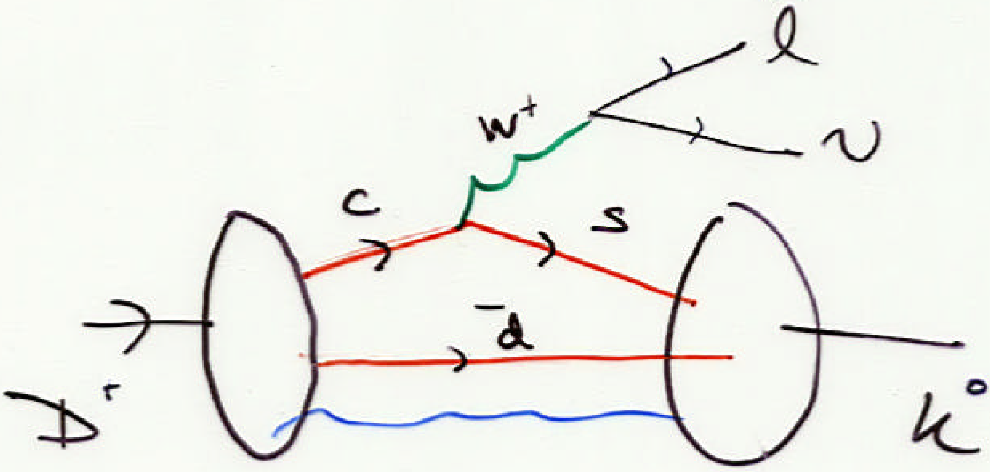
$$J^\mu(0) = j^\mu(0)$$

interaction picture

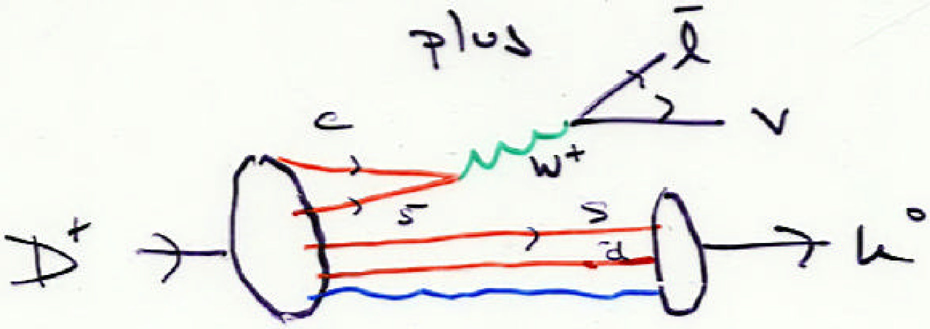
Light-Cone Wavefunctions:

B, D

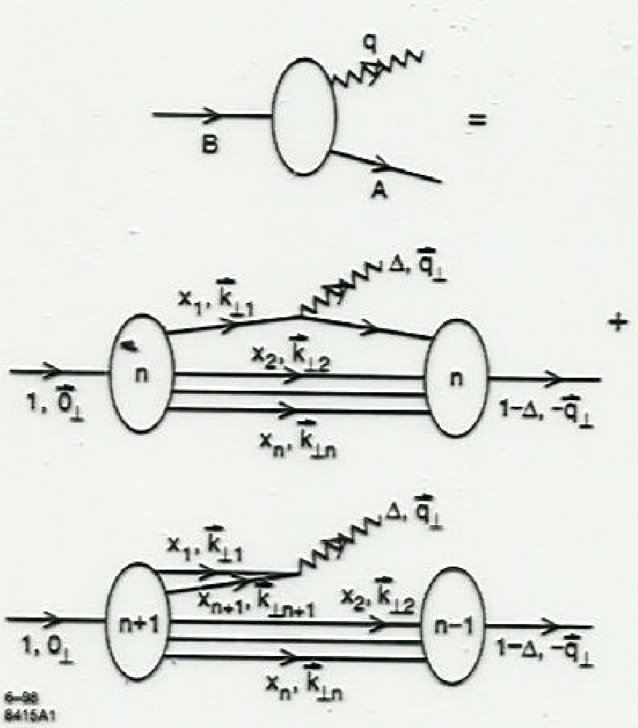
exclusive decays



plus



New phenomena in weak decays

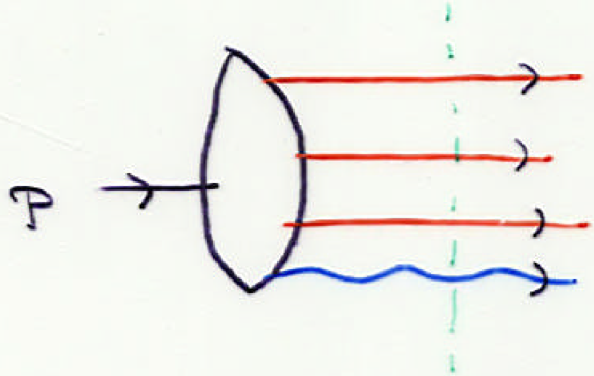


Exact representation of electroweak decays and time-like form factors in the light-cone Fock representation.

Light-Cone Wavefunctions and QCD Phenomena

Non-Perturbative QCD

$\{\Psi_n\}$: translation: hadrons $\Rightarrow$ q, g



$x_1, \vec{k}_{\perp 1}, \lambda_1$

$$x_i = \frac{k_i^+}{P^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

fixed $\tau = t + z/c$ Dirac

$$|\Psi\rangle = \sum_n |n\rangle \, \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

free q, g basis

$$\sum_{i=1}^{n} x_i = 1 \, , \quad \sum_{i=1}^{n} \vec{k}_{\perp i} = 0$$

"Light-cone Fock expansion"

boost invariant Frame-indep.

$\tau = t + z/c$

Dirac
Bjorken, Kogut, Soper
Lepage + SJB
Pauli + SJB

Equation of motion

$$i \frac{\partial}{\partial \tau} |\Psi_H\rangle = P^- |\Psi_H\rangle = \frac{M_H^2 + \vec{P}_\perp^2}{P^+} |\Psi_H\rangle$$

$$H_{LC} = P^- P^+ - P_\perp^2$$

$P^+, P_\perp$
kinematical

$$\boxed{H_{LC} |\Psi_H\rangle = M_H^2 |\Psi_H\rangle}$$

⇒ eigenvalue problem for LC Hamiltonian

Insert complete set of H^0_{LC} color-singlet eigenstates

$$\sum_n |n\rangle\langle n| = I$$

$$\sum_n \langle m| H_{LC} |n\rangle\langle n|\Psi_H\rangle = M_H^2 \langle m|\Psi_H\rangle$$

DLCQ

⇒ Heisenberg matrix form of eigenvalue problem

$$|\Psi_H\rangle = \sum_n |n\rangle\langle n|\Psi_H\rangle = \sum_n |n\rangle \psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

⇒ LC Fock expansion of eigenstate $|\Psi_H\rangle$

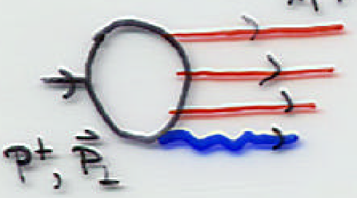
$*$ Given $\{\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\}$

wavefunction known for all P^μ !

