Open- and Hidden-Charm Tetra-Quark Mesons and Related — $D_{s0}^{+}(2317)$, $D_0(2400)$ and $X(3872)$ —

K. Terasaki
YITP, Kyoto U.,
ITP, Kanazawa U.

§1. Introduction – Why Tetra-Quark Mesons ?

§2. Classification of Tetra-Quark Mesons

§3. Conventional Open-Charm Scalar Mesons

§4. Summary and Discussion

§1. Introduction – Why Tetra-Quark Mesons ?

$\underline{D_{s0}^{+}(2317)}$

- **In inclusive $e^{+}e^{-}$ annihilation :**

$$R(D_{s0}^{+})|_{\rm CLEO} = \frac{\Gamma(D_{s0}^{+}(2317) \to D_{s}^{*+}\gamma)}{\Gamma(D_{s0}^{+}(2317) \to D_{s}^{+}\pi^{0})}\Big|_{\rm CLEO} < 0.059$$

$$\text{c.f.} \quad R(D_{s}^{*+})|_{\rm exp}^{-1} = \frac{\Gamma(D_{s}^{*+} \to D_{s}^{+}\pi^{0})}{\Gamma(D_{s}^{*+} \to D_{s}^{+}\gamma)}\Big|_{\rm exp} = 0.062 \pm 0.008 \quad \text{PDG06}$$

I-conserving ST int.	$\gg$	EM interaction	$\gg$	I-nonconserving int.
$(\sim O(1))$		$(\sim O(\sqrt{\alpha}))$		$(\epsilon \sim O(\alpha))$
$\Downarrow$		$\Downarrow$		$\Downarrow$
$\Gamma(D_{s0}^{+}(2317) \to D_{s}^{+}\pi^{0})$	$\gg$	$\Gamma(D_{s0}^{+}(2317) \to D_{s}^{*+}\gamma)$		
		$\Gamma(D_{s}^{*+} \to D_{s}^{+}\gamma)$	$\gg$	$\Gamma(D_{s}^{*+} \to D_{s}^{+}\pi^{0})$

$\Rightarrow$ $D_{s0}^{+}(2317)$ **should be an <u>iso-triplet</u> charm-strange scalar state.**

$$\Downarrow$$

$\hat{F}_{I}^{+} \sim [cn][\bar{s}\bar{n}]_{I=1}$ **K.T., PRD <u>68</u>, 011501(R) (2003)**

- ◇ **Iso-singlet partner:** $\Gamma(\hat{F}_{0}^{+} \to D_{s}^{*+}\gamma) \gg \Gamma(\hat{F}_{0}^{+} \to D_{s}^{+}\pi^{0})$
- ◇ **Conventional $\{c\bar{s}\}$:** $\Gamma(D_{s0}^{*+} \to D_{s}^{*+}\gamma) \gg \Gamma(D_{s0}^{*+} \to D_{s}^{+}\pi^{0})$

Hayashigaki and K. T., PTP <u>114</u>, 1191 (2005); hep-ph/0410393

(Current algebra for $D_{s}^{+}\pi^{0}$ decays and VMD for radiative decays)

- **In B decays :** $D_{s0}^{+}(2317) = \tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}] \neq \tilde{D}_{s0}^{+}(2317)[D_s^{*+}\gamma]$
  - Belle **PRL 91, 262002(2003)**

$$\begin{cases} Br(B \to \bar{D}\tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}]) = (8.5^{+2.1}_{-1.9} \pm 2.6) \times 10^{-4} \\ \boxed{Br(B \to \bar{D}\tilde{D}_{s0}^{+}(2317)[D_s^{*+}\gamma]) = (2.5^{+2.0}_{-1.8}(< 7.5)) \times 10^{-4}} \end{cases}$$

  - Babar

$$\begin{cases} Br(B_u^{+} \to \bar{D}^{0}\tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}])_{\rm Babar} = (1.0 \pm 0.3 \pm 0.1^{+0.4}_{-0.2}) \times 10^{-3} \\ Br(B_d^{0} \to D^{-}\tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}])_{\rm Babar} = (1.8 \pm 0.4 \pm 0.3^{+0.6}_{-0.4}) \times 10^{-3} \end{cases}$$

$$\Updownarrow$$

◇ **Much smaller than** $Br(B \to \bar{D}D_s^{+})_{\rm exp} = (1.4 \pm 0.4) \times 10^{-2}$

(A color favored spectator decay)

♡ $$\begin{cases} D_{s0}^{+}(2317) = \tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}] & \Rightarrow \quad [cn][\bar{s}\bar{n}]_{I=1} \sim \hat{F}_I^{+} \\ \tilde{D}_{s0}^{+}(2317)[D_s^{*+}\gamma] & \Rightarrow \quad [cn][\bar{s}\bar{n}]_{I=0} \sim \hat{F}_0^{+} \end{cases}$$

$\underline{D_0}$

- $(m_{D_0^0})_{\rm exp} = \mathbf{2352 \pm 50}$ **MeV**, $\Gamma(D_0^0)_{\rm exp} = \mathbf{261 \pm 50}$ **MeV** **PDG07**

