

**Fragmentation Function studied  
with e<sup>+</sup>-e<sup>-</sup> data and  
its impact on the nucleon spin structure  
analysis**

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# Contents

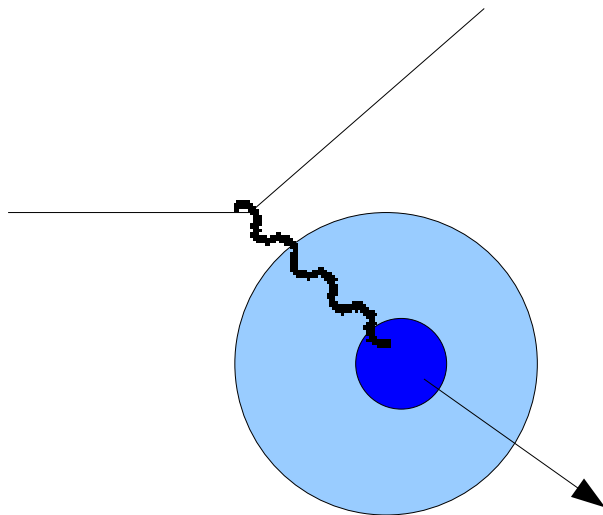
- Fragmentation and parton distribution function
- Resent fragmentation function analysis
- Impact on the proton spin structure analysis
  - Strange quark component and kaon fragmentation
- Quark transversity and Collins fragmentation function
- Fragmentation function analysis in Belle
- Summary

# Parton distribution and Fragmentation functions

$q(x)$

Probability of finding a parton with momentum fraction  $x$  of a parent proton  $q$

Deep inelastic scattering (DIS)

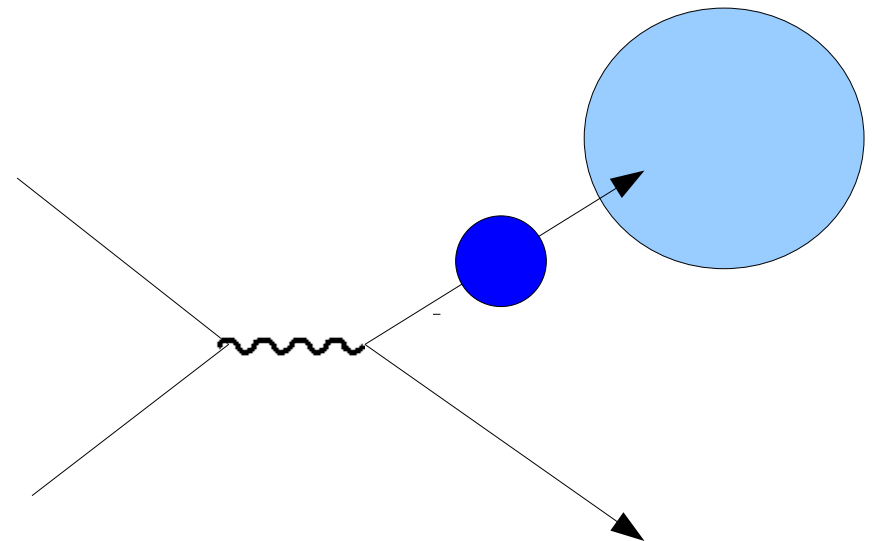


$$\sum_q \int_0^1 dx x \cdot q(x) = 1$$

$D_q^h(z)$

Probability of fragmenting to a hadron  $h$  with energy fraction  $z$  of a parent parton  $q$

Single inclusive anihilation (SIA)



$$\sum_h \int_0^1 dz z D_q^h(z) = 1$$

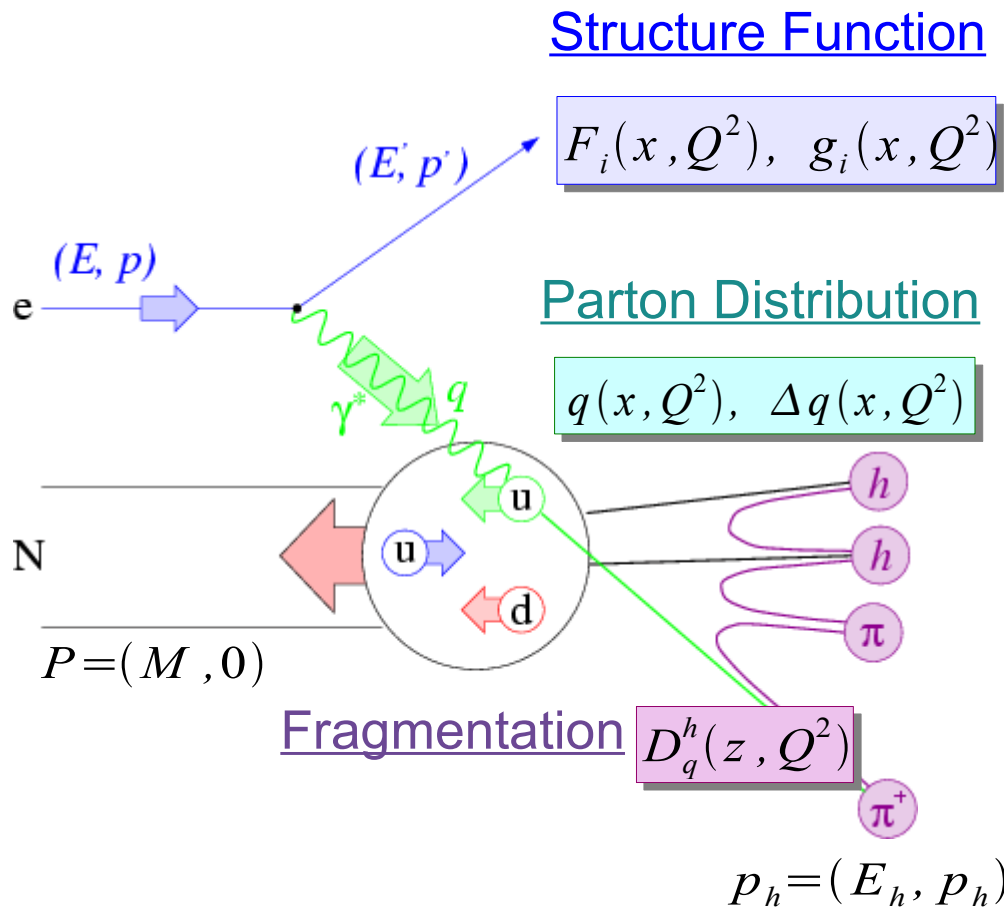
# Semi-inclusive DIS

$$\vec{l} + \vec{N} \rightarrow l' + h + X$$

$$\frac{d^3 \sigma^h}{dx dz dQ^2} \propto \sum_i e_q^2 q(x, Q^2) D_q^h(z, Q^2)$$

$$\frac{d^3 \Delta \sigma^h}{dx dz dQ^2} \propto \sum_i e_q^2 \Delta q(x, Q^2) D_q^h(z, Q^2)$$

$$z = \frac{P \cdot p_h}{P \cdot q} = \frac{E_h}{\nu}$$



FF and PDF are equally important to analyze high energy process with hadron production

## Flavor Tagging:

Hadron carries information on quark flavor through fragmentation function

# Fragmentation function parametrization

Parametrize functional form:

$$D(z) \propto N_i z^\alpha (1-z)^\beta \left[ 1 + \gamma (1-z)^\delta \right]$$

N: 2<sup>nd</sup> moment of D(z)