relative coordinates

$$|P^+, \vec{P}_\perp\rangle = \sum_n \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \prod_j \frac{1}{\sqrt{x_j}}$$
$$|x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

absolute coordinates



In equal-time theory (instant form)

boosts mix with interactions

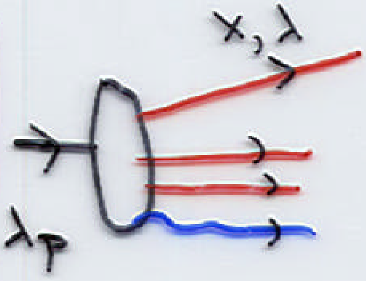
changing $\vec{P} \to \vec{P}'$ as complicated

as solving $H|\Psi\rangle = E|\Psi\rangle$

Light-Cone Wavefunctions

encode all helicity, transversity distributions

$$q_{\lambda/\lambda_p} = \sum \int \Bigg| \quad \Bigg|^2$$



$q_{\lambda/\lambda_p}(x, \Lambda)$ $\begin{cases} \text{transversity: density matrix} \\ \text{light-cone helicity} \end{cases}$

$$= \sum_{n,q} \int \left| \Psi^{(\Lambda)}_{n,\lambda_p}(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2 \prod_{j=1} dx_j \prod_{j=1} d^2 k_{\perp j}$$

$$\delta\left(\sum_i^n x_i - 1\right) \delta\left(\sum_i^n \vec{k}_{\perp i}\right)$$

$$\delta(x - x_q)\, \delta_{\lambda, \lambda_q}$$

Factorization
Light-cone Scheme

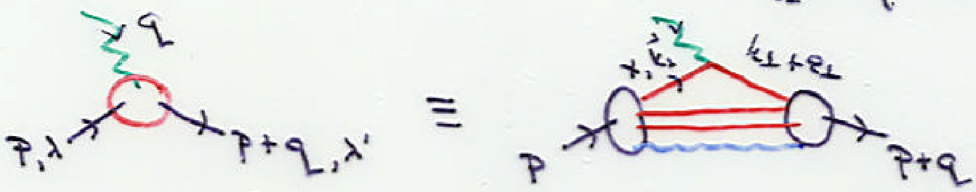
$$\Theta(\Lambda^2 - \mathcal{M}_n^2)$$

Why Light-Cone Wave Functions?

$$\boxed{\Psi_n(x, \vec{k}_\perp, \lambda)}$$

Form Factors calculable from overlap of L.C. W.F's

$q_\perp^2 = -q^2 = Q^2$



Sum over Ψ_n, e_q

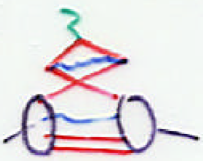
$$F_{\lambda\lambda'}(q^2) = \langle p+q, \lambda' | \frac{J^+(0)}{p^+} | p, \lambda \rangle$$

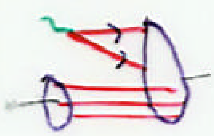
$$= \sum_q e_q \sum_n \int [d^2k_\perp] \int [dx]\, \Psi_{n,\lambda'}(x, \vec{k}'_\perp, \lambda')\, \Psi_{n,\lambda}(x, \vec{k}_\perp, \lambda)$$

Drell-Yan-West
Drell-SLO

$$\vec{k}'_\perp = \begin{cases} \vec{k}_\perp + (1-x)\vec{q}_\perp & \text{Struck } q \\ \vec{k}_{\perp i} - x_i \vec{q}_\perp & \text{Spectators} \end{cases}$$

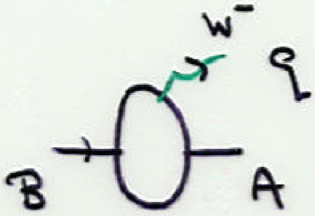
Remarks: $\tau = 0$: $j^+ = J^+$ free current

L.C. : $(k^+ > 0)$  $= 0$.

L.C. : $(q^+ = 0)$  $= 0$ good current J^+

L.C. : $(A^+ = 0)$ no ghosts

$B \to A\, \ell^- \bar{\nu}$



Light-cone coordinates

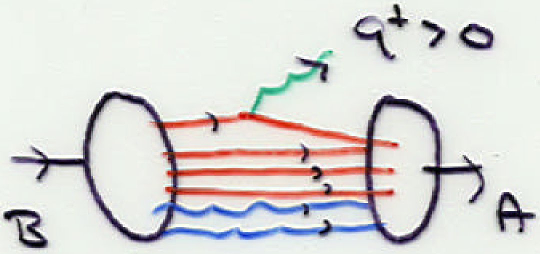
$$P_B = (P^+, \vec{P}_\perp, P^-)$$

$$= \left(P^+, \vec{0}_\perp, \frac{M_B^2}{P^+}\right)$$

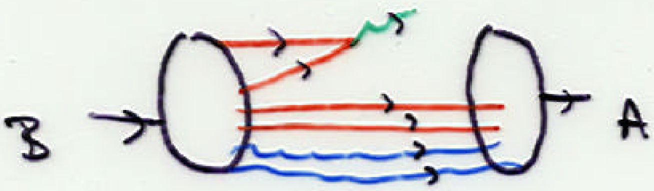
timelike decays require $q^+ = \Delta p^+ > 0$

$$q = \left(q^+, q_\perp, \frac{q^2 + q_\perp^2}{q^+}\right)$$

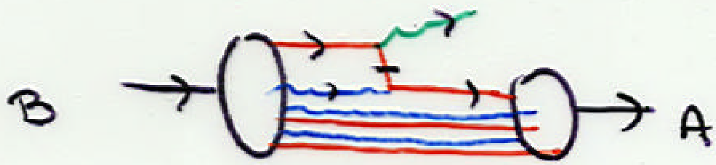
$$= \left(\Delta p^+, q_\perp, \frac{q^2 + q_\perp^2}{\Delta p^+}\right)$$



$\Delta n = 0$



$\Delta n = 2$



LC "instantaneous"

$\Delta n = 1$

Semi-leptonic Decays:

$$P_B^\mu = (P^+, P^-, \vec{P}_\perp) = \left(P^+, \frac{M_B^2}{P^+}, \vec{0}_\perp\right)$$

$$q = \left(\Delta P^+, \frac{q^2 + q_\perp^2}{\Delta P^+}, \vec{q}_\perp\right)$$

$$P_A = \left((1-\Delta)P^+, \frac{M_A^2 + \vec{q}_\perp^{\,2}}{(1-\Delta)P^+}, -\vec{q}_\perp\right)$$

$$M_B^2 = \frac{q^2 + q_\perp^2}{\Delta} + \frac{M_A^2 + q_\perp^2}{1-\Delta}$$

Lorentz-invariance: indep of P^+, $\vec{q}_\perp$

Choose $\vec{q}_\perp = 0$:

$$\boxed{M_B^2 = \frac{q^2}{\Delta} + \frac{M_A^2}{1-\Delta}}$$

SJB
D.S. Hwang

$$\langle A|J^\mu|B\rangle_{\Delta n=0} = \sum_{n,\ \lambda}\prod_{i=1}^{n}\int_\Delta^1 dx_1 \int_0^1 dx_{i(i\neq 1)} \int \frac{d^2\vec{k}_{\perp i}}{2(2\pi)^3}\,\delta\left(1-\sum_{j=1}^{n}x_j\right)\,\delta^{(2)}\left(\sum_{j=1}^{n}\vec{k}_{\perp j}\right)$$
$$\times\psi^\dagger_{A(n)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\ j^\mu\ \psi_{B(n)}(x_i,\vec{k}_{\perp i},\lambda_i), \quad (1)$$

$$\begin{cases} x'_1=\frac{x_1-\Delta}{1-\Delta}, & \vec{k}'_{\perp 1}=\vec{k}_{\perp 1}-\frac{1-x_1}{1-\Delta}\vec{q}_\perp \quad \text{for the struck quark} \\ x'_i=\frac{x_i}{1-\Delta}, & \vec{k}'_{\perp i}=\vec{k}_{\perp i}+\frac{x_i}{1-\Delta}\vec{q}_\perp \quad \text{for the } (n-1) \text{ spectators.}\end{cases} \quad (2)$$

$$\langle A|J^\mu|B\rangle_{\Delta n=-2} = \sum_{n\ \lambda}\int_0^\Delta dx_1\int_0^1 dx_{n+1}\int\frac{d^2\vec{k}_{\perp 1}}{2(2\pi)^3}\int\frac{d^2\vec{k}_{\perp n+1}}{2(2\pi)^3}\prod_{i=2}^{n}\int_0^1 dx_i\int\frac{d^2\vec{k}_{\perp i}}{2(2\pi)^3}$$
$$\times\delta\left(1-\sum_{j=1}^{n+1}x_j\right)\,\delta^{(2)}\left(\sum_{j=1}^{n+1}\vec{k}_{\perp j}\right)$$
$$\times\psi^\dagger_{A(n-1)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\ j^\mu\ \psi_{B(n+1)}(\{x_1,x_i,x_{n+1}=\Delta-x_1\},$$
$$\{\vec{k}_{\perp 1},\vec{k}_{\perp i},\vec{k}_{\perp n+1}=\vec{q}_\perp-\vec{k}_{\perp 1}\},\{\lambda_1,\lambda_i,\lambda_{n+1}=-\lambda_1\}). \quad (3)$$

$$x'_i=\frac{x_i}{1-\Delta},\qquad \vec{k}'_{\perp i}=\vec{k}_{\perp i}+\frac{x_i}{1-\Delta}\vec{q}_\perp \quad (4)$$

