**QCD ($3+1$)**

| Sector | Class | 0 | 04 | 08 | 01 | 05 | 09 | 04 | 08 | 01 | 05 | 09 | 04 | 08 | 01 | 05 | 09 | 04 | 08 | 01 | 05 | 09 |
|--------|-------|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1      | 0     | 0 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 2      | 0     | 0 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 3      | 0     | 0 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 4      | 0     | 0 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 5      | 0     | 0 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 6      | 0     | 0 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |

**MCQ:** \[ \langle n \mid H_{\text{MCQ}} \mid m \rangle \]

\[
(M^2 - \sum_{i} \frac{k_i^2 + m_i^2}{x}) \psi_n = \sum_{m} \langle n \mid H_{\text{MCQ}} \mid m \rangle \psi_m
\]

**PBC:** \[ K^+ = \frac{2\pi}{L} n, \quad \bar{K}^0 = \frac{2\pi}{L} \bar{n} \]
Higgs QCD: New $g \rightarrow g'$ interaction

\[ \frac{g'}{4\pi^2} \]

\[ \frac{1}{(u'^+)^2} \frac{g'}{4\pi^2} \]

\[ \text{Vertex Factor} \]

\[ \text{Color Factor} \]

\[ \frac{v}{(p_1 - p_2) \cdot q} \]

\[ \frac{1}{(p_1 - p_2) \cdot q} \]

\[ + \text{ cycle permutations} \]

\[ \frac{v}{(p_1 - p_2) \cdot q} \]

\[ \frac{1}{(p_1 - p_2) \cdot q} \]

\[ \text{Instaneous Fermi} \]

\[ \text{Instaneous gluon} \]

\[ \chi = \chi \cdot \chi \cdot \chi \]

\[ \text{Figure 54: Graphical rules for QCD in light-cone perturbation theory.} \]
For 1+1 Theories, \( k \) cuts off Fock states

\[
\chi_c = \frac{k^+}{p^+} = \frac{n_i}{k} = \left\{ \frac{1}{k}, \frac{2}{k}, \ldots, \frac{k-1}{k} \right\}
\]

Sample states at finite resolution

Continuous limit: \( k \rightarrow \infty \).

For fixed \( k \), \( \sum n_i = k \).

\[ x_i = \frac{n_i}{k} \]

QED (1+1): No dynamical photons in \( A^0 = 0 \) gauge.

\[
V = \frac{\alpha^2}{2} \left[ \frac{1}{(k^-)^2} - \frac{1}{(k^+)^2} \right]
\]

From second order:

\[
\frac{n^+}{m^+} + \frac{2}{m^2} + \frac{3}{m^2} + \ldots
\]
6. The renormalized spectrum of invariant masses. — The invariant masses $M_i/M_1$ as calculated with the full Fock space of the massive representation for $K = 16$ is plotted versus all values of the coupling constant $\lambda$. — Note the qualitatively different parts of the spectrum. Many quasi-crossings are not resolved graphically despite the small step in the calculation, $\Delta \lambda = 0.01$. 

$\prod_{\alpha=1}^{9999} \approx 10^{-\alpha}$
Figure 5. a-c) Three states in $N = 3$ baryon spectrum. $2K = 2$: d) First $P = 2$ state.
DLCQ (3+1)

Calculations in a renormalizable quantum field theory model

Simplified Jakow model with Pauli-Villars regularization

Invariant mass cutoff to regularize phase space

$\frac{1}{2} (k^2 + m^2) < \Lambda^2$

See also
- Faddeev, Yakubov, Walt
- Penning, Moore

SCh, McCarta, Hille

$M_0$ -- dynamic fermion

$M_0$ -- bosons

Analytic path for non-dynamic fermion
DLCQ ($3+1$)

Periodic boundary condition

\[-L < x^- < L, \quad -L_\perp < x_\perp < L_\perp\]

\[\Rightarrow \text{Discrete moments:}\]

\[p_\perp^i = \frac{\pi}{L_\perp} n_i, \quad p_{\perp i}^L = \frac{\pi}{L_\perp} (n_x, n_y)\]

\[p^+ = \frac{\pi}{L} k, \quad \Sigma n_i = k\]

\[\chi^i = \frac{p_\perp^i}{p^+} = \frac{n_i}{k} > 0\]

\[\text{Each state number limited by } k\]

\[\sum_i \frac{m_i^2 + p_{\perp i}^2}{x_i^2} \leq \Lambda^2 \text{ limit } n_x, n_y\]

\[\text{Continuous limit } k \rightarrow \infty\]

Applications to Yukawa theory ($3+1$)

H. I. H., McCorm., 878
Figure 1: The boson distribution function $f_B$ at various numerical resolutions, with $\langle \phi^2(0) \rangle = 1$, $k^2 = 50\mu^2$, and $\mu_d^2 = 10\mu^2$. The solid line is the parameterized fit.