Constraint: no sensitivity on quark and anti-quark separation  
weak quark flavor sensitivity ( especially for light quarks: u, d, s )  
Favored – dis-favored fragmentation

$$D^{disfav} = (1-z) \cdot D^{fav}$$

Fit the available data from the experiments:  
Determine the parameters ( and possibly their uncertainty )

- |   |                                    |
|---|------------------------------------|
| - S. <b>Kretzer</b>   | Phys. Rev. D 62, 054001 (2000)     |
| - B.A. <b>Kniehl</b> , G. <b>Kramer</b> and B. <b>Pötter</b>                  | Nucl. Phys. B582, 514 (2000)       |
| - M. <b>Hirai</b> , S. <b>Kumano</b> , T.-H. <b>Nagai</b> , K. <b>Sudoh</b> . | Phys. Rev. D75, 094009 (2007)      |
| - <b>D.</b> de Florian, R. <b>Sassot</b> , M. <b>Stratmann</b>                | Phys. Rev. D75, 114010 (2007)      |
|   | Phys. Rev. Lett. 76, 074033 (2007) |

Including Hadron multiplicity in SIDIS and hadron cross-section in p-p  
(allow to loose some constraint)

# Available data for FF analysis

DSS, Phys. Rev. D75, 114010 (2007),

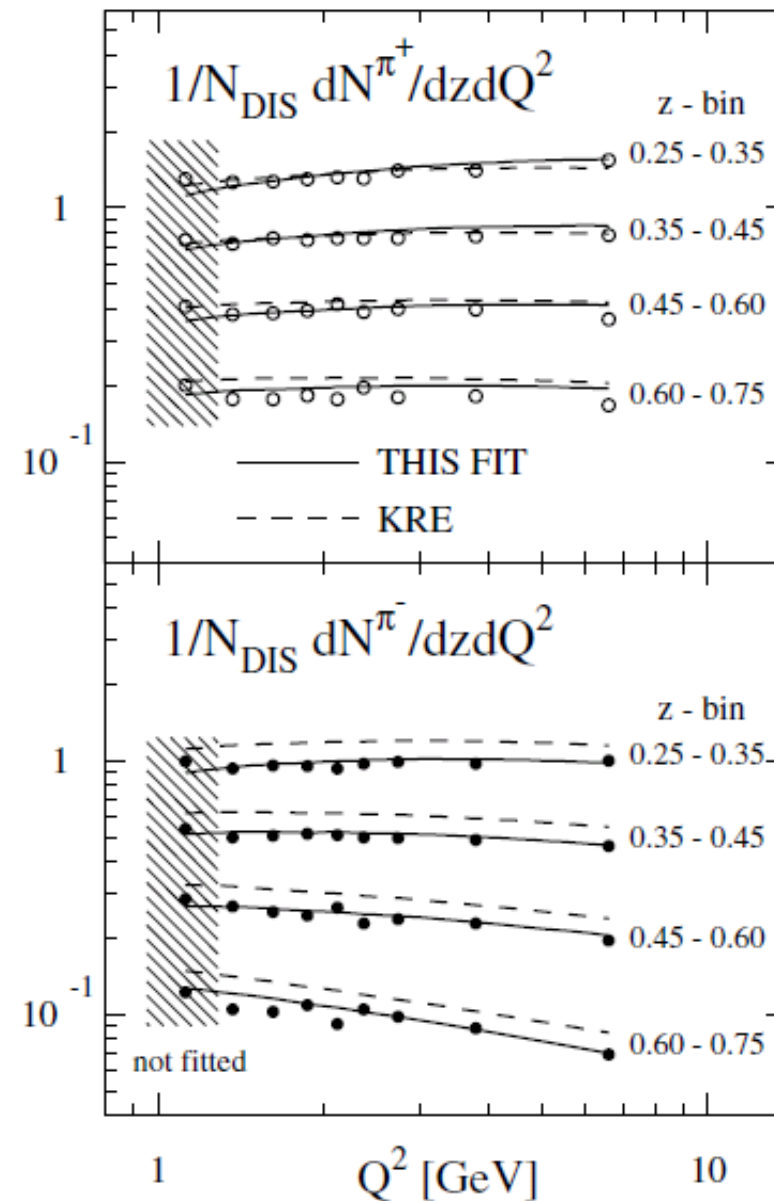
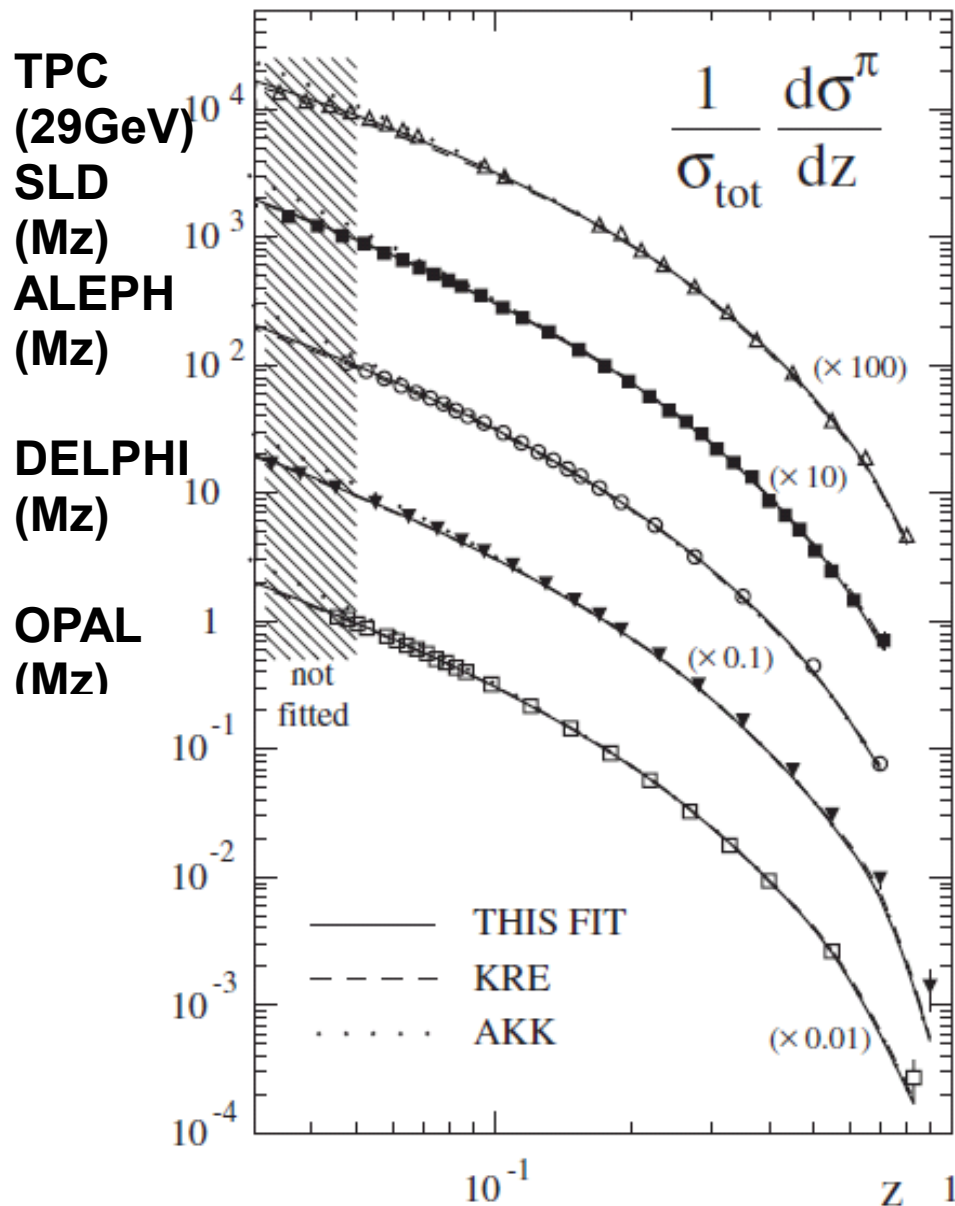
pion		Relative	Data	
Experiment	Data type	normalization in fit	points fitted	$\chi^2$
TPC [15] 29GeV	Inclusive	0.94	17	18.5
	“uds tag”	0.94	9	1.9
	“c tag”	0.94	9	5.7
	“b tag”	0.94	9	7.4
TASSO [38]	Inclusive (34 GeV)	0.94	11	30.1
	Inclusive (44 GeV)	0.94	7	20.5
SLD [16]	Inclusive	1.008	28	14.0
	“uds tag”	1.008	17	11.6
	“c tag”	1.008	17	11.1
	“b tag”	1.008	17	33.2
ALEPH [11]	Inclusive	0.97	22	38.3
DELPHI [12]	Inclusive	1.0	17	42.3
	“uds tag”	1.0	17	26.4
	“b tag”	1.0	17	42.8
OPAL [13,14]	Inclusive	1.0	21	9.2
	“u tag”	1.10	5	11.8
	“d tag”	1.10	5	9.0
	“s tag”	1.10	5	49.8
	“c tag”	1.10	5	38.3
	“b tag”	1.10	5	73.0
HERMES [17]	$\pi^+$	1.03	32	67.4
	$\pi^-$	1.03	32	120.8
PHENIX [18]	$\pi^0$	1.09	23	76.4
STAR [22]	$\pi^0, \langle \eta \rangle = 3.3$	1.05	4	3.4
	$\pi^0, \langle \eta \rangle = 3.7$	1.05	5	9.8
BRAHMS [21]	$\pi^+, \langle \eta \rangle = 2.95$	1.0	18	28.2
	$\pi^-, \langle \eta \rangle = 2.95$	1.0	18	43.0
Total			392	843.7