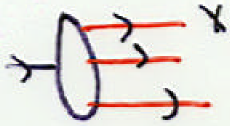
$\Delta\to 0$ recover DYW

J^- $\delta(x)$ contrib

see also Choi + Ji

Intuition on LC Wavefunctions

$0 < x_i < 1$

Ψ  $x_1, \vec{k}_{\perp 1}, \lambda_1$

$\sum_i x_i = 1 \;, \; \sum_i k_{\perp i} = 0$

$$\Psi = \frac{\Gamma(x_i, k_{\perp i})}{M^2 - \sum_i \frac{k_\perp^2 + m^2}{x_i} + i\epsilon}$$

$\nwarrow \; m_i^2 = \left(\sum_i k_i^\mu\right)^2$

Ψ peaks at $x_i = \frac{m_{\perp i}}{\sum m_{\perp i}}$ $\quad (m_{\perp i}^2 = k_{\perp i}^2 + m_i^2)$

"equal velocity" $\Leftrightarrow$ minimum inv. mass

$$x_i = \frac{k_i^+}{P^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

$$\Delta y_i = \ln x_i = y_P - y_i$$

Non-relativistic:

$x_i = \frac{k^0 + k^z}{M} \simeq \frac{m + k^z}{M}$ $\quad$ peaks at $\frac{m}{M}$

$x \Rightarrow 0$ is $k_z \Rightarrow -\infty$ for $m_\perp \neq 0$.

Canonical Form of Light-Cone Wavefunctions

$$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) = \frac{\Gamma_n(x_i, \vec{k}_{\perp i}, \lambda_i)}{M^2 - \sum_{i=1}^{n} \left(\frac{m^2 + k_\perp^2}{x}\right)_i}$$

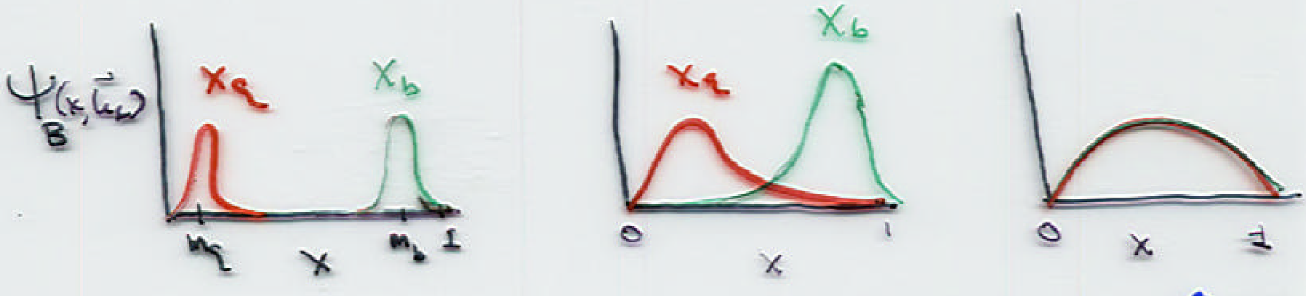
$$\Gamma_n = \int V_{nm} \otimes \psi_m$$

D S Hwang + SJB

$$\frac{1}{M^2 - \sum_{i=1}^{n} \frac{m_{\perp i}^2}{x_i}} = \frac{1}{M^2\left[\delta^2 - \sum_{i=1}^{n} \frac{(x_i - \hat{x}_i)^2}{x_i}\right]}$$

$$\delta^2 = \frac{-M + \sum_i m_\perp}{M} = \mathcal{O}\left(\frac{\epsilon_{BE}}{M}\right)$$

$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j m_{\perp j}}$$ $\Big\{$ equal rapidity, minimum m_n^2



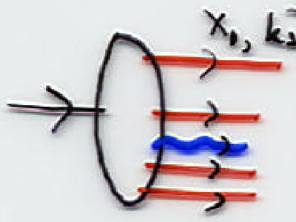
$(k_\perp \to 0)$ $\quad$ $(k_\perp \sim$ few GeV$)$ $\quad$ $(k_\perp \to \infty)$

heavy quark limit

Use QCD Eqn. of Motion to evolve to high $k_\perp$

SJB
+
DS Hwang

Canonical Form for Light-Cone W.F.'s



$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$ $\qquad \sum x_i = 1$, $\quad \sum k_\perp = 0$

EQM: $$\left[M^2 - \sum_{i=1}^{n} \left(\frac{m^2 + k_\perp^2}{x} \right)_i \right] \psi_n = \int \sum V_{nm} \times \psi_m$$

$$\therefore \quad \psi_n \equiv \frac{\Gamma_n(x, k_\perp)}{M^2 - \sum_{i=1}^{n} \left(\frac{m_\perp^2}{x} \right)_i} \qquad m_\perp = \sqrt{m^2 + k_\perp^2}$$

Maximum at $x_i = \hat{x}_i = \frac{m_{\perp i}}{\sum m_{\perp i}}$ (equal rapidit

In fact $$\boxed{\psi_n \equiv \frac{\tilde{\Gamma}_n(x, \vec{k}_\perp)}{\frac{\delta^2}{1+\delta^2} + \sum_i \frac{(x_i - \hat{x}_i)^2}{x_i}}}$$

$$\delta^2 = \left(\sum m_{\perp i} - M \right) / M \sim \epsilon_{BE} / M$$

* ψ_n: Sharply peaked around $x_i \sim \frac{m_{\perp i}}{\sum m_{\perp i}}$ for small δ
Spread controlled by lightest parton.
$\Delta x \sim p_{Bohr} / M$

Lepage +SJB

<u>Calculate high momentum tail of $\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$</u>

$$|\Psi^{(\mu)}\rangle = \sum_n |n\rangle\langle n|\Psi\rangle \;\Theta(\mu^2 - \mathcal{M}_n^2)$$

$$\equiv P^{(\mu)}|\Psi\rangle$$

$$\mathcal{M}_n^2 = \sum_{i=1}^{n}\left(\frac{k_\perp^2 + m^2}{x}\right)_i$$

Feshbach

$$H|\Psi\rangle = M^2|\Psi\rangle \qquad P^{(\mu)} + Q^{(\mu)} = 1$$

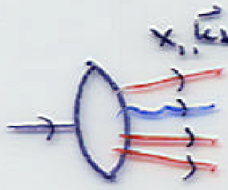
$$PHP|\Psi\rangle + PHQ|\Psi\rangle = M^2 P|\Psi\rangle$$

$$QHP|\Psi\rangle + QHQ|\Psi\rangle = M^2 Q|\Psi\rangle$$

$$\therefore \; |\Psi\rangle = |\Psi^{(\mu)}\rangle + \frac{1}{M^2 - QHQ} QH_I|\Psi^{(\mu)}\rangle$$

high mass, $\mathcal{M}_n^2 > \mu^2$

* short distance states only



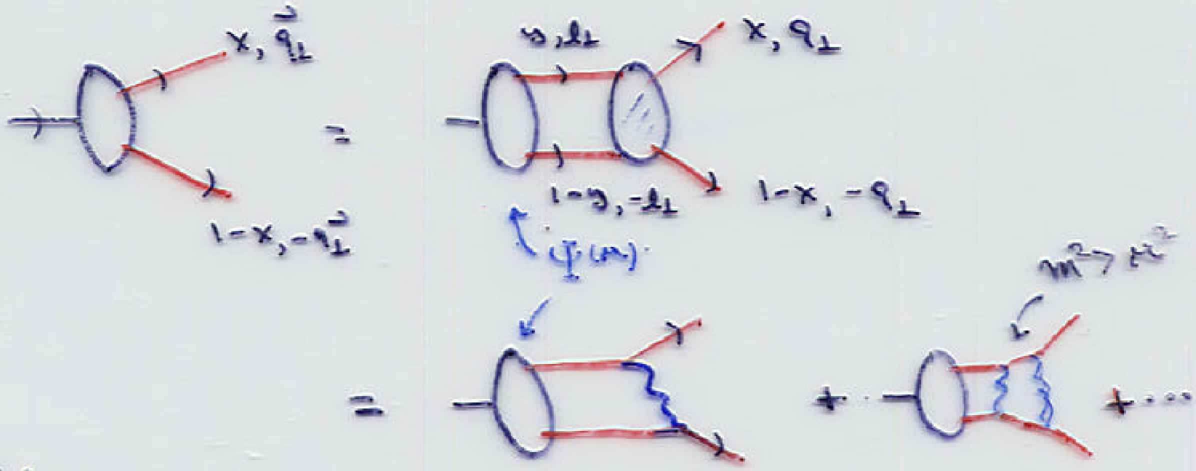
$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Calculate
- high $\vec{k}_\perp$
- $x \to 1$
- $x \to 0$
- heavy quark fluctuations

from PQCD

Example:

meson wavefunction at large transverse momentum



Q: $m_n^2 > \Lambda^2$:

$$|\Psi\rangle = \frac{1}{M^2 - QHQ + i\epsilon} \, Q H_I \, |\Psi^{(M)}\rangle$$

$$\psi(x, q_\perp) = \frac{1}{M^2 - \frac{q_\perp^2 + m^2}{x(1-x)}} \int_0^1 dy \int d^2\ell_\perp \, V(x, q_\perp; y, \ell_\perp) \, \psi^{(M)}(y, \ell_\perp)$$

$$\cong - \frac{x(1-x)}{q_\perp^2} \alpha_s(q_\perp^2) \int_0^1 dy \, V_{1g}(x, y) \, \phi(y, q_\perp^2)$$