Structure Function $f_B(y)$

for Fermion eigenstate

$M = M_\alpha, \quad \langle : \phi^2(0) : \rangle = \frac{1}{2}$
Figure 8: The boson distribution function $f_B(y, q_{\perp})$ with $K = 21$, $N_{\perp} = 7$, $\langle \phi^2(0) \rangle = 1$, $\Lambda^2 = 25\mu^2$, and $\mu^2 = 10\mu^2$. The transverse momentum is varied with $q_y$ fixed at zero.

$F_B(y, q_{\perp}, q_{\perp} = 0)$

summed over all Fock states
Figure 4: The one-boson amplitude $\psi^{(1,0)}$ as a function of longitudinal momentum fraction $y$ and one transverse momentum component $q_z$ in the $q_y = 0$ plane. The parameter values are $K = 17$, $N = 4$, $\mu^2 = 10\mu^2$, $\Lambda^2 = 50\mu^2$, and $\langle \phi_0 | \phi_0 \rangle = 1$.

$$\psi(y, q_x, q_y = 0)$$

Lowest Fock state.
DLCQ (3+1)

- Model shows good convergence
- Reliable calculation
  - Structure functions, vertex elements, distribution amplitudes, etc.
- PV regularization renders theory finite
  - Broken SUSY regularization

Encouraging for ACC (3+1)

- Feynman gauge formulation of the
  - P. Srishti, 060

- Chiral properties?

- Heavy Quarkonia application
  - Calm deset component
Apply \( \mathcal{PCD} \) to

\[ B \to \text{Baryon Pair} \]

"Non-Factorizable"

\( \delta_g \) Oxen

"Annihilation"

Similar to

"timelike \( F_p(1') \)"

novel behavior

Low mass dibaryons

Study new decay states

\( (\Lambda F) \) \( (\Lambda \Lambda F) \) \( (5 \Lambda F) \)
Two contributions to sea quark distribution:

- Extrinsic (photons-gluon from $g + \pi \rightarrow q\bar{q}$)
- Intrinsic

Initial state factorization evolution:

$Q^2 \gg m^2$

$C_t \propto \frac{1}{m^2 R^2}$

$\alpha_s$ ($x_0, Q^2$)

$V_{ts}$

Thay

576, Vex, Sita, Perso

Regular, O'Brien
Higher Particle-Number Fock States

- required by relativity, $q^2 = 0$.

Example: **Intrinsic Clebsch in B**

\[ \langle x \rangle \sim \frac{x^2}{M^2} \frac{d^4}{dx^4} (M^2) \]

* OPE analysis by Fronz, Polyakov, Shifner

* EMC measurement \[ \text{IC in proton} \]

\[ c(x, Q) \]

\[ Q^2 = 75 \text{GeV}^2 \]

\[ x = 0.4 \]

* $\gamma B \bar{c}c$ in \( B(x, \vec{k}) \)

Hoye, Ref: Sati, S.

\[ x_c = \frac{m_{L}}{\Sigma m_{i}} \]

\[ \rho \bar{c}c / B \sim 4 \times \rho \bar{c}c / p ! \]
Color - Octet Intrinsic Charm

\[ \text{QCD:} \quad e^+ \rightarrow (e^+e^-e^-) \rightarrow e^+m^++m^-e^- \]

\[ P_{\text{pro}} \sim \left( \frac{M_{\text{Bohr}}}{M_{\text{pro}}} \right)^4 \times a^4 \]

QCD

Only suppressed by \( \frac{1}{m_c^2} \) \( \gtrsim \) color octet

Frank et al.

\[ <x_{c\bar{c}}>_H \Rightarrow \]

\[ T^{++} (q=0) \]

\[ T^{++} \Rightarrow \frac{G^{\nu} G^{+\mu} G_{\mu \nu}}{M_c^2} \left[ A_{\mu}, A_{\nu} \right] \]

\[ \frac{1}{M} = \frac{1}{M_1 + M_2} \]

\[ M_{\text{Bohr}} \sim Z \frac{M_{\text{Bohr}}}{M_{\text{Bohr}}} \]

Move \( Z \) in \( B \)!
Consequences for Intrinsic Charmonium

\[ J/\psi \rightarrow \pi \pi', \, \psi' \rightarrow \pi \pi \]

\[ J/\psi \rightarrow 0^+ \rightarrow c \rightarrow \bar{c} \rightarrow \pi \rightarrow J/\psi \quad \text{Korliner 8d8} \]

\[ \gamma \rightarrow J/\psi \times \text{ Spectrum} \quad \text{Hoy} \]

\[ \pi \rho \rightarrow J^{\pm} \times, \quad \Sigma_{p \rightarrow \Lambda} \times \quad \text{Hagen et al.} \]

Consequences for B-decays

\[ B^0 \rightarrow J/\psi \]

\[ B^0 \rightarrow \pi \rightarrow D^0 \]

\[ B^0 \rightarrow s \rightarrow k \pi \]

Leading CKM

Evolution of CKM

Hoy

CBA, Belle

bump of low \( J/\psi \)

Grossen

8d8
Both produce bump in $M_X$ spectrum

$\sim 2$ GeV

$B \rightarrow \pi' \pi$

$$ln' = (\sim g_3) + (\sim g_{ee}) + \ldots$$
Use color-octet intrinsic charm to evade the CKM hierarchy