BELLE Collaboration, hep-ex/0307021

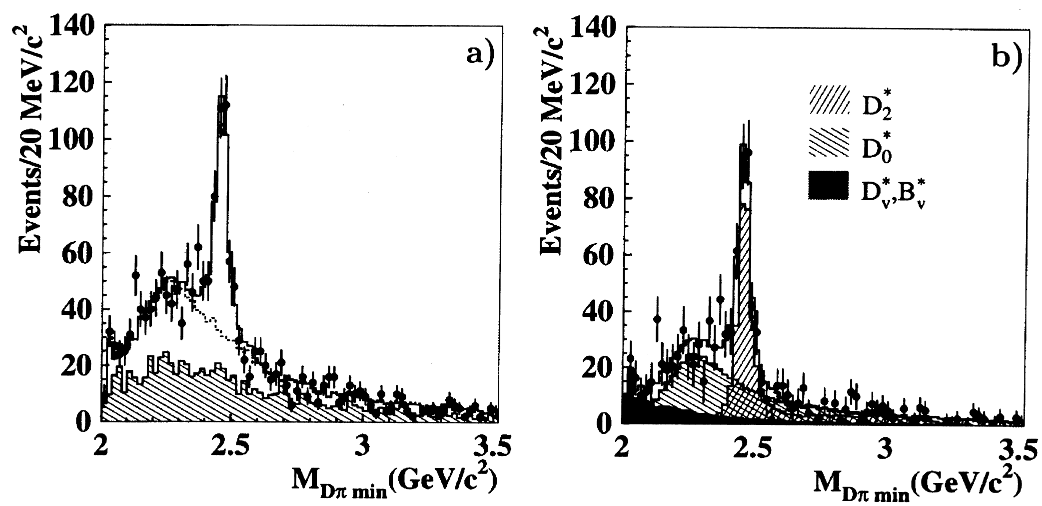

FIG. 3: a)The minimal $D\pi$ mass distribution of $B^- \to D^+\pi^-\pi^-$ candidates. The points with error bars correspond to the signal box events, while the hatched histogram shows the background obtained from the sidebands. The open histogram is the result of a fit while the dashed one shows the fit function in the case when the narrow resonance amplitude is set to zero. b)The background-subtracted $D\pi$ mass distribution. The points with error bars correspond to the signal box events, hatched histograms show different contributions, the open histogram shows the coherent sum of all contributions.

◇ **More precise (re-)analyses in $D\pi$ mass distribution**

$\Rightarrow$ **Find a structure:** $\left\{ \begin{array}{ll} D_0^* & \text{with } \Gamma(D_0^*) \sim 50 \text{ MeV} \\ \hat{D} & \text{with } \Gamma(\hat{D}_0) \lesssim 10 \text{ MeV} \end{array} \right.$ **(discussed later)**

$X(3872)$

- Angular analysis $\Rightarrow J^P(X(3872)) = 1^+$ Belle, hep-ex/0505038
- Decays into opposite G-parity states;

$$\frac{\Gamma(X(3872) \to \pi^+\pi^- J/\psi)}{\Gamma(X(3872) \to \pi^+\pi^-\pi^0 J/\psi)} = 1.0 \pm 0.4 \pm 0.3$$ Belle, hep-ex/0505037

$$\Downarrow$$

- Two (approximately) degenerate states
  - Unitarized chiral model : $D\bar{D}^* \cdots$ molecules

    Gamermann and Oset, EPJ A33, 119 (2007)
  - Tetra-quark :
    - $\{[cn](\bar{c}\bar{n}) \oplus (cn)[\bar{c}\bar{n}]\}$ (with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$) K. T., PTP 118, 821 (2007)

      $C_{\{\eta_c\omega\}}(\overline{60}_f) \sim [cn](\bar{c}\bar{n})_{I=0}$ and $C_{\{\eta_c\omega\}}(60_f) \sim (cn)[\bar{c}\bar{n}]_{I=0}$

      mix with each other to form G-parity eigenstates

$$\Rightarrow \begin{cases} C_{\{\eta_c\omega\}}(-) \sim C_{\{\eta_c\omega\}}(\overline{60}_f) - C_{\{\eta_c\omega\}}(60_f) \\ C_{\{\eta_c\omega\}}(+) \sim C_{\{\eta_c\omega\}}(\overline{60}_f) + C_{\{\eta_c\omega\}}(60_f) \end{cases}$$

$$X(3872) \overset{\Downarrow}{=} \begin{cases} C_{\{\eta_c\omega\}}(-) \to \pi^+\pi^- J/\psi, \\ C_{\{\eta_c\omega\}}(+) \to \pi^+\pi^-\pi^0 J/\psi, \end{cases}$$

(discuused later)

where $C_{\{PV\}}$ has the same flavor as the cript-exotic PV system.

$Z(4430)^+$, $Z_1(4050)^+$, $Z_2(4250)^+$

$$\left.\begin{array}{ll} Z(4430)^+ & \to \psi'\pi^+ \\ & \not\to J/\psi\pi^+ \\ Z_1(4050)^+ & \to \chi_{c1}\pi^+ \\ Z_2(4250)^+ & \to \chi_{c1}\pi^+ \end{array}\right\} \Rightarrow \underline{\text{exotic}}, \text{ if } \left\{\begin{array}{l} \text{isospin is conserved} \\ \text{and} \\ \text{the OZI-rule works.} \end{array}\right.$$

$X(3740)$, $Y(3940)$, $Y(4260)$, $Y(4360)$, $Y(4660)$, $Z(3930)$, $\cdots$

$$\left\{\begin{array}{l} \text{Charmonium (radially/orbitally excited) ?} \\ \text{Hybrid ?} \\ \text{Molecule ?} \\ \text{Tetra-quark ?} \end{array}\right.$$

§2. Classification of Tetra-Quark Mesons

- **Light four-quark mesons:** Jaffe, PRD 15, 267 and 281 (1977)

  Symmetry property of flavor wavefunctions:

$\{qq\bar{q}\bar{q}\}$	$=$	$[qq][\bar{q}\bar{q}]$ $\oplus$	$(qq)(\bar{q}\bar{q})$ $\oplus$	$\{[qq](\bar{q}\bar{q}) \oplus (qq)[\bar{q}\bar{q}]\}$	
$SU_f(3)$		$\bar{\mathbf{3}}_f \times \mathbf{3}_f$	$6_f \times \bar{6}_f$	$\bar{\mathbf{3}}_f \times \bar{\mathbf{6}}_f \oplus \mathbf{6}_f \times \mathbf{3}_f$	
Color		$\bar{\mathbf{3}}_c \times \mathbf{3}_c$ $(\mathbf{6}_c \times \bar{\mathbf{6}}_c)$	$\bar{3}_c \times 3_c$ $(6_c \times \bar{6}_c)$	$\bar{\mathbf{3}}_c \times \mathbf{3}_c$ $(\mathbf{6}_c \times \bar{\mathbf{6}}_c)$	(Large mixing)
J^P		0^+ $(0^+, 1^+, 2^+)$	$0^+, 1^+, 2^+$ (0^+)	1^+ (1^+)	
		$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	
		$a_0(980)$, $f_0(980)$ $\kappa(800)$ $f_0(600)$	No $(K\pi)_{I=3/2}$ scalar resonance at $E \lesssim 1.8$ GeV	???	