$\left(\#\frac{\ell_\perp^2}{q_\perp^2}\right)$

where $\phi(y, Q^2) = \int^{Q^2} \frac{d^2\ell_\perp}{16\pi^3} \psi(y, \ell_\perp^2)$ "distribution amplitude"

log dep on Q^2.

Lepage
SLAC

Characteristic Scaling of L.C. Wavefunctions

QCD Eqn. of Motion:

$$(M^2 - \mathcal{M}_n^2)\,\Psi_n(x, \vec{k}_\perp) = \Gamma_n(x, k_\perp)$$

$$= \int V_{nm} \otimes \Psi_m$$

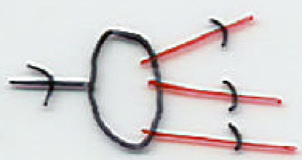
$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \left(\frac{k_\perp^2 + m^2}{x} \right)_i$$

$$\therefore \quad \Psi_n(x, \vec{k}_\perp) = \frac{\Gamma_n(x, \vec{k}_\perp)}{M^2 - \mathcal{M}_n^2}$$

At large $\mathcal{M}_n^2$ $(\vec{k}_\perp \to \infty,\ x \to 0)$

M 

$$\Psi_{q\bar{q}/M} \sim \frac{f_M}{\mathcal{M}_2^2}$$

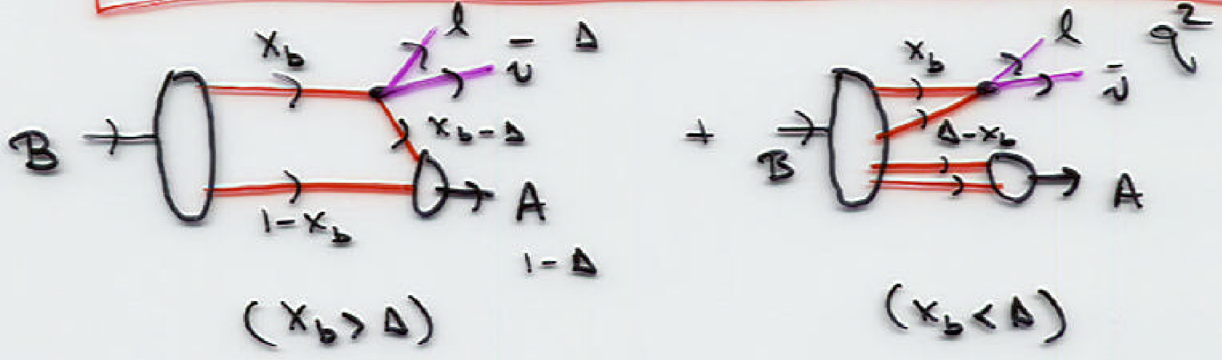
B 

$$\Psi_{qqq/B} \sim \frac{f_B}{(\mathcal{M}_3^2)^2}$$

$\therefore$ Intrinsic Wavefunctions: finite norm $\int |\Psi_I|^2 \,\Pi\, d^2k_\perp\, dx < \infty$

D-S Hwang
SJB

$B \to$ meson Form Factors: $F_{B\to A}(q^2)$



Both contributions necessary for Lorentz invariance!

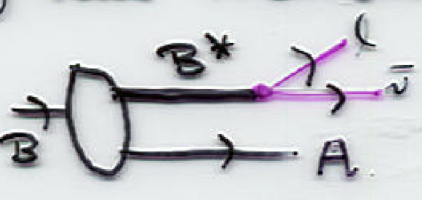
$(x_b>\Delta)$: Calculate as convolutions of ψ_B and ψ_A

e.g. $$\int_\Delta^1 \psi_B(x_b,\vec{k}_\perp)\,\psi_A\left(\frac{x_b-\Delta}{1-\Delta},\vec{k}_\perp\right)dx_b\, d^2k_\perp$$

Choose $q_\perp=0$, $q^+=\Delta P_B^+$, $\Delta \simeq 1-\frac{M_A^2}{M_B^2-q^2}$

Typical form: powers of $1-\Delta = \frac{M_A^2}{M_B^2-q^2}$

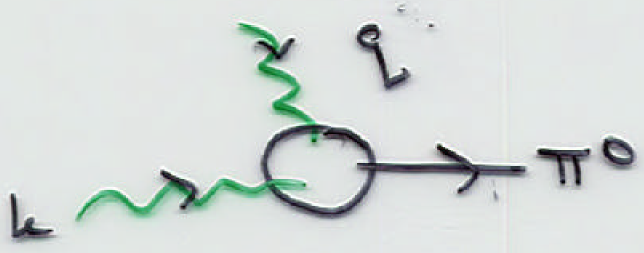
$(x_b<\Delta)$ term resembles (Desndrea, et al)

*


use $B^*\to BA$ coupling.

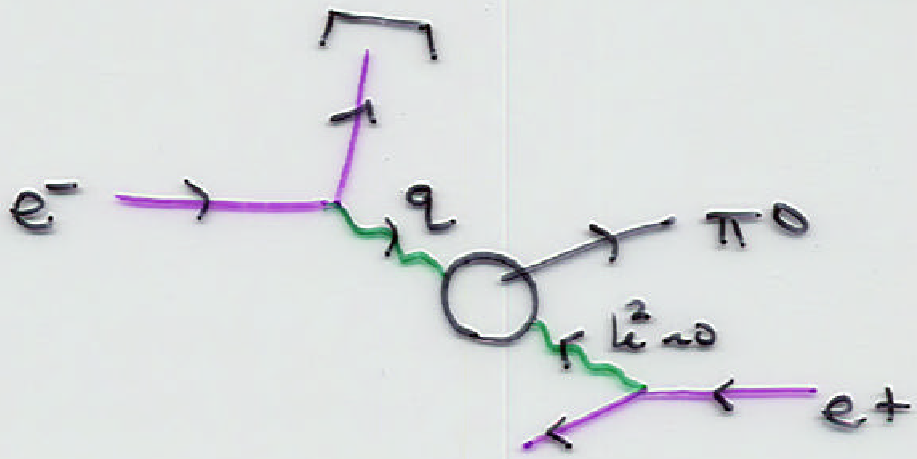
Photon-to-Pion

Transition Form Factor



$F_{\gamma\pi}(q^2)$

* Measure in e^+e^- colliders



CLEO

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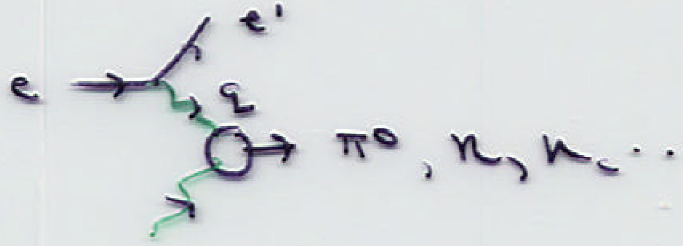
Simplest example of exclusive process