Phys. Rev. Lett. 76, 074033 (2007)

kaon		Relative	Data		
Experiment	Data type	normalization in fit	points fitted	$\chi^2$	
TPC [15] 29GeV	Inclusive	0.94	12	9.5	
	SLD [16]	Inclusive	0.983	18	14.4
		“uds tag”	0.983	10	14.4
		“c tag”	0.983	10	17.2
	“b tag”	0.983	10	15.2	
ALEPH [11]	Inclusive	0.97	13	12.3	
DELPHI [12]	Inclusive	1.0	12	1.0	
	“uds tag”	1.0	12	2.3	
	“b tag”	1.0	12	4.3	
OPAL [14]	“u tag”	1.10	5	6.5	
	“d tag”	1.10	5	9.9	
	“s tag”	1.10	5	36.8	
	“c tag”	1.10	5	44.9	
	“b tag”	1.10	5	18.6	
HERMES [17]	$K^+$	1.03	24	15.0	
	$K^-$	1.03	24	79.3	
STAR [20]	$K_S^0$	0.95	14	40.0	
BRAHMS [21]	$K^+, \langle \eta \rangle = 2.95$	1.0	18	28.8	
	$K^-, \langle \eta \rangle = 2.95$	1.0	18	21.5	
Total			232	394.1	

# Pion Fragmentation Analysis

DSS, Phys. Rev. D75, 114010 (2007)



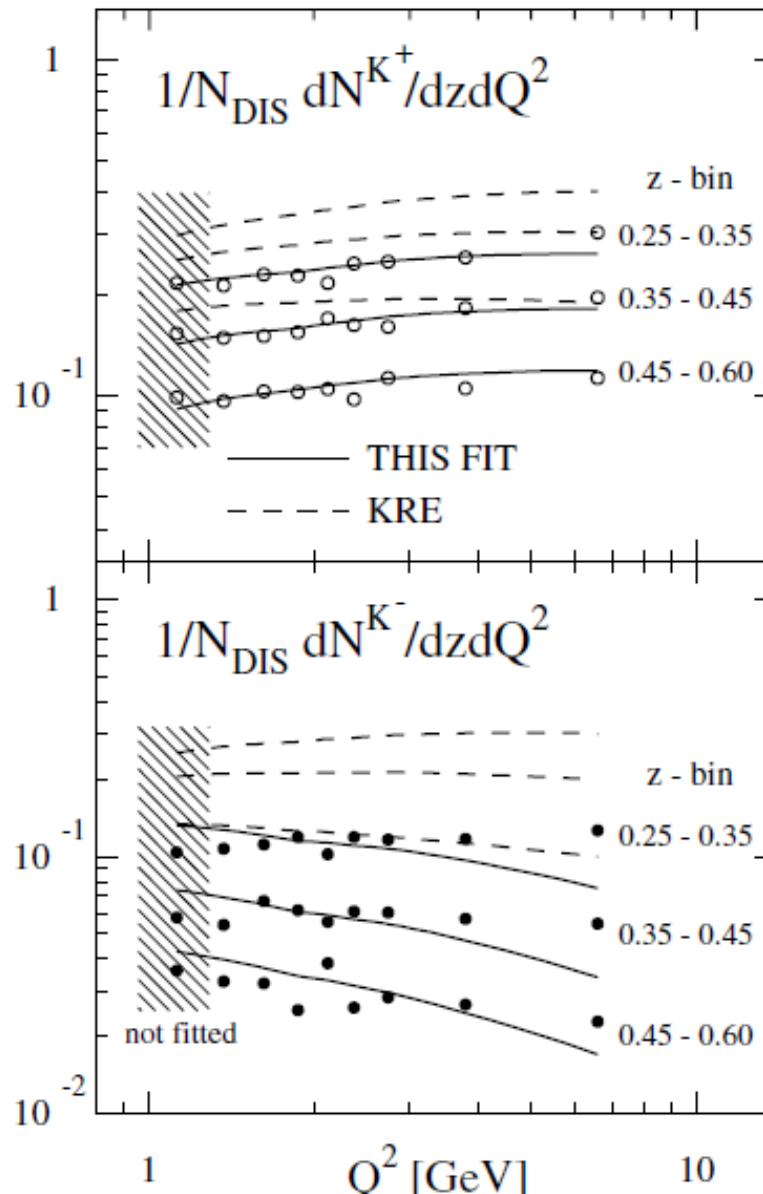
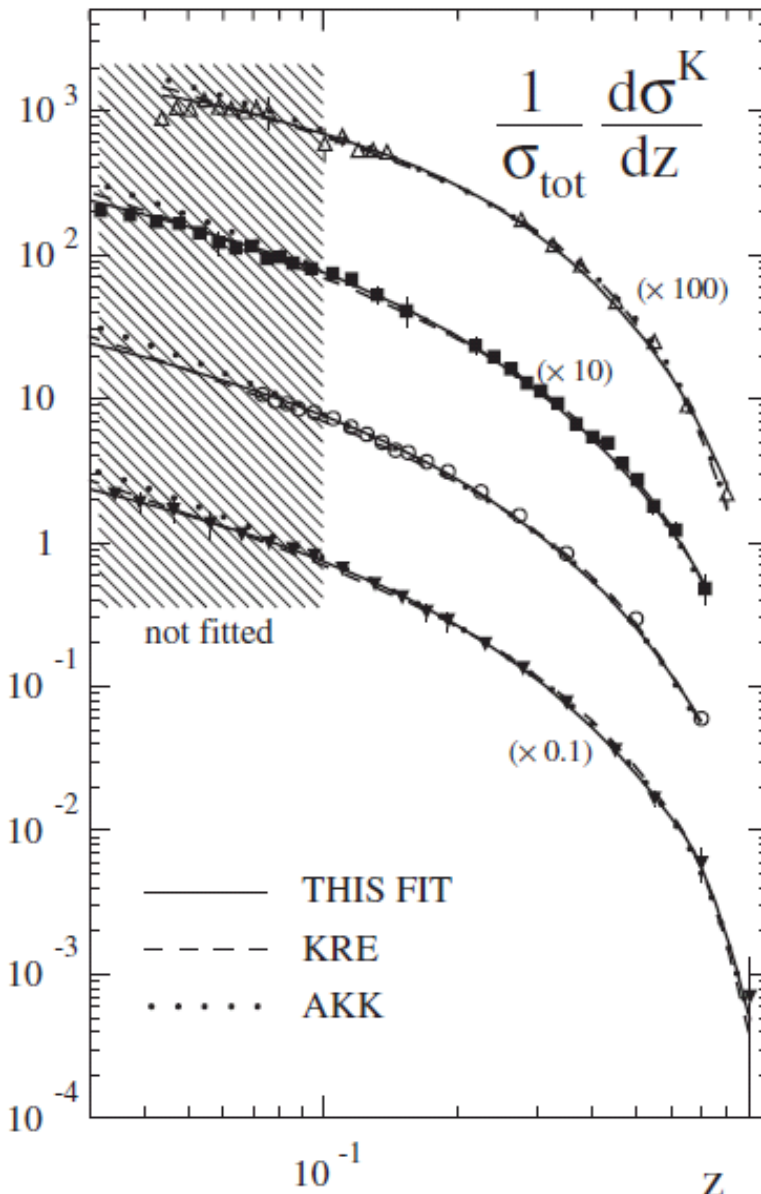
+ charged and neutral pions in pp