- Large mixing between $\bar{\mathbf{3}}_c \times \mathbf{3}_c$ and $\mathbf{6}_c \times \bar{\mathbf{6}}_c$ $\Leftarrow$ Non-perturbative QCD
- Broad widths of light scalar tetra-quark mesons
  ($f_0(600) = \hat{\sigma}$ and $a_0 = \hat{\delta}^s$) (discussed later)

– **Light scalar $[qq][\bar{q}\bar{q}]$ mesons:** Jaffe, PRD 15, 267 and 281 (1977)

	$S=1$	$S=0$	
$I=1$		$\left.\begin{array}{l}\hat{\delta}^s \\ \hat{\delta}^{s*}\end{array}\right\} \sim [ns][\bar{n}\bar{s}]_{I=1}$	
$I=\frac{1}{2}$	$\left.\begin{array}{l}\hat{\kappa} \\ \hat{\kappa}^*\end{array}\right\} \sim [ud][\bar{n}\bar{s}]$		
$I=0$		$\left.\begin{array}{l}\hat{\sigma}^s \\ \hat{\sigma}^{s*}\end{array}\right\} \sim [ns][\bar{n}\bar{s}]_{I=0}$	$\left.\begin{array}{l}\hat{\sigma} \\ \hat{\sigma}^*\end{array}\right\} \sim [ud][\bar{u}\bar{d}]$
(Mass (‡) (GeV)	0.90 1.60*	1.10 1.80*	0.65 1.45*)
Candidate	$\kappa(800)$ –	$a_0(980)$, $f_0(980)$ –	$f_0(600)$ –

(‡) MIT bag model:

$\Rightarrow$ ♡ Mass hierarchy of the observed low-lying scalar mesons
♡ Approximate degeneracy between $a_0(980)$ and $f_0(980)$

- **Tetra-quark mesons with charm quark(s), ($Q = u,\ d,\ s,\ c$):**

$$\begin{array}{lcccc} \{QQ\bar{Q}\bar{Q}\} = & [QQ][\bar{Q}\bar{Q}] & \oplus\ (QQ)(\bar{Q}\bar{Q}) & \oplus\ \{[QQ](\bar{Q}\bar{Q}) \oplus (QQ)[\bar{Q}\bar{Q}]\} & \\ SU_f(4) & \mathbf{36}_f & \mathbf{100}_f & \overline{\mathbf{60}}_f \oplus \mathbf{60}_f & \\ J^P & 0^+ & 0^+,\ 1^+,\ 2^+ & 1^+ & \Leftarrow\ \bar{\mathbf{3}}_c \times \mathbf{3}_c \\ & (0^+,\ 1^+,\ 2^+) & (0^+) & (1^+) & \Leftarrow\ (\mathbf{6}_c \times \bar{\mathbf{6}}_c) \end{array}$$

  - $[QQ][\bar{Q}\bar{Q}] \sim \mathbf{20}''_f \oplus \mathbf{15}_f \oplus \mathbf{1}_f$ **of** $SU_f(4)$ **with** $J^P = 0^+,\ (0^+, 1^+, 2^+)$
  - $(QQ)(\bar{Q}\bar{Q}) \sim \mathbf{84}_f \oplus \mathbf{15}_f \oplus \mathbf{1}_f$ **of** $SU_f(4)$ **with** $J^P = 0^+,\ 1^+,\ 2^+,\ (0^+)$
  - $\left\{ \begin{array}{l} [QQ](\bar{Q}\bar{Q}) \sim \overline{\mathbf{60}}_f = \overline{\mathbf{45}}_f \oplus \mathbf{15}_f \\ (QQ)[\bar{Q}\bar{Q}] \sim \mathbf{60}_f = \mathbf{45}_f \oplus \mathbf{15}_f \end{array} \right\}$ **with** $J^P = 1^+,\ (1^+)$

    States with the same quantum numbers in the $[QQ](\bar{Q}\bar{Q})$ **and** $(QQ)[\bar{Q}\bar{Q}]$ **can mix with each other.**

- **Open-charm tetra-quark mesons:**
  - $\{cq\bar{q}\bar{q}\}$

$$
\begin{array}{lccccc}
\{cq\bar{q}\bar{q}\} & = & [cq][\bar{q}\bar{q}] & \oplus & (cq)(\bar{q}\bar{q}) & \oplus & \{[cq](\bar{q}\bar{q}) \oplus (cq)[\bar{q}\bar{q}]\} \\
SU_f(3) & & \mathbf{3}_f \times \mathbf{3}_f & & \mathbf{3}_f \times \bar{\mathbf{6}}_f & & \mathbf{3}_f \times \bar{\mathbf{6}}_f \oplus \mathbf{3}_f \times \mathbf{3}_f \\
\text{Color} & \left\{ \begin{array}{l} \\ \\ \end{array} \right. & \begin{array}{c} \bar{\mathbf{3}}_c \times \mathbf{3}_c \\ (\mathbf{6}_c \times \bar{\mathbf{6}}_c) \end{array} & & \begin{array}{c} \bar{\mathbf{3}}_c \times \mathbf{3}_c \\ (\mathbf{6}_c \times \bar{\mathbf{6}}_c) \end{array} & & \begin{array}{c} \bar{\mathbf{3}}_c \times \mathbf{3}_c \\ (\mathbf{6}_c \times \bar{\mathbf{6}}_c) \end{array} \\
J^P & \left\{ \begin{array}{l} \\ \\ \end{array} \right. & \begin{array}{c} 0^+ \\ (0^+,\ 1^+,\ 2^+) \end{array} & & \begin{array}{c} 0^+,\ 1^+,\ 2^+ \\ (0^+) \end{array} & & \begin{array}{c} 1^+ \\ (1^+) \\ \Downarrow \end{array}
\end{array}
$$

Narrow resonance with $J^P = 1^+$: $D_{s1}(2460)$, $D_{s1}(2530)$?