SJB Lepage

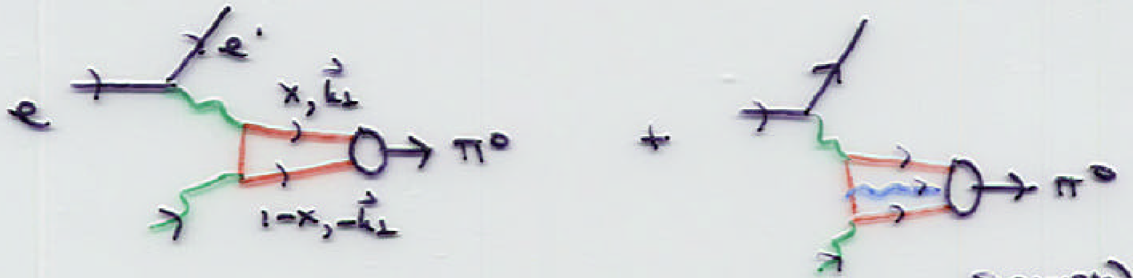
$q^2 = -Q^2$



$$\frac{d\sigma}{dQ^2} \sim \frac{1}{Q^6}$$

$Q^2 \gg \Lambda^2_{QCD}$

$F_{\gamma\pi^0}(Q^2)$



suppressed by $\frac{1}{Q^2}$

$$F_{\gamma\pi^0}(Q^2) = \frac{1}{Q^2} 2\sqrt{N_c}\,(e_u^2 - e_d^2) \int^1 \frac{dx}{x(1-x)}\, \phi_\pi(x, \tilde{Q})$$

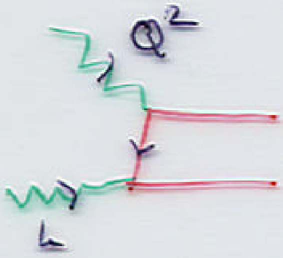
Pion distribution amplitude

$$\phi_\pi(x, \tilde{Q}) = \int^{\tilde{Q}} \frac{d^2k_\perp}{16\pi^3}\, \psi^{(\tilde{Q})}_{q\bar{q}}(x, \vec{k}_\perp)$$

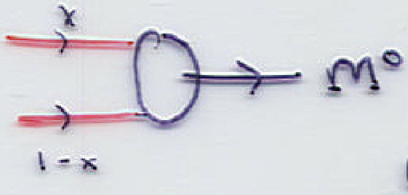
$$\int_0^1 dx\; \phi_\pi(x, Q) = \frac{F_\pi}{2\sqrt{3}}$$

SJB Lepage

PQCD : $\boxed{F_{\gamma\to M^0}(Q^2) \sim \frac{1}{Q^2}\int_0^1 \frac{dx}{1-x}\, \phi_M(x,\tilde{Q})}$



T_H



$$\phi_M(x,Q) = \int^{k_\perp^2 < \tilde{Q}^2} d^2k_\perp\, \psi_{q\bar{q}}(x,\vec{k}_\perp)$$

* $T_H(\gamma^*\gamma \to \underbrace{q\bar{q}}_{\text{collinear}}) \sim \frac{1}{Q^2(1-x)}$

$\mathcal{O}(Q)^n$

* Higher Fock states : $\frac{1}{Q^4}$ 

Other diagrams $\mathcal{O}(\alpha_s(Q^2))$!

* $\phi_M(x,Q) = \sum_{n=0}^{\infty} a_n P_n(x) \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n}$ 

log evolution

* $\lambda_M = \lambda_q + \lambda_{\bar{q}} = 0$.

** Small part of Fock state dominates

$$\phi_M \sim \psi(x, b_\perp \sim \tfrac{1}{Q})$$

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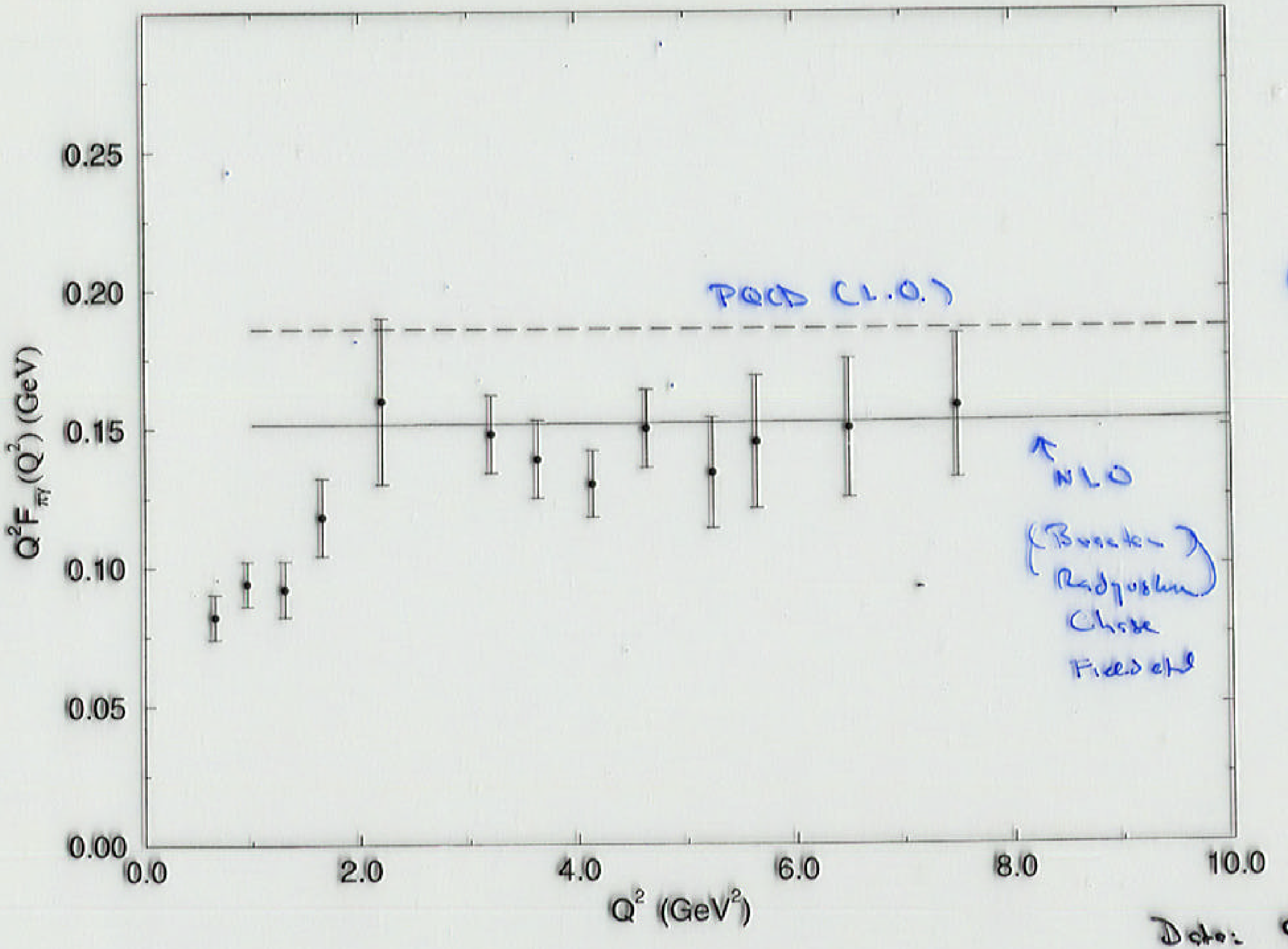
D. Robertson

C.R. Ji: 4 Pang & JB



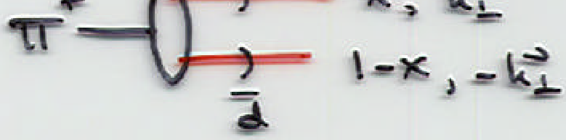
$\phi_\pi = x(1-x)\sqrt{3} f_\pi$

Data: CLEO (97) Savinov et al

G. P. Lepage
SJB

Pion Distribution Amplitude

$$\phi_\pi(x, Q^2) = \int \frac{d^2 k_\perp}{16\pi^3}\, \psi^{(Q)}_{q\bar{q}/\pi}(x, \vec{k}_\perp)$$



$$\sim \psi_{q\bar{q}/\pi}\left(x, b_\perp \sim O\left(\tfrac{1}{Q}\right)\right)$$

$$\phi_\pi(x, Q) = \int \frac{dz^- P_\pi^+}{4\pi}\, e^{i x P_\pi^+ z^-/2}$$

$$\left. \langle 0 | \bar{\psi}(0) \frac{\gamma^+ \gamma_5}{2\sqrt{2 n_c}} \psi(z) | \pi \rangle^{(Q)} \right|_{z^+ = \vec{z}_\perp = 0}$$

$$P \exp \int_0^1 ds\, i g\, A(sz)\cdot z = 1 \quad \text{in } A^+ = 0 \text{ gauge}$$

$$= \int \frac{dk^-}{2\pi}\, \psi_{BS}(k, P)$$

obeys: OPE, RGE, Evolution Eqs.