# Data used for kaon fragmentation

DSS, Phys. Rev. D75, 114010 (2007)

TPC  
(29GeV)  
SLD  
(Mz)  
ALEPH  
(Mz)  
DELPHI  
(Mz)



HERMES

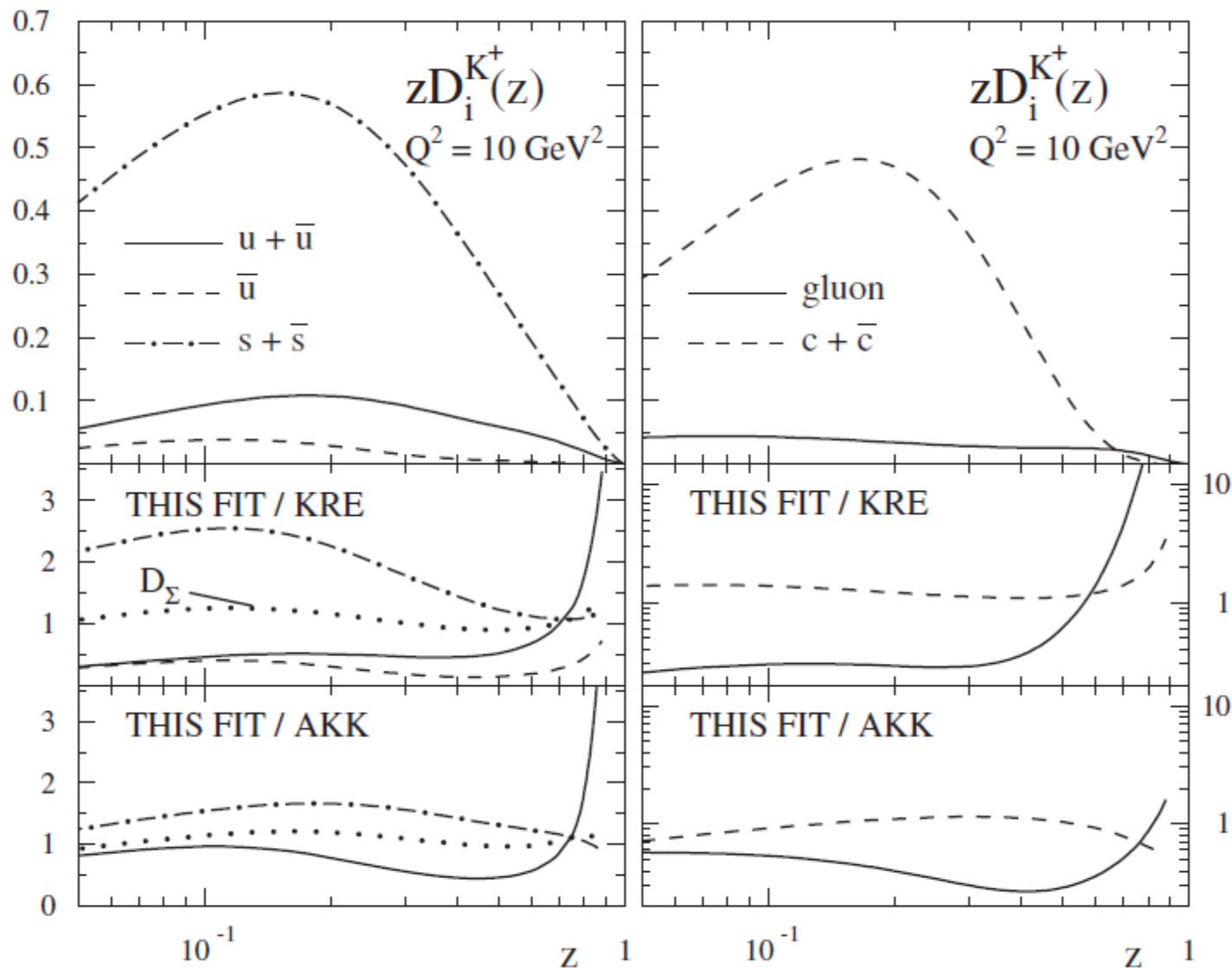
HERMES

+ charged kaon production in pp scattering



# Extracted kaon fragmentation function

DSS, Phys. Rev. D75, 114010 (2007)