Chang + Hou

New pattern of exclusive decays
Enhances role
Interferes with standard contributions

Other examples of IC in B decays

Seen at CLEO? dump at low P_T

Missour
Region Point
The "$J/\psi \rightarrow \rho \pi$" Puzzle
and Intrinsic Charge

$B \left[ J/\psi \rightarrow \rho \pi \right] = 1.28 \pm 0.10 \%$

$B \left[ \psi'(2S) \rightarrow \rho \pi \right] < 3.6 \times 10^{-5}$

PQCD: $B \left[ \psi' \rightarrow \rho \pi \right] < \frac{1}{50}$ expected value

$Q \bar{Q} \rightarrow \rho \pi$ supposed by "heaven helicity cone."

Some problem: $\rho \pi^*$
Intrinsic charm \( P = \{ u\bar{d} c\bar{c} \} \)

\( P^+ \to J/\psi \pi^+ \)

(\( \lambda = +1 \))

minimizes \( J^{PC} = 1^- \)

close match to \( J/\psi \pi^+ \)

\( J/\psi \pi^+ \)

suppressed coupling to \( \psi(28) \)

because \( c\bar{c} \) node in radial \( \psi(28) \) wavefunction.
Conformal Symmetry and
Baryon Distribution Amplitudes

V. Braun, S. Derkachov,
A. Manashov, G. Korchemsky

\[ q^+ q^+ q^+ \Rightarrow \phi_\lambda^{3/2} (x_i, \mu^2) \]

\[ q^+ q^- q^+ \Rightarrow \{ \phi_N^{\lambda=\frac{1}{2}} (x_i, \mu^2), \phi_A^{\lambda=\frac{1}{2}} (x_i, \mu^2) \} \]

\[ \sum_{i=1}^{3} x_i = 1 \]

\[ \phi_N^{\lambda=\frac{1}{2}} (x_i, \mu^2) = x_1 x_2 x_3 \sum_{N=0}^{\infty} a_N \frac{d^{N+1}}{d\beta (\mu)} \left( \frac{1-2x_3}{\beta} \right) \]

Scalar diquark

(\(\uparrow \downarrow \uparrow\))

1 2 3

Scalar diquark

(\(\uparrow \downarrow \uparrow\))

1 2 3

expansion in conformal polynomials

\( p_N (1-2x_3) \pm p_N (1-2x_i) \)

Jacobi Polynomials \( p_N^{(1,2)} \)

\( p_N \)
Light-Cone Fock Representation of Hadrons

\[ |P\rangle = \sum_n |a_n \rangle \Psi_n (x_i, \xi_i, \lambda_i) \]

1. Explicit solutions using "DLQD"
2. Calculate structure functions
3. Calculate regge behaviour using "ladder relations"
4. \( x \to 0 \), BFKL
5. \( x \to 1 \), constraints
6. Properties of heavy quark sector \( s(x) \neq \bar{s}(x) \)
7. Extensive vs intrinsic
8. Physics of Q, Q\bar{Q}, anomaly

QCD (1+1), "collinear" QCD
SIB, Pauli, Heinsius
Antonuccio, Dalleo

Mueller, SIB, Antonuccio, Dalleo
Legere, SIB, Burgeat, Sch"oder
Koga, "Me" Sch"oder
SIB, Schwend
Atomic Physics Analog to B, D decay

\[ \mu (\mu^{-}) \rightarrow e^{+}e^{-} \rightarrow e^{+}e^{-} \rightarrow e^{+}e^{-} \]

"atomic alchemy"

* observe sudden emergence of moving atom

\[ B^+ \rightarrow e^+e^- \]

Semi-leptonic B decay

* Calculations require \( \Psi_B, \Psi_D \)
at large and soft momenta

* Abelian Correspondence Principle

QCD \( \rightarrow \) QED
Light-Cone Wavefunctions in QCD

Essential quantities in QCD calculations at amplitude level

\[ x_i, k_i, \beta_i \]

\[ x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{p^0 + p^z} \]

Fixed \( \tau = t + \frac{q}{c} \)

\( \psi_{\eta/p}(x_i, k_i^2, \beta_i) \)

\[ \sum_{i=1}^2 x_i = 1, \quad \sum_{i=1}^2 k_i^z = 0 \]

Relativistic representation of hadrons in terms of quark and gluon degrees of freedom.
Summary

* Exact Formula for $B \rightarrow l^+ l^- M$

  - Higher Feild Stokes required for Lorentz Invariance

* PQCD Analysis of Exclusive B-decays

  - Rigorous factorization formulae for
    - Some contributions
    - Analyzable contributions controlled by Sudakov suppression
    - $\Lambda_s$ freezes in IR

* Light-cone wavefunctions + Distributor Amplitude

  - Computable using DLEQ, non-pert methods
  - Measurable in $B \rightarrow \pi^+ \pi^-$, $Df$ sectors

* Intrinsic Charm and Strongness

  - Evidence of CKM
  - Novel Consequences