- Universal Distribution Amplitude

$$\phi_H(x_i, Q), \quad \sum_i x_i = 1$$

Evolution Equation, RGE, OPE
for log Q^2 variation of $\phi(x, Q)$:

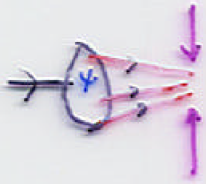
$$\frac{\partial}{\partial \log Q}\phi(x, Q) = \int dy V(x, y; \alpha_s)\phi(y, Q)$$

Lepage S&B

$$\phi(x, Q) = \sum_n a_n \; P_n(x) \log^{-\gamma_n} \frac{Q^2}{\Lambda^2_{QCD}}$$

OPE:
Lepage S&B
Frishma
Sachraj

- Exclusive amplitudes
at large momentum transfer
dominated by "small"
color-singlet Fock component:



$b_\perp = \frac{1}{Q}$

$$\phi(x, Q) \sim \psi_{valence}(x, b_\perp = \frac{1}{Q})$$

- Color Transparency

Frankfurt
Strikman et al
Miller et al
Ralston

Mueller
S&B

Bertsch, Gunion
Goldhaber S&B

Lepage, SJB

Evolution Eqn. for Distribution Amplitudes

$$\phi_M(x,Q) = \int \frac{d^2k_\perp}{16\pi^3}\, \psi^{(Q)}_{q\bar{q}}(x,\vec{k}_\perp)\, \Theta\left(Q^2 - \frac{k_\perp^2}{x(1-x)}\right)$$

$$= x_1 x_2\, \tilde{\phi}$$

$$\boxed{x_1 x_2 \frac{\partial}{\partial \ln Q^2} \tilde{\phi}_M(x,Q) = \frac{\alpha_s(Q^2)}{4\pi} \int_0^1 dy\, V(x,y)\, \tilde{\phi}_M(y,Q)}$$

$*$ $$V(x,y) = 2C_F\; x_1 y_2\, \theta(y_1 - x_1)\left(\delta_{h_1\bar{h}_2} + \frac{\Delta}{y_1 - x_1}\right) + (1 \leftrightarrow 2)$$

$$\Delta\tilde{\phi} = \tilde{\phi}(y_1, Q) - \tilde{\phi}(x_1, Q)$$

$$x_1 = x, \quad x_2 = 1 - x$$

$\delta_{h_1\bar{h}_2}$ = 1 opp hel

$\Rightarrow$ favors opp hel.

For baryons:

$*$ $$V(x_i, y_i) = 2x_1x_2x_3 \sum_{i\neq j} \theta(y_i - x_i)\,\delta(x_k - y_k)\,\frac{y_j}{x_j}\left(\frac{\delta_{h_i\bar{h}_j}}{x_i + x_j} + \frac{\Delta}{y_i - x_i}\right)$$

$\phi_M(x_i, Q)$

Hadron Distribution Amplitude

- key non-perturbative input to hadronic exclusive processes at large momentum transfer

Rigorous results

$$\phi(x_i, Q) \equiv \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-n\gamma_n/2\beta_0}$$

$$\int \prod_{i=1}^{n} \left\{ d^2 k_\perp^{(i)}\, \Theta(Q^2 - M_n^2) \right\} \delta^{(2)}\left(\Sigma k_\perp^{(i)}\right) \psi^{val}(x_i, k_\perp^{(i)}$$

$$* \quad \phi_M(x_i, Q) = x_1 x_2 \sum_n a_n\, C_n^{3/2}(x_1 - x_2) \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n/2\beta_0}$$

Lepage
SJB
Efremov Rad

non-perturbative input

from conformal symmetry

$$\zeta(Q^2) = \frac{1}{2\pi} \int^{Q^2} \frac{d\ell^2}{\ell^2}\, \alpha_s(\ell^2)$$

deviations from conformal symmetry

$$e^{-\gamma_n \zeta(Q^2)} \Rightarrow \left(\ln Q^2/\Lambda^2\right)^{-\gamma_n/2\beta_0}$$

"Asymptotic" Distribution Amplitude

$$\phi_{asympt}(x) = \sqrt{3}\; x(1-x)\, f_\pi$$

- Leading anomalous dimension in OPE
- Solution to QCD evolution eqn at $Q^2 \to \infty$