  * Mass of $\bar{3}_c \times 3_c$ $<$ Mass of $6_c \times \bar{6}_c$
  * Small mixing between $\bar{3}_c \times 3_c$ and $6_c \times \bar{6}_c$
  * Narrow widths of $[cq][\bar{q}\bar{q}]$ mesons with $\bar{3}_c \times 3_c$

    $\Uparrow$

    Small overlap of color and spin wavefunctions (discussed later)

  - The other open-charm tetra-quark mesons:

$$
\left. \begin{array}{l} (cc)[\bar{q}\bar{q}] \\ (cc)[\bar{q}\bar{c}] \end{array} \right\} \text{ and their anti-particles with } J^P = 1^+
$$

– **Open-charm scalar $[cq][\bar{q}\bar{q}]$ mesons (with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$):**

K. T., PRD 68, 011501(R) (2003)

$I = 1$	$I = \frac{1}{2}$	$I = 0$	Mass (‡) (GeV)	Candidate
$\hat{F}_I \sim [cn][\bar{s}\bar{n}]_{I=1}$			**2.32**	$D_{s0}^{+}(2317) = \tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^0]$;
		$\hat{F}_0^{+} \sim [cn][\bar{s}\bar{n}]_{I=0}$	**2.32**	$\tilde{D}_{s0}^{+}(2317)[D_s^{*+}\gamma]$
	$\hat{D} \sim [cn][\bar{u}\bar{d}]$ $\hat{D}^s \sim [cs][\bar{n}\bar{s}]$		**2.22** **2.42**	
		$\hat{E}^0 \sim [cs][\bar{u}\bar{d}]$	**2.32**	

(‡) **Quark counting with**

$$\begin{cases} \Delta_{sn} = m_s - m_n \simeq m_{D_s} - m_D \simeq 0.1 \text{ GeV} \\ m_{\hat{F}_I^{+}} = m_{D_{s0}} \simeq 2.32 \text{ GeV (input data)} \end{cases}$$

- Narrow width of $\hat{F}_I^+ = D_{s0}^+(2317) \Leftarrow$ Small rate for $\hat{F}_I^+ \to D_s^+ \pi^0$

$\nearrow$

Small overlap of color and spin wavefunctions (w.f.) :

$$
\begin{array}{c}
\hat{F}_I \\ \Downarrow \\ |[cn]_{\bar{3}_c}^{1_s}[\bar{s}\bar{n}]_{3_c}^{1_s}\rangle_{1_c}^{1_s}
\end{array}
= \overbrace{-\sqrt{\frac{1}{4}}\times\sqrt{\frac{1}{3}}}^{\beta_0(\hat{F}_I)}\,|\overset{D_s^+}{\overset{\Downarrow}{\{c\bar{s}\}_{1_c}^{1_s}}}\overset{\pi^0}{\overset{\Downarrow}{\{n\bar{n}\}_{1_c}^{1_s}}}\rangle_{1_c}^{1_s} + \overbrace{\sqrt{\frac{3}{4}}\times\sqrt{\frac{1}{3}}}^{\beta_1(\hat{F}_I)}\,|\overset{D_s^{*+}}{\overset{\Downarrow}{\{c\bar{s}\}_{1_c}^{3_s}}}\overset{\rho^0}{\overset{\Downarrow}{\{n\bar{n}\}_{1_c}^{3_s}}}\rangle_{1_c}^{1_s}
$$

$$
-\underset{\text{spin}}{\underset{\Uparrow}{\sqrt{\frac{1}{4}}}}\times\underset{\text{color}}{\underset{\Uparrow}{\sqrt{\frac{2}{3}}}}\,|\{c\bar{s}\}_{8_c}^{1_s}\{n\bar{n}\}_{8_c}^{1_s}\rangle_{1_c}^{1_s} + \underset{\text{spin}}{\underset{\Uparrow}{\sqrt{\frac{3}{4}}}}\times\underset{\text{color}}{\underset{\Uparrow}{\sqrt{\frac{2}{3}}}}\,|\{c\bar{s}\}_{8_c}^{3_s}\{n\bar{n}\}_{8_c}^{3_s}\rangle_{1_c}^{1_s}
$$

- Broad $\hat{\delta}^s = a_0(980)$
  - Large mixing between $\bar{3}_c \times 3_c$ and $6_c \times \bar{6}_c$
  - Full reshuffling of the decomposition in the light mesons

non-perturbative QCD

$\Rightarrow$ Large overlap of color and spin w.f. between $\hat{\delta}^s = a_0(980)$ and $\eta\pi$

$$1 \sim \beta_0(\hat{\delta}^s) \gg \beta_0(\hat{F}_I) = \beta_0$$

- **Dominant decays of scalar $[cq][\bar{q}\bar{q}]$ mesons.**

$$\left\{\begin{array}{l}\text{Current algebra} \oplus \text{broken } SU_f(4) \text{ symmetry with } \beta_0(\hat{F}_{I^+}) \simeq 1/12 \\ \text{Input data: } \Gamma(a_0 \to \eta\pi)_{\text{exp}} = 50 - 100 \text{ MeV}\end{array}\right.$$

Parent(GeV)	Final State	Rate(MeV)
$\hat{F}_I^{++}(2.32)$	$D_s^+\pi^+$	$\sim 2-5$
$\hat{F}_I^{+}(2.32)$	$D_s^+\pi^0$	
$\hat{F}_I^{0}(2.32)$	$D_s^+\pi^-$	
$\hat{D}^+(2.24)$	$D^0\pi^+$	$\sim 2-5$
	$D^+\pi^0$	$\sim 1-2$
$\hat{D}^0(2.24)$	$D^+\pi^-$	$\sim 2-5$
	$D^0\pi^0$	$\sim 1-2$
$\hat{D}^s(2.40)$	$D^{*+}\gamma$	$\ll 2-5$
$\hat{F}_0^+(2.32)$	$D_s^{*+}\gamma$	$\sim 0.002(*)$
$\hat{E}^0(2.32)$	$\langle D\bar{K}\rangle$	(weak int.)