DSS:  $e^+e^-$ , SIDIS, pp

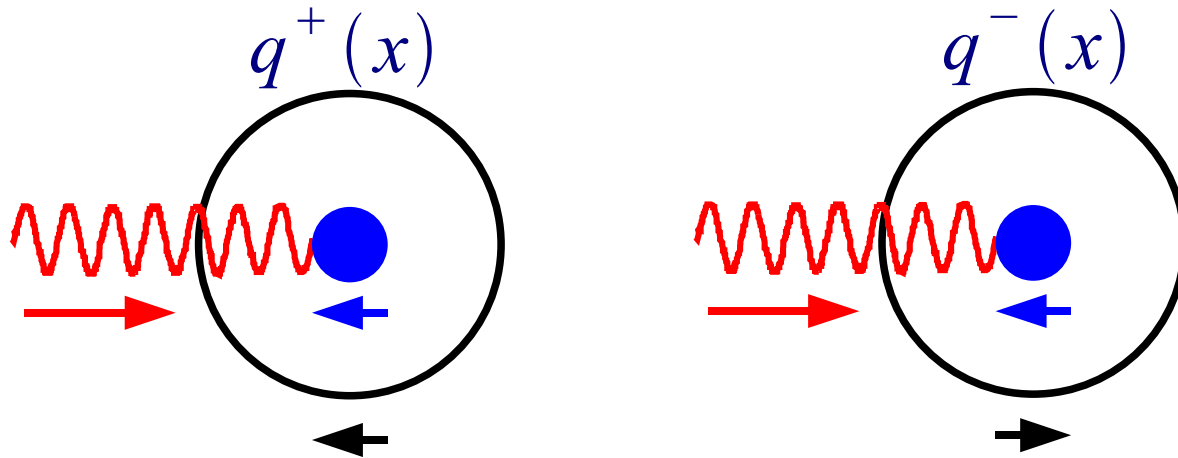
Kretzer:  $e^+e^-$

AKK:  $e^+e^-$

# Impact on the nucleon spin analysis

- strange quark spin component  $\Delta s$

# Helicity distribution function $\Delta q$



$$F_1(x) = \frac{1}{2} \sum_q e_q^2 q(x)$$

$$q(x) = q^+(x) + q^-(x)$$

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x)$$

$$\Delta q(x) = q^+(x) - q^-(x)$$

**Spin sum rule:** 
$$\sum_q \int_0^1 \Delta q(x) = \frac{1}{2}$$

# Proton spin problem

*Nucl. Phys. B328 (1989) 1, Phys. Lett. B206 (1988) 364*

## Polarized DIS:

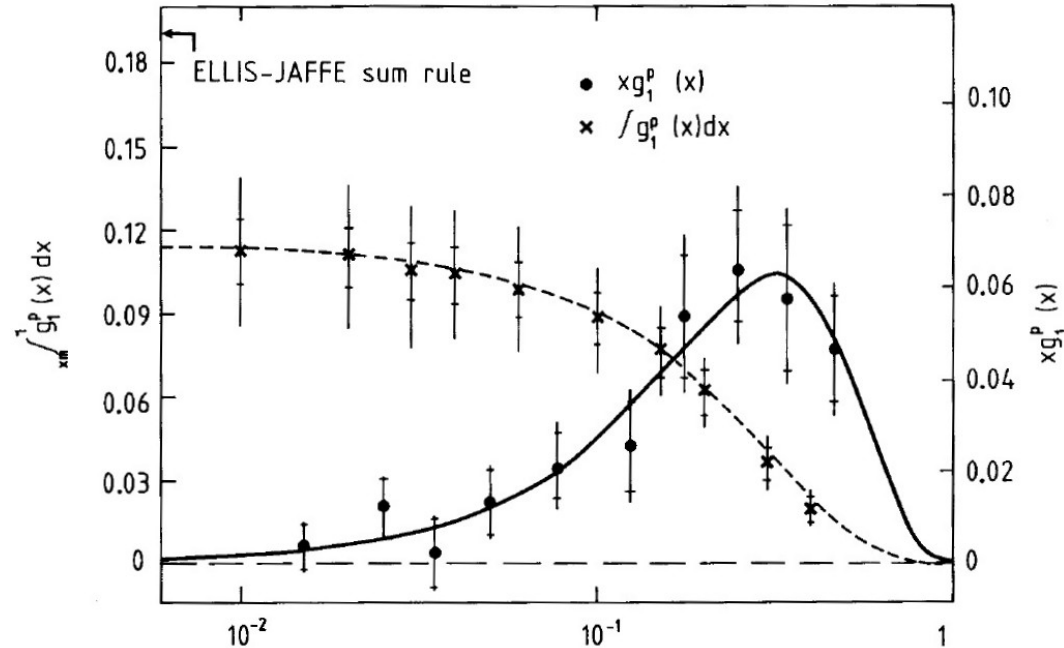
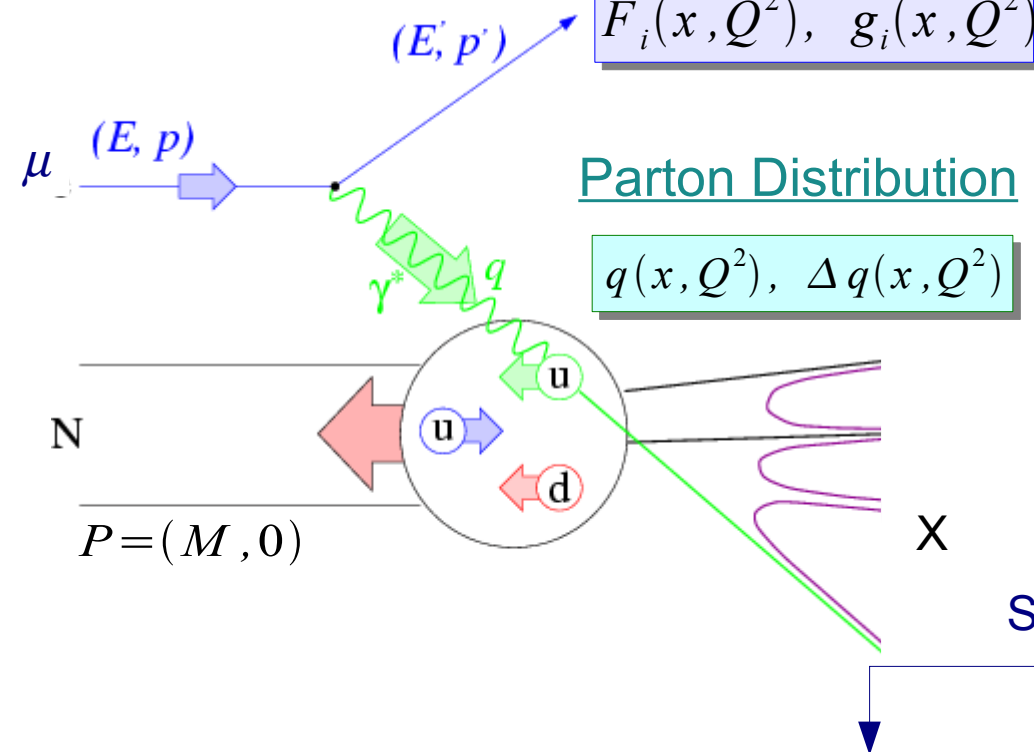
$$\vec{\mu} + \vec{N} \rightarrow \mu' + X$$