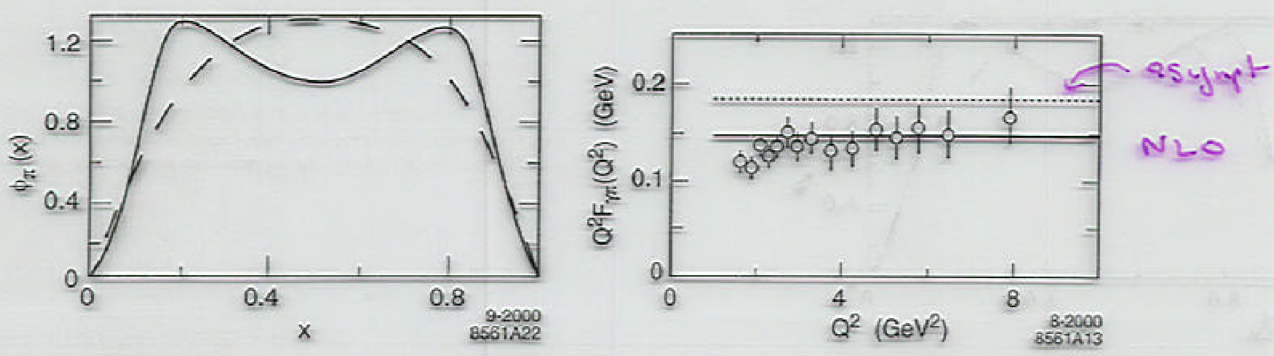
SJB + GPL

- $\frac{\partial}{\partial \ln Q^2} \phi(x, Q^2) = \int dy\, V(x,y)\, \phi(y$

- Non-perturbative methods
  DLCQ + Transverse lattice

  Dalley
  Burkardt

- Normalized by $\pi \to e\nu$



— transverse lattice / DLCQ

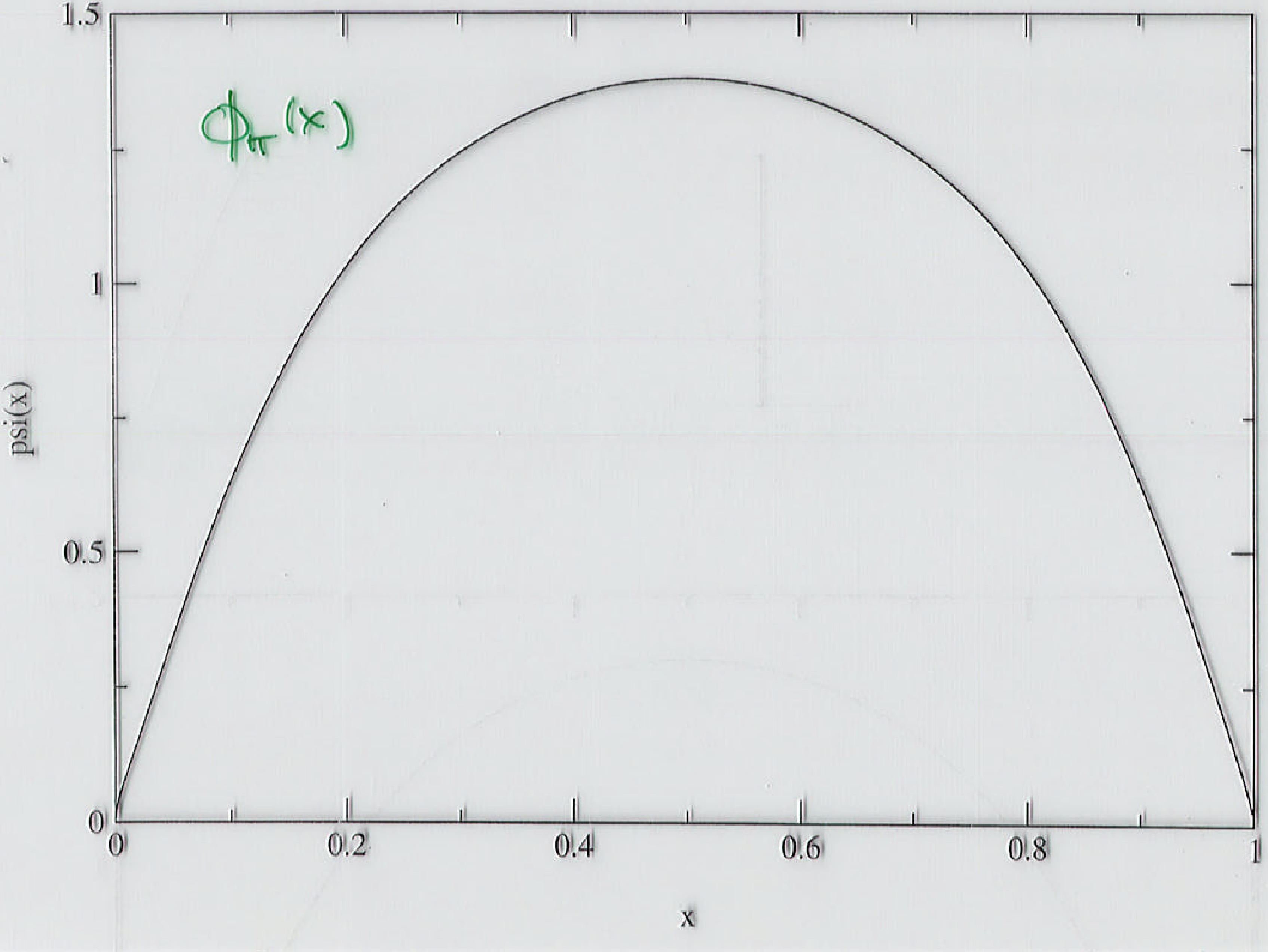
Dalley

- - - Asymptotic dist. ampl.

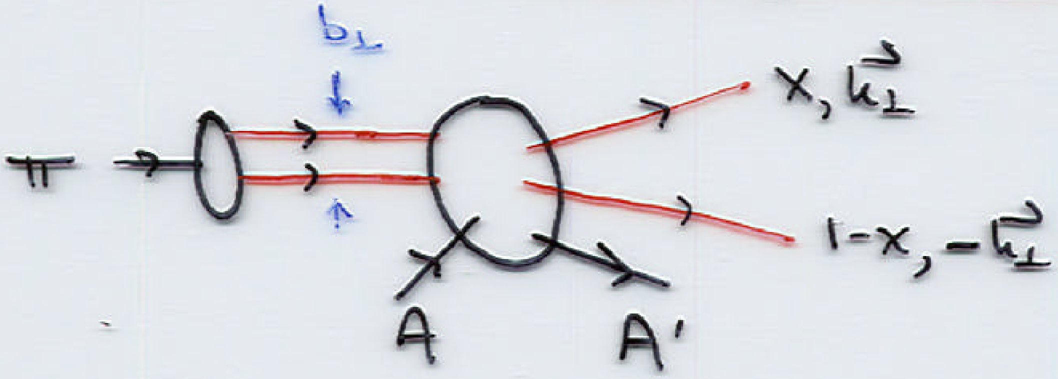
Burkardt

Transverse Lattice + DLCQ

Burkardt + Seal



Test of Color Transparency and Measurement of $\Psi_\pi(x, k_\perp)$



* "Nuclear Filter"

  Small color-singlet components pass
  large components absorbed — A. Mueller, SJB

* Diffractive production of di-jets
  nucleus left intact

* Jet distributions measure $\Psi_\pi(x, \vec{k_\perp})$ — G. Bertsch, J. Gunion, SJB, F. Goldhaber; Frankfurt, Miller, Strikman

* E791 Fermilab — D. Ashery, R. Weiss-Babai et al

DI - JETS ANALYSIS

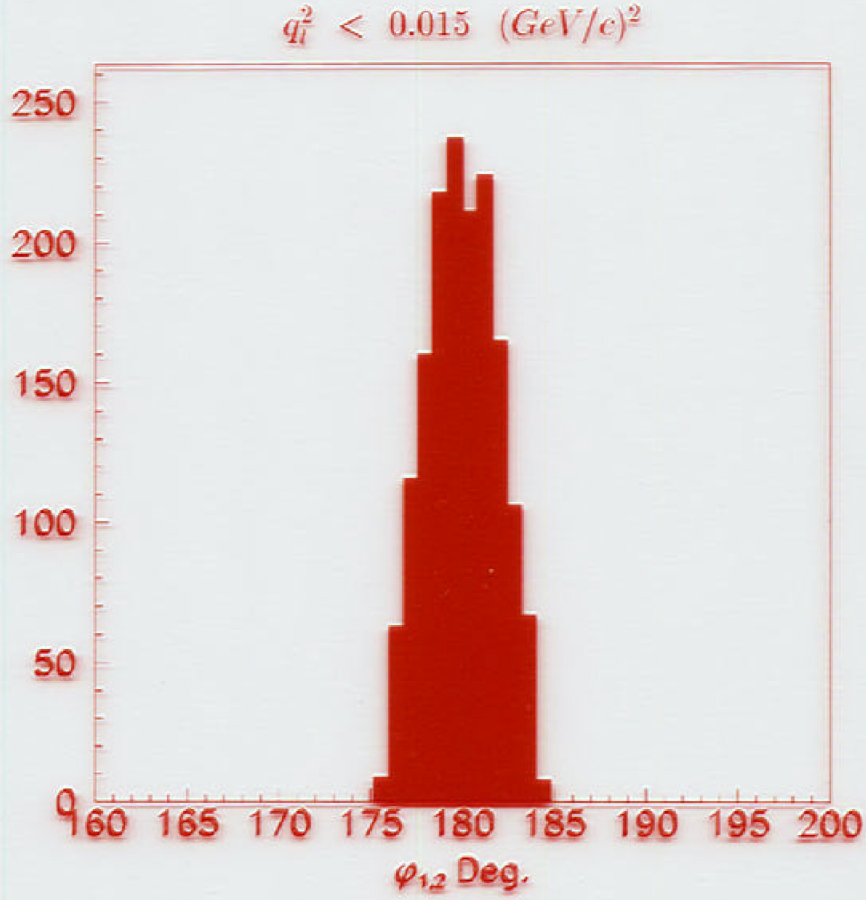
Used $\sim 1/3$ of E791 data

Basic Cuts:

1. $\Sigma p_z > 450$ GeV/c (in charged tracks)

2. Jet Finder - JADE Algorithm

3. Select Di-Jet events



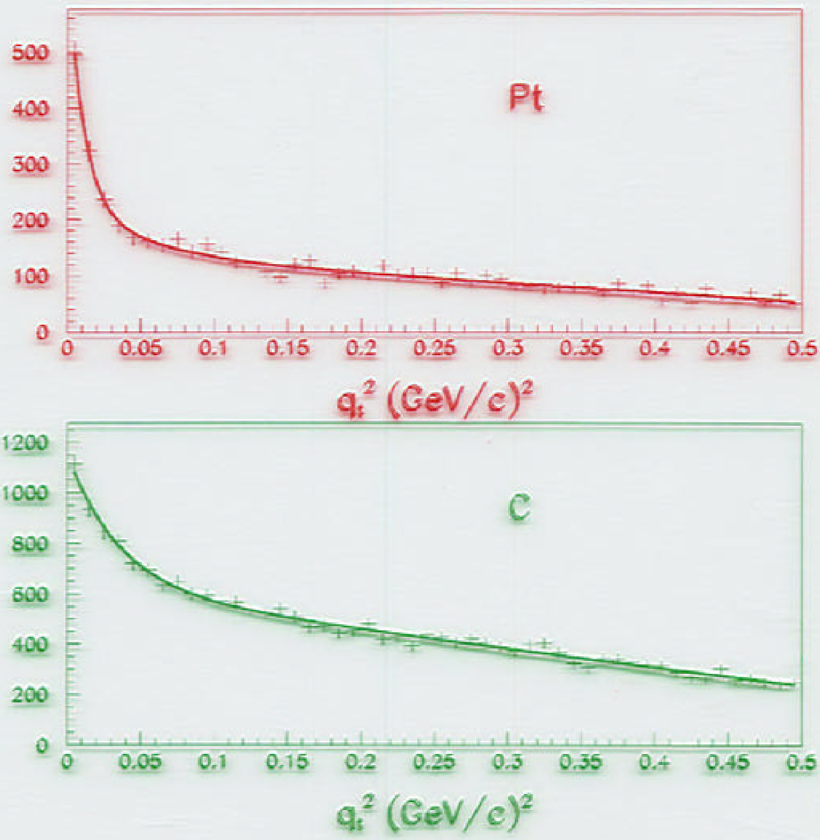
E791

DIFFRACTIVE DI - JETS

$\pi A \to Jet\ Jet\ A$

- Diffractive DI-JETS are identified through the $e^{-bq_t^2}$ dependence of their yield.
- $b = \frac{<R^2>}{3}$, R is the nuclear radius: $R_C = 2.44 fm$, $R_{Pt} = 5.27 fm$
- $q_t^2 = t - t_{min}$, is the square of the transverse momentum transfer to the nucleus.