(∗) Vector Meson Dominance (VMD) $\oplus$ Broken $SU_f(4)$

with $\beta_1(\hat{F}_I^+) = 1/4$

- **Open-charm axial-vector $[cq](\bar{q}\bar{q}) \pm (cq)[\bar{q}\bar{q}]$ mesons with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$**

S	$I=3/2$	$I=1$	$I=1/2$	$I=0$	Candidate
2			$E_{\{FK^*\}}$		
1		$\left.\begin{array}{l} E_{\{\pi F^*\}}(\overline{60}_f) \\ E_{\{\pi F^*\}}(60_f) \end{array}\right\}$		$\left.\begin{array}{l} C_{\{\eta^0 F^*\}}(\overline{60}_f) \\ C_{\{\eta^0 F^*\}}(60_f) \end{array}\right\}$ $C_{\{\eta^s F^*\}}(\overline{60}_f)$ $C_{\{\eta^s F^*\}}(60_f)$	$D_{s1}^+(2460)$? $D_{s1}^+(2536)$?
0	$E_{\{\pi D^*\}^{3/2}}(\overline{60}_f)$		$C_{\{\pi D^*\}^{1/2}}(60_f)$ $C_{\{\eta^0 D^*\}}(\overline{60}_f)$ $C_{\{\eta^s D^*\}}(\overline{60}_f)$ $C_{\{\eta^s D^*\}}(60_f)$		
-1		$E_{\{\bar{K}D^*\}^1}(\overline{60}_f)$		$E_{\{\bar{K}D^*\}^0}(60_f)$	

? $\left\{ \begin{array}{l} \spadesuit \text{ Large mass difference — conventional and/or tetra ?} \\ \heartsuit \ \dfrac{\Gamma(D_{s1}^+(2460) \to D_s^+\gamma)}{\Gamma(D_{s1}(2460) \to D_s^{*+}\pi^0)}\Big|_{\text{exp}} = 0.31 \pm 0.06 \\ \heartsuit \ D_{s1}(2536) \text{ is very narrow, although it has } \underline{\text{strong}} \text{ decay modes.} \end{array} \right.$

- **Hidden-charm tetra-quark mesons:**

$$\begin{array}{lccccc}
\{cq\bar{c}\bar{q}\} & = & [cq][\bar{c}\bar{q}] & \oplus & (cq)(\bar{c}\bar{q}) & \oplus & \{[cq](\bar{c}\bar{q}) \oplus (cq)[\bar{c}\bar{q}]\} \\
SU_f(3) & & \mathbf{3_f} \times \mathbf{3_f} & & \mathbf{3_f} \times \bar{\mathbf{3}}_\mathbf{f} & & \mathbf{3_f} \times \bar{\mathbf{3}}_\mathbf{f} \oplus \mathbf{3_f} \times \bar{\mathbf{3}}_f \\
\text{Color} & \left\{ \begin{array}{c} \\ \\ \end{array} \right. & \begin{array}{c} \bar{\mathbf{3}}_\mathbf{c} \times \mathbf{3_c} \\ (\mathbf{6_c} \times \bar{\mathbf{6}}_\mathbf{c}) \end{array} & & \begin{array}{c} \bar{\mathbf{3}}_\mathbf{c} \times \mathbf{3_c} \\ (\mathbf{6_c} \times \bar{\mathbf{6}}_\mathbf{c}) \end{array} & & \begin{array}{c} \bar{\mathbf{3}}_\mathbf{c} \times \mathbf{3_c} \\ (\mathbf{6_c} \times \bar{\mathbf{6}}_\mathbf{c}) \end{array} \\
J^P & \left\{ \begin{array}{c} \\ \\ \end{array} \right. & \begin{array}{c} 0^+ \\ (0^+, 1^+, 2^+) \end{array} & & \begin{array}{c} 0^+, 1^+, 2^+ \\ (0^+) \end{array} & & \begin{array}{c} 1^+ \\ (1^+) \end{array}
\end{array}$$

- Mass of $\bar{\mathbf{3}}_c \times \mathbf{3}_c$ $<$ Mass of $\mathbf{6}_c \times \bar{\mathbf{6}}_c$
- Small mixing between $\bar{\mathbf{3}}_\mathbf{c} \times \mathbf{3_c}$ and $\mathbf{6_c} \times \bar{\mathbf{6}}_\mathbf{c}$
- Narrow widths of $[cq][\bar{c}\bar{q}]$ mesons with $\bar{\mathbf{3}}_\mathbf{c} \times \mathbf{3_c}$

  $\Uparrow$

  Small overlap of color and spin wavefunctions

– **Hidden-charm scalar $[cq][\bar{c}\bar{q}]$ mesons (with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$):**

Strangeness (S)	1	0		
$I = 1$		$\hat{\delta}^c \sim [nc][\bar{n}\bar{c}]_{I=1}$		
$I = \frac{1}{2}$	$\hat{\kappa}^c \sim [nc][\bar{s}\bar{c}]$			
$I = 0$			$\hat{\sigma}^c \sim [nc][\bar{c}\bar{n}]_{I=0}$	$\hat{\sigma}^{sc} \sim [sc][\bar{c}\bar{s}]$
Mass (GeV) (‡)	$\sim$3.4	$\sim$3.3	$\sim$3.3	$\sim$3.5
OZI-allowed Decay	$\eta_c K$	$\eta_c \pi$	$\eta_c \eta$	$\eta_c \eta$
Threshold (GeV)	3.48	3.12	3.53	3.53