Structure Function

$$F_i(x, Q^2), g_i(x, Q^2)$$

Parton Distribution

$$q(x, Q^2), \Delta q(x, Q^2)$$



$$\int_0^1 dx g_1^p(x) = \frac{1}{9} a_0 + \frac{1}{12} a_3 + \frac{1}{36} a_8$$

$$= 0.126 \pm 0.01 \pm 0.015$$

SU(3) flavor symmetry

$$a_3 = 1.26, a_8 = 0.58$$

$$\sum_q \Delta q \sim 0.1 \quad \Delta s \sim -0.19 \quad \Delta G > 0$$

# Updated results on $g_1$ and $\Delta s$



Phys. Rev. D 75 (2007) 012007

## Procedure and assumption:

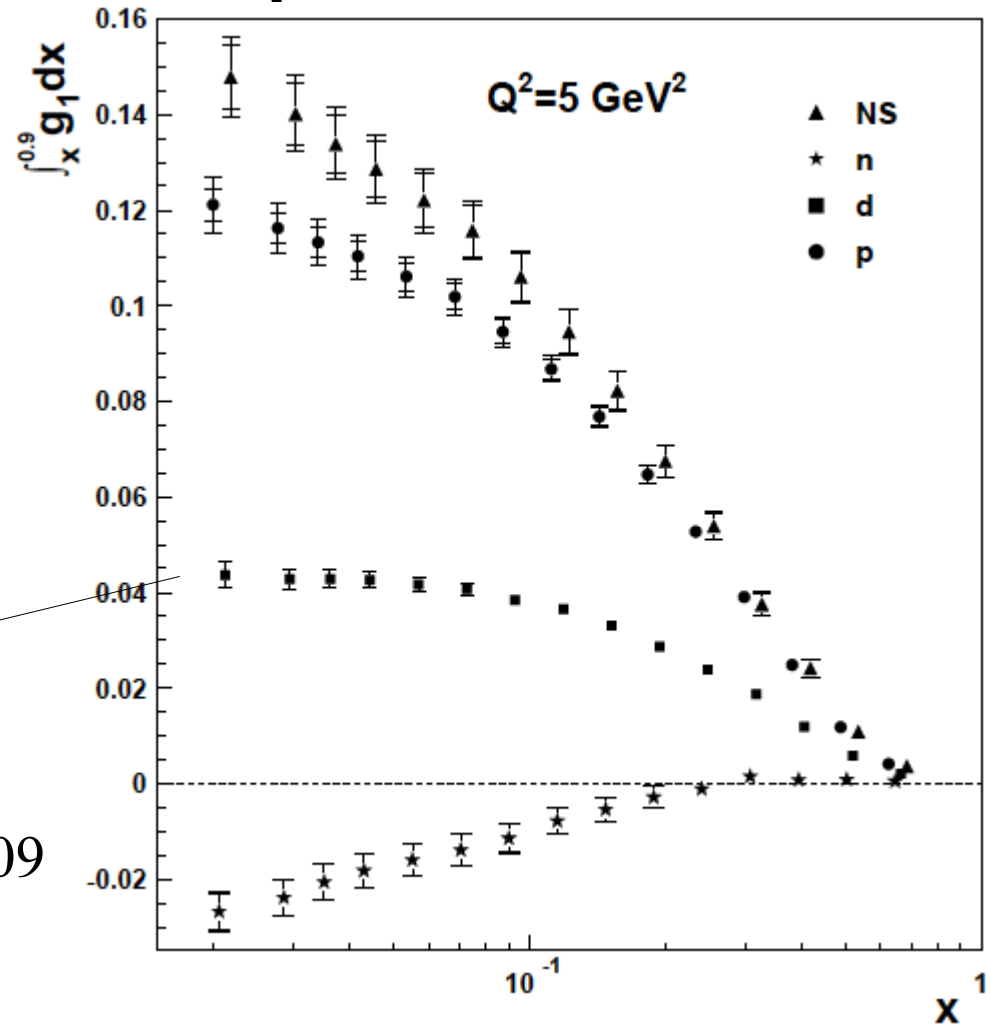
- $Q^2$  evolution to  $5\text{GeV}^2$  with NLO pQCD
- High  $x$  contribution = 0
- Saturation in the lower  $x$  region
- **SU(3) flavor symmetry:**

$$\int_{0.021}^{0.9} g_1^d(x, 5\text{GeV}^2) dx =$$

$$0.0436 \pm 0.0012 \pm 0.018 \pm 0.0008 \pm 0.0026$$



$$\Delta s(5\text{GeV}^2) = -0.085 \pm 0.013 \pm 0.008 \pm 0.009$$



Phys. Lett. B 647 (2007) 8:

Assuming **SU(3) flavor symmetry**

$$\Delta s(\infty) = -0.08 \pm 0.01 \pm 0.02$$

**No Fragmentation function involved.**

# Semi-inclusive DIS

$$\vec{l} + \vec{N} \rightarrow l' + h + X$$

Structure Function

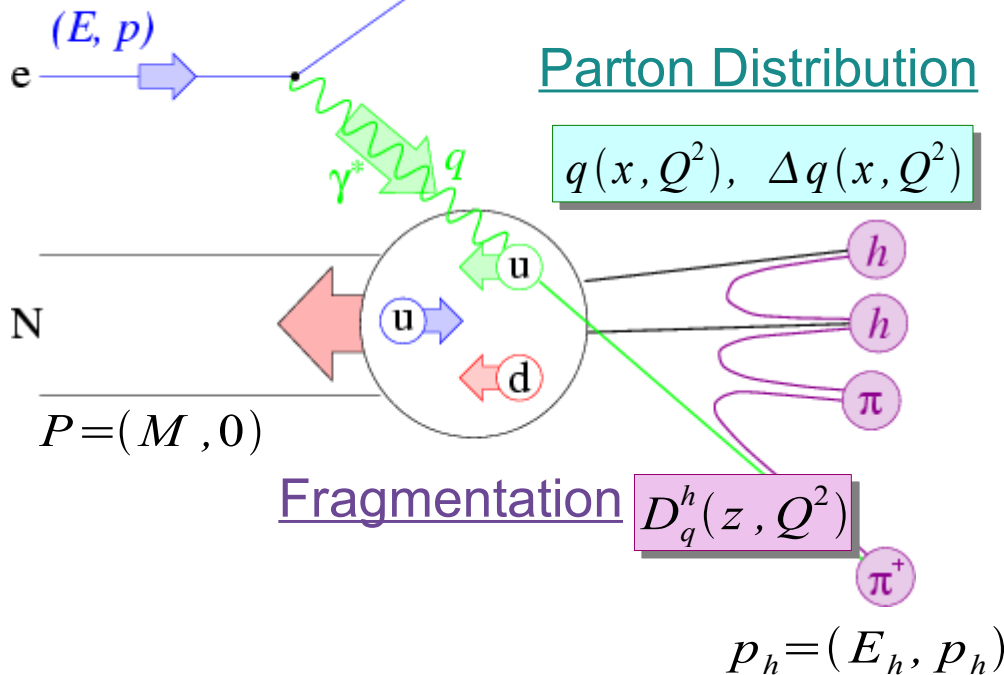
$$F_i(x, Q^2), g_i(x, Q^2)$$

Parton Distribution

$$q(x, Q^2), \Delta q(x, Q^2)$$

Fragmentation  $D_q^h(z, Q^2)$

$$p_h = (E_h, p_h)$$



$$\frac{d^3 \sigma^h}{dx dz dQ^2} \propto \sum_i e_q^2 q(x, Q^2) D_q^h(z, Q^2)$$

$$\frac{d^3 \Delta \sigma^h}{dx dz dQ^2} \propto \sum_i e_q^2 \Delta q(x, Q^2) D_q^h(z, Q^2)$$

$$z = \frac{P \cdot p_h}{P \cdot q} = \frac{E_h}{\nu}$$

FF and PDF are equally important to analyze high energy process with hadron production