$\vec{q}_\perp$ = mom. transfer to nucleus $= -\vec{k}_{\perp 1} - \vec{k}_{\perp 2}$

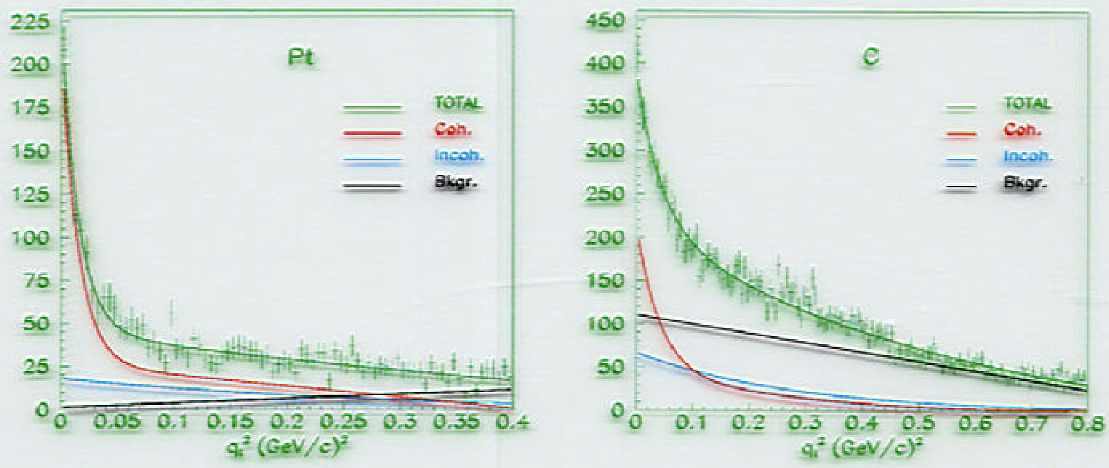


A DEPENDENCE OF DIFFRACTIVE YIELD

1. Combine the reconstructed MC distributions of Pt + N for the coherent and incoherent interactions with the nucleus.
2. Fit the combination to the data of Pt.
3. Repeat for carbon.
4. The parameters of the fit give the total normalization factor between MC and DATA and coherent/incoherent dissociation.

- Integrate over the diffractive peaks (of Pt and C) in the generated MC multiplied by the total normalization factor between MC and DATA.

$$1.5 \leq k_t < 2.0 \; GeV/c$$

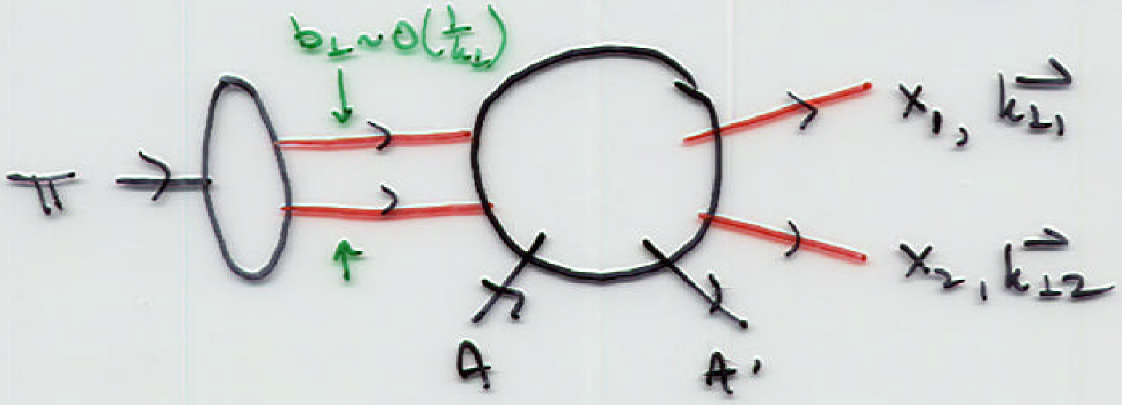


$$\boxed{\text{Measure } \psi_H^{val}(x, k_\perp) \text{ via Diffractive Dissociation}}$$

$$\boxed{\pi A \to q\bar{q} A'}$$

Bertsch, Goldhaber, Gunion, SJB

Frankfurt, Miller, Strikman



reconstruct jets

$$x_1 + x_2 \cong 1$$

$$\vec{q}_\perp = -\vec{k}_{\perp 1} - \vec{k}_{\perp 2} < R_A^{-1}$$

$\vec{k_\perp}$ large: color transparency $\mathcal{M}_A \sim A^1 \mathcal{M}_1$

$q_\perp$ small $\quad \frac{d\sigma}{dq_\perp^2} \propto e^{-q_\perp^2 R_A^2/3}$

* $\int \frac{d\sigma}{dq_\perp^2} dq_\perp^2 \sim \frac{A^2}{R_A^2} \sim A^{4/3}$

* $x, \vec{k_\perp}$ dist $\Rightarrow \left|\frac{\partial^2}{\partial k_\perp^2} \psi(x, k_\perp^2)\right|^2$

E791 DATA - THE $q\bar{q}$ MOMENTUM WAVE FUNCTION AS MEASURED BY THE DI-JETS

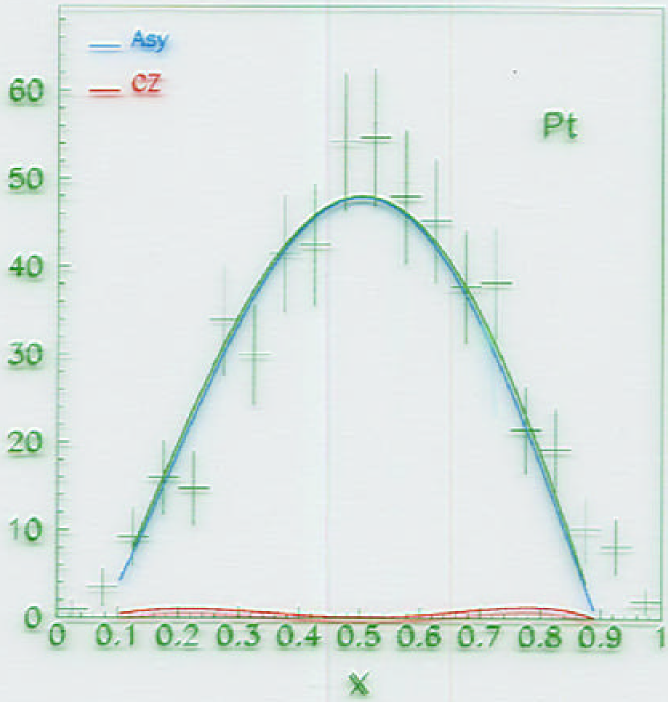
- Use the diffractive di-jets to extract the momentum x distribution.
- Fit to a combination of the two wave function simulations.

$$\phi^2 \equiv \alpha\phi^2_{Asy} + \beta\phi^2_{CZ}$$

$\phi_{Asympt}(x, Q^2)$ $\Rightarrow$ for $\frac{x(1-x)}{\sqrt{3}}$

Soln to PQCD evol. eqn.

$$k_t > 1.5GeV/c\ ;\ Q^2 \sim 10GeV^2$$



$\phi_{\pi}^{Asympt} \propto x(1-x)$

PRELIMINARY: > 90% Asymptotic W.F.

Same as seen in $F_{\gamma \to \pi^0}(Q^2)$