(‡) Quark counting with
$$\begin{cases} \Delta_{cs} = m_c - m_s \simeq m_{\eta_c} - m_{D_s} \simeq 1 \text{ GeV} \\ \Delta_{sn} = m_s - m_n \simeq m_{D_s} - m_D \simeq 0.1 \text{ GeV} \\ m_{\hat{F}_I^+} = m_{D_{s0}} \simeq 2.32 \text{ GeV (input data)} \end{cases}$$

Masses much lower than the other results:

∗ Unitarized chiral model: Gamermann et al., PRD 76, 074016(2007)

$$m_{X(3700)} \gg m_{\hat{\sigma}^c}$$

∗ Preceding $[cq][\bar{c}\bar{q}] \Leftarrow X(3872)$ as the input data:

$$m_{X(3723)} \gg m_{\hat{\sigma}^c}$$

Maiani et al., PRD 71, 014028 (2005)

$*$ **OZI-rule-allowed strong decay :**

$\left\{\begin{array}{l}\textbf{Current algebra} \oplus \textbf{Broken } SU_f(4) \textbf{ with } \beta_0(\hat{\delta}^c) \simeq \beta_0(\hat{F}_I^+) \\ \textbf{Input data: } \Gamma(\hat{F}_I^+(=D_{s0}^+(2317)) \to D_s^+\pi^0) \sim 2-5\ \text{MeV}\end{array}\right.$

- $\Gamma(\hat{\delta}^c \to \eta_c\pi) \sim 2-5$ **MeV**
- **No other OZI-rule allowed decay of** $[cq][\bar{c}\bar{q}]$ **scalar mesons**

$$\Downarrow$$

- **The** $[cq][\bar{c}\bar{q}]$ **mesons are narrow.**
- **Radiative decays are important.**

$*$ **Radiative decays of scalar $[cq][\bar{c}\bar{q}]$ mesons (with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$):**

$$\left\{ \begin{array}{l} \textbf{Vector meson dominance} \oplus \textbf{broken } SU_f(4) \textbf{ with} \\ \qquad |\beta_1| \simeq |\beta_1(\textbf{hidden-charm})| \simeq |\beta_1(\hat{F}_I^+)| \simeq \sqrt{\tfrac{1}{4}} \\ \textbf{Input data: } \Gamma(\phi \to a_0(980)\gamma)_{\rm exp} = 0.32 \pm 0.03 \textbf{ keV} \end{array} \right.$$

Decay	Pole	$\Gamma(S \to V\gamma)$ (in keV)
$\hat{F}_I^+ \to D_s^{*+}\gamma$	ρ^0	$\sim$ 20
$\hat{F}_0^+ \to D_s^{*+}\gamma$	ω	$\sim$ 2
$\hat{\kappa}^c \to K^*\gamma$	ψ	$\sim$ 100
$\hat{\delta}^c \to \rho\gamma$	ψ	$\sim$ 100
$\hat{\delta}^c \to \psi\gamma$	ρ^0	$\sim$ 30 − 40
$\hat{\sigma}^c \to \omega\gamma$	ψ	$\sim$ 100
$\hat{\sigma}^c \to \psi\gamma$	ω	$\sim$ 3 − 4
$\hat{\sigma}^{sc} \to \phi\gamma$	ψ	$\sim$ 100
$\hat{\sigma}^{sc} \to \psi\gamma$	ϕ	$\sim$ 20

· $\Gamma(\hat{\sigma}^c \to \omega\gamma) \gg \Gamma(\hat{\sigma}^c \to \psi\gamma) \Leftrightarrow \Gamma(X(3700) \to \psi\gamma) \sim 20$ **keV**

Unitarized model (loop contributions):

Gamermann et al., PRC 76, 055205 (2007)

- **Hidden-charm axial-vector $[cq](\bar{c}\bar{q}) \oplus (cq)[\bar{c}\bar{q}]$ mesons with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$**

S	1	0			-1
$I=1$		$C_{\{\eta_c\rho\}}(\pm)$			
$I=1/2$	$C_{\{\eta_c K^*\}}(\overline{60}_f)$ $C_{\{\eta_c K^*\}}(60_f)$				$C_{\{\eta_c \bar{K}^*\}}(\overline{60}_f)$ $C_{\{\eta_c \bar{K}^*\}}(60_f)$
$I=0$			$C_{\{\eta_c\omega\}}(\pm)$	$C_{\{\eta_c\phi\}}(\pm)$	
Mass $(*)$ (GeV)	~ 4.0	$\sim 3.9(\ddagger)$		~ 4.1	~ 4.0
Candidate			$X(3872)$		

$(*)$ Quark counting, $(\ddagger)$ Input data

$$\left.\begin{array}{l} C_{\{\eta_c\omega\}}(\overline{60}_f) \sim \{[cn](\bar{c}\bar{n})\}_{I=0} \\ C_{\{\eta_c\omega\}}(60_f) \sim \{(cn)[\bar{c}\bar{n}]\}_{I=0} \end{array}\right\rangle \text{ are not eigenstates of } G\text{-parity}$$

$\Downarrow$ $C_{\{\eta_c\omega\}}(60_f)$-$C_{\{\eta_c\omega\}}(\overline{60}_f)$ mixing to form G-parity eigenstates

Two opposite G-parity eigenstates with (approximately) degenerate masses

$$\Rightarrow X(3872) = \begin{cases} C_{\{\eta_c\omega\}}(+) \rightarrow J/\psi\pi\pi\pi, \\ C_{\{\eta_c\omega\}}(-) \rightarrow J/\psi\pi\pi, \quad \text{(Search for } \rightarrow J/\psi\pi^0\pi^0 \text{)} \end{cases}$$

§3. Conventional Open-Charm Scalar Mesons

- Current algebra $\oplus$ broken $SU_f(4)$ ($\Leftarrow$ 20 -30 % deviation of s.w.f. overlap)

  Input data: $\begin{cases} m_{K_0^*} = 1412 \pm 6 \text{ MeV}, \\ \Gamma_{K_0^*} = 294 \pm 23 \text{ MeV}, \quad Br(K_0^* \to K\pi) = 93 \pm 10 \text{ \%} \end{cases}$