Flavor Tagging:

Hadron carries information on quark flavor through fragmentation function

# Helicity distribution with SIDIS data

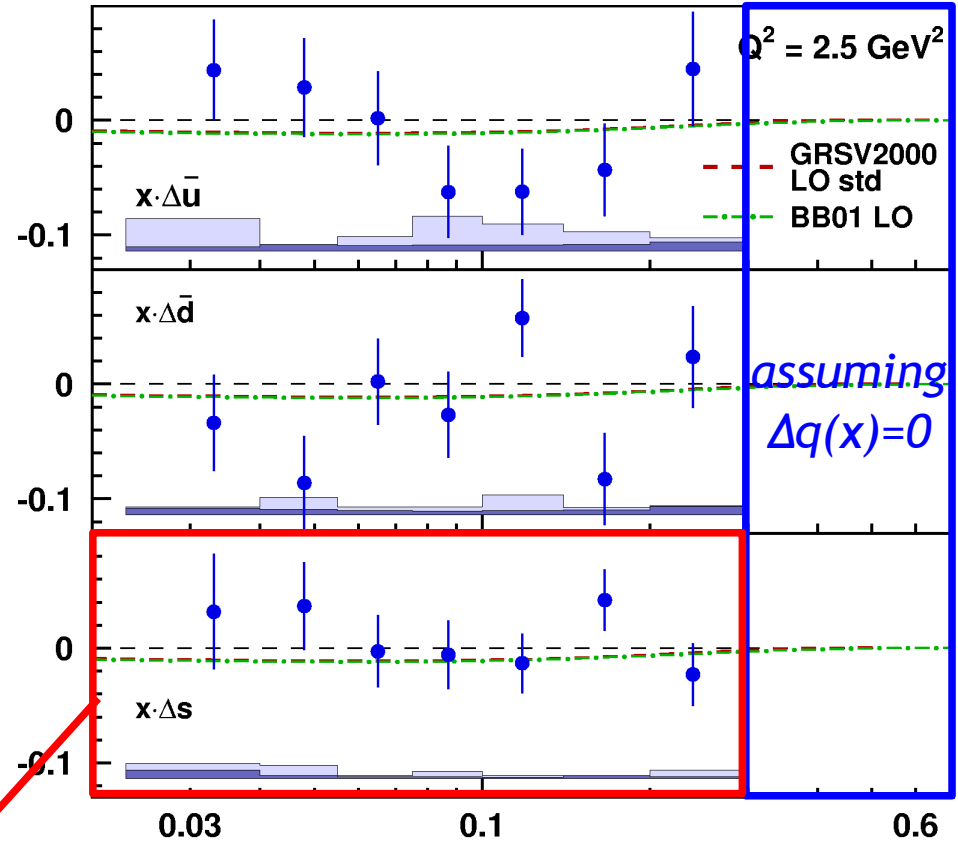
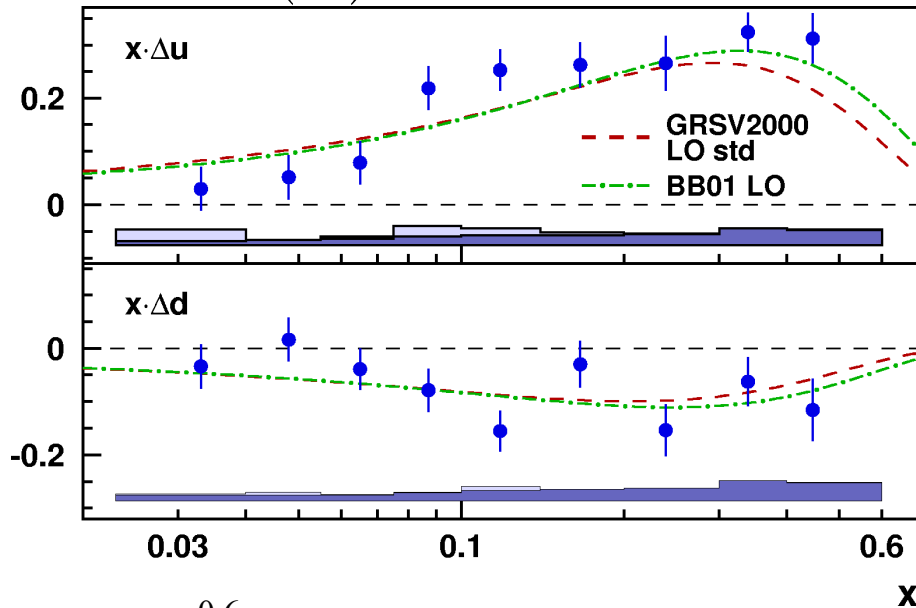