  $\underline{D_0^* \sim \{c\bar{n}\}}$

  – Mass: $m_{D_0^*} \simeq 2.35$ GeV (tentative)

  – Width: $\Gamma(D_0^*) \simeq \Gamma(D_0^* \to D\pi) \sim 50$ MeV $\ll \Gamma(D_0)_{\text{Belle}}$

  ♢ Re-analyze the $D\pi$ mass distribution just below the D_2^* peak
  Find $\begin{cases} \text{the conventional } D_0^* \text{ resonance with} \sim 50 \text{ MeV width} \\ \text{the very narrow tetra-quark } \hat{D} \text{ meson} \end{cases}$
  Kill models [e.g. W. A. Bardeen et al., PRD 68, 054024 (2003)]
  providing $\Gamma(\{D_{s0}^+\}_{I=0} \to D_s^+\pi^0) \gg \Gamma(\{D_{s0}^+\}_{I=0} \to D_s^{*+}\gamma)$,
  for example, $\Gamma(D_{s0}^{*+} \to D_s^+\pi^0) \gtrsim 20$ keV
  $\Rightarrow \Gamma(D_0^* \to D\pi^0) \gtrsim 500 \text{ MeV} > \Gamma(D_0^*)_{\text{Belle}} \gg \Gamma(D_0^*)$

  $\underline{D_{s0}^{*+} \sim \{c\bar{s}\}}$

  – Mass: $m_{D_{s0}^{*+}} \simeq m_{D_0^*} + \Delta_{sn} \simeq 2.45$ GeV

  – Width: $\Gamma(D_{s0}^*) \simeq \Gamma(D_{s0}^* \to DK) \sim 40$ MeV

  ♢ Search for a mildly broad DK peak in the region $2.4 - 2.5$ GeV

K.T. and B.H. McKellar, PTP 114, 205 (2005)

§4. Summary and Discussion

- Signals of existence of tetra-quark mesons :
  - Ratio of decay rates

$$\left.\frac{\Gamma(D_{s0}^{+}(2317) \to D_s^{*+}\gamma)}{\Gamma(D_{s0}^{+}(2317) \to D_s^{+}\pi^0)}\right|_{\text{CLEO}} < 0.059.$$

$\Rightarrow D_{s0}^{+}(2317) = \hat{F}_I^{+} \sim [cn][\bar{s}\bar{n}]_{I=1}$

$\Rightarrow$ Existence of $\hat{F}_I^{++}$ and $\hat{F}_I^{0}$

$\diamond$ $\left\{\begin{array}{l}\text{Suppression of } \hat{F}_I^{++} \text{ and } \hat{F}_I^{0} \text{ production in } e^{+}\text{-}e^{-} \text{ annihilation} \\ \qquad \text{AIP Conf. Proc. } \underline{1030}\text{, 190 (2008); hep-ph/0804.2295} \\ \text{Search for } \hat{F}_I^{++} \text{ and } \hat{F}_I^{0} \text{ in } B \text{ decays}\end{array}\right.$

$$\left.\begin{array}{l} Br(B_u^{+} \to D^{-}\hat{F}_I^{++}[D_s^{+}\pi^{+}]) \\ Br(B_d^{0} \to \bar{D}^{0}\hat{F}_I^{0}[D_s^{+}\pi^{-}]) \end{array}\right\} \sim 10^{-3}$$

(K.T., PTP 116, 435 (2006); hep-ph/0604207)

  - Two decays of $X(3872)$ with opposite G-parities
    - $X(3872) \sim \{[cn](\bar{c}\bar{n}) \pm (cn)[\bar{c}\bar{n}]\}_{I=0}$

- **Tetra-quark mesons with charm quark(s), ($Q = u, d, s, c$):**

$$\{QQ\bar{Q}\bar{Q}\} = [QQ][\bar{Q}\bar{Q}] \oplus (QQ)(\bar{Q}\bar{Q}) \oplus \{[QQ](\bar{Q}\bar{Q}) \oplus (QQ)[\bar{Q}\bar{Q}]\}$$

	$[QQ][\bar{Q}\bar{Q}]$	$(QQ)(\bar{Q}\bar{Q})$	$\{[QQ](\bar{Q}\bar{Q}) \oplus (QQ)[\bar{Q}\bar{Q}]\}$	
$SU_f(4)$	$\mathbf{36_f}$	$\mathbf{100_f}$	$\overline{\mathbf{60}}_\mathbf{f} \oplus \mathbf{60_f}$	
J^P	0^+	$0^+, 1^+, 2^+$	1^+	$\Leftarrow \bar{\mathbf{3}}_c \times \mathbf{3}_c$
	$(0^+, 1^+, 2^+)$	(0^+)	(1^+)	$\Leftarrow (\mathbf{6}_c \times \bar{\mathbf{6}}_c)$

  - **Open- and hidden-charm scalar $[QQ][\bar{Q}\bar{Q}]$ with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$**
  - $(QQ)(\bar{Q}\bar{Q}) \Leftarrow$ **No $(K\pi)_{I=3/2}$ scalar resonance** $\subset (qq)(\bar{q}\bar{q})$
  - **Open- and hidden-charm axial-vector $[QQ](\bar{Q}\bar{Q}) \oplus (QQ)[\bar{Q}\bar{Q}]$ with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$**

    Opposite G-parity eigenstates with same flavor

$$\Rightarrow X(3872) = \begin{cases} C_{\{\eta_c\omega\}}(+) \rightarrow J/\psi\pi\pi\pi, \\ C_{\{\eta_c\omega\}}(-) \rightarrow J/\psi\pi\pi, \end{cases}$$

  - **Narrow resonances because of $\underbrace{\text{small overlap}}_{\Uparrow\ \bar{\mathbf{3}}_c \times \mathbf{3}_c}$ of color and spin w.f.**

– **Hidden-charm scalar tetra-quark mesons :**

  * **Narrow** $\Leftarrow \Gamma(\hat{\delta}^c \to \eta_c \pi) < 10$ **MeV**

    (The other decays are OZI-rule suppressed.)

  * **Radiative decays**

    $\Gamma(\hat{\sigma}^c \to \omega\gamma) \gg \Gamma(\hat{\sigma}^c \to \psi\gamma) \sim 3 - 4$ **keV** **(VMD)**

    $$\Updownarrow$$

    $\Gamma(X(3700) \to \psi\gamma) \sim 20$ **keV** **(Unitarized chiral model)**

- **Conventional open-charm scalar mesons:**

  – **Re-analyze the $D\pi$ enhancement just below the D_2^* peak**

  $$\Rightarrow \left\{ \begin{array}{l} \text{Find } D_0^* \sim \{c\bar{n}\} \text{ with } \Gamma(D_0^*) \sim 50 \text{ MeV} \\ \qquad\qquad \Rightarrow \text{Reject an artificially large } \Gamma(D_{s0}^{*+} \to D_s^+ \pi^0) \\ \text{Find } \hat{D}_0 \sim [cn][\bar{u}\bar{d}] \text{ with } \Gamma(\hat{D}) \lesssim 10 \text{ MeV} \end{array} \right.$$

  K.T. and McKellar, PTP <u>114</u>, 205 (2005)

  – **Search for the conventional charm-strange meson $D_{s0}^{*+} \sim \{c\bar{s}\}$**

  * **In DK mass distribution in exclusive $\underline{B \to \bar{D}DK}$ decays**

    $$\Uparrow$$

    spectator decay

  * **Find D_{s0}^{*+} as a mildly broad ($\sim 40 - 50$ MeV width) resonance in the $2.4 - 2.5$ GeV region**

Appendix

A-I. Hard Pion Technique

- A hard pion technique in the infinite momentum frame (IMF; $p \to \infty$):

Oneda and K.T., PTP Suppl., No.82, 1 (1985)

$$M(A \to B\pi) \simeq \left(\frac{m_A^2 - m_B^2}{f_\pi}\right)\langle B|A_\pi|A\rangle,$$

where
- $*$ A_π is the axial counterpart of isospin ($I = V_\pi$),
- $*$ $\langle B|A_{\bar{\pi}}|A\rangle$ is given by

$$\langle B(\mathbf{q}))|A_{\bar{\pi}}|A(\mathbf{p})\rangle_{\mathbf{k}\to 0,\mathbf{p}\to\infty} = (2\pi)^3\delta(\mathbf{p}-\mathbf{q})\langle B|A_{\bar{\pi}}|A\rangle\sqrt{N_A N_B}$$

- Application:

Compare $\hat{\delta}^c \to \eta_c\pi$ (and $\hat{F}_I^+ \to D_s^+\pi^0$) with $\hat{\delta}^{s+} = a_0(980) \to \eta\pi^+$

  - Asymptotic $SU_f(4)$:

$$\langle\eta_c|A_{\pi^-}|\hat{\delta}^{c+}\rangle = \sqrt{2}\langle\eta_c|A_{\pi^0}|\hat{\delta}^{c0}\rangle = \langle\eta_c|A_{\pi^+}|\hat{\delta}^{c-}\rangle = -\sqrt{2}\langle\eta_c|A_{\eta^s}|\hat{\sigma}^{sc}\rangle$$
$$\simeq \sqrt{2}\langle D_s^+|A_{\pi^0}|\hat{F}_I^+\rangle \simeq \langle\eta^s|A_{\pi^-}|\hat{\delta}^{s+}\rangle\beta_0.$$

  - Overlap of color and spin w.f. : $|\beta_0| \simeq |\beta_0(\hat{\delta}^c)| \simeq |\beta_0(\hat{F}_I)| \simeq \sqrt{\frac{1}{12}}$
  - Input data:

$$\left.\begin{array}{l} |\langle\eta^s|A_{\pi^-}|\hat{\delta}^{s+}\rangle| \\ \sim 0.68 - 0.96 \end{array}\right\} \Leftarrow \left\{\begin{array}{l} \Gamma(a_0(980) \to \eta\pi^+)_{\exp} \sim 50 - 100\ \mathrm{MeV} \\ \eta\text{-}\eta' \text{ mixing angle: } \theta_P \simeq -20^\circ \end{array}\right.$$

A-II. Vector Meson Dominance

- **V**ector **M**eson **D**ominance (VMD) with a broken flavor $SU_f(4)$:

$(V = \rho^0, \omega, \phi, \psi)$

– **Amplitude :**

$$M(S \to V\gamma) = F_{\mu\nu}(\gamma) G^{\mu\nu}(V) A(S \to V\gamma),$$

$$A(S \to V\gamma) = \sum_{V' = \rho,\, \omega,\, \phi,\, \psi} \frac{X_{V'}(0)}{m_{V'}^2} A(S \to VV')$$

– **Photon-vector meson coupling strengths on the photon mass-shell:**

$$X_\rho(0) = 0.033 \pm 0.003,\ X_\omega(0) = 0.011 \pm 0.001,$$
$$X_\phi(0) = -0.018 \pm 0.004,\ X_\psi(0) = 0.051 \pm 0.012$$

(K.T., NC 66A, 475 (1981))

– $SU_f(4)$ **relation among** SVV' **coupling strengths:**

$$A(\hat{\kappa}^{c+} \to K^{*+}\psi) = A(\hat{\kappa}^{c0} \to K^{*0}\psi) = A(\hat{\delta}^{c+} \to \rho^+\psi)$$
$$= A(\hat{\delta}^{c0} \to \rho^0\psi) = -A(\hat{\sigma}^{c+} \to \omega\psi) = -A(\hat{\sigma}^{sc+} \to \phi\psi) = \cdots$$
$$\simeq A(\hat{F}_I^+ \to \rho^0 D_s^{*+}) = \cdots \simeq A(\hat{\delta}^{s0} \to \rho^0\phi)\beta_1,$$