$$\vec{e} + \vec{N} \rightarrow e' + h + X$$

- LO extraction -

Phys. Rev D 71 (2005) 012003

$\Delta \bar{s}(x) = 0$  assumed



$$\Delta q = \int_{0.023}^{0.6} dx \Delta q(x) \quad Q^2 = 2.5 \text{ GeV}^2$$

$$\Delta u = 0.601 \pm 0.039 \pm 0.049$$

$$\Delta d = -0.226 \pm 0.039 \pm 0.050$$

$$\Delta \bar{u} = -0.002 \pm 0.036 \pm 0.023$$

$$\Delta \bar{d} = -0.054 \pm 0.033 \pm 0.011$$

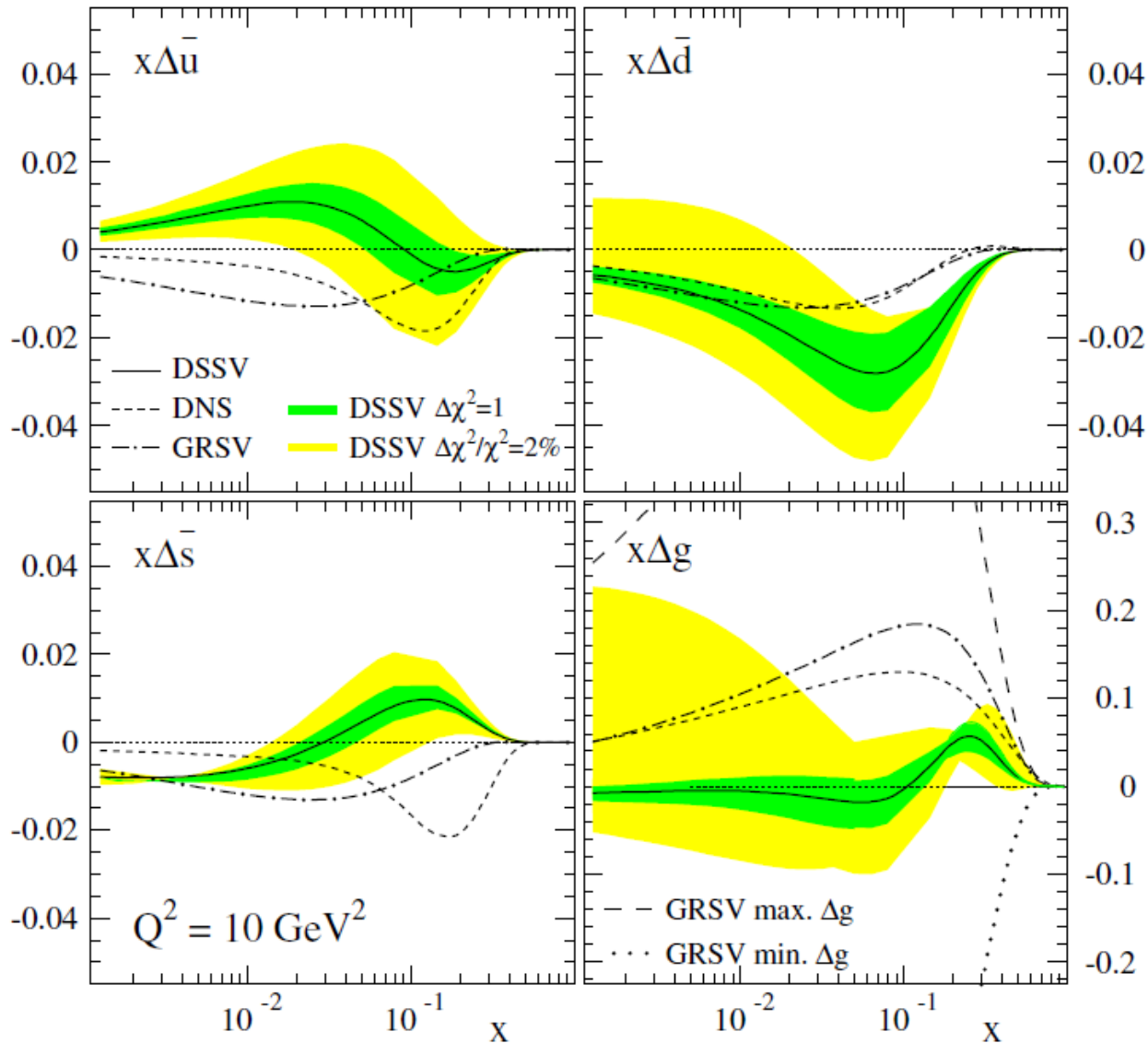
$$\Delta s = 0.028 \pm 0.033 \pm 0.009$$

- without SU(3) symmetry assumption
- only partial moments are available
- "LO" extraction
- String model based MC for FF ( $\langle Q^2 \rangle \sim 2.5 \text{ GeV}^2$ )



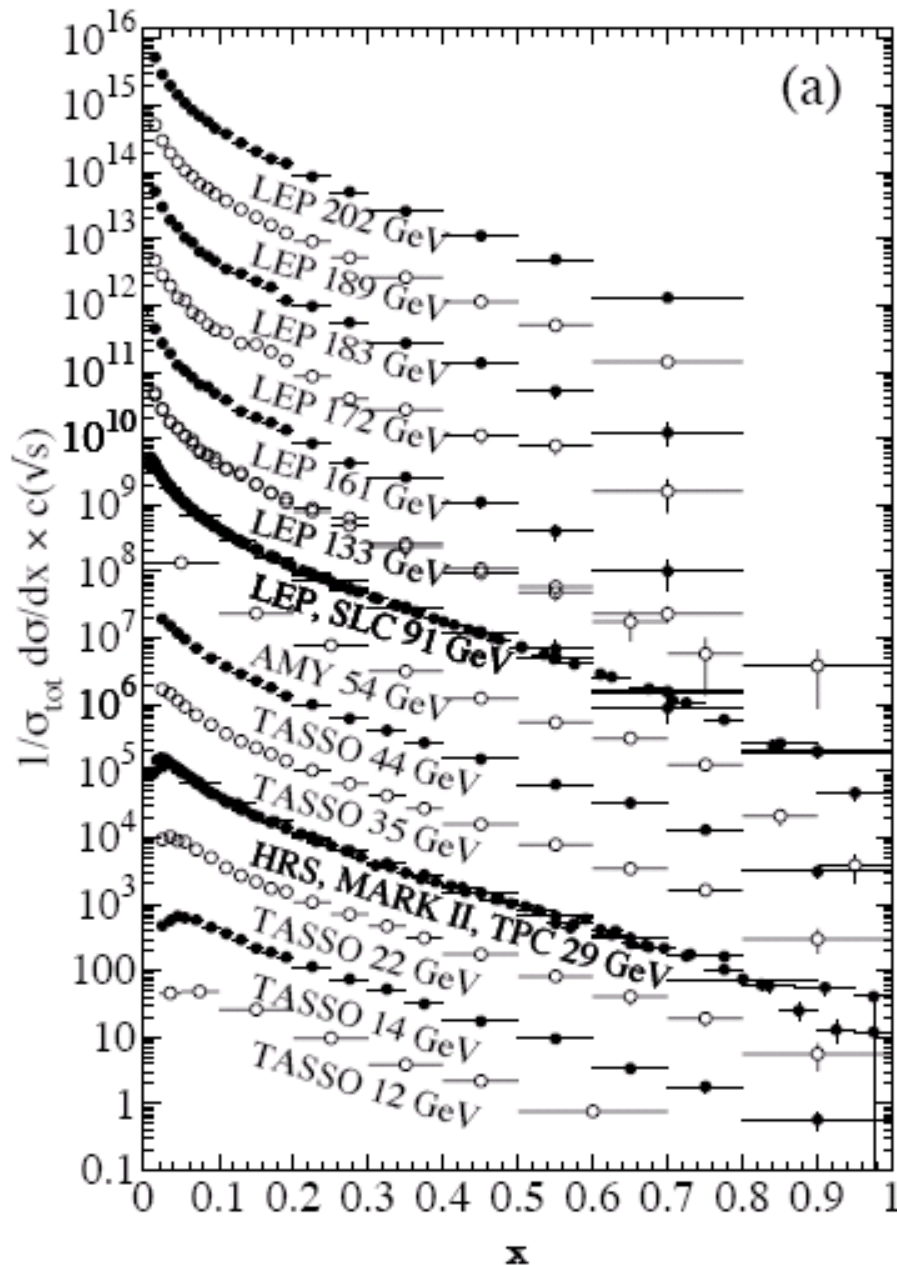
# Helicity distribution from QCD fit

D. de Florian, R. Sassot, M. Stratmann, and W. Vogelsang,  
Phys. Rev. Lett. 101, 072001 (2008)



	Data	FF
DSSV	DIS, SIDIS pp	DSS
DNS	DIS SIDIS	Kretzer
GRSV	DIS	-----

# Belle and Fragmentation function



- Fragmentation function at Belle (on going)
- $\sqrt{s} \sim 10.52$  GeV
  - Cover the lowest scale
  - Gluon FF from scale violation
  - Similar scale to pol. DIS
- Light quarks: u, d, s
- pion, kaon

# Fragmentation function analysis in Belle

- Collins Fragmentation Function
  - Belle collaboration, Phys. Rev. D78 (2008) 032011
  - Transverse momentum dependence:  $k_T$
  - Vector meson
- Di-hadron fragmentation function
  - published
  - on-going
  - plan/idea
- Unpolarized Fragmentation function
  - Charged pion, charged kaon, proton
    - $\sqrt{s} = 10.8$  GeV: cover lower  $Q^2$  ( for gluon ), similar  $Q^2$  with SIDIS and p-p
  - Other mesons:  $\pi^0$ , eta,  $f_0$ , phi, Ks, .....
  - Barions: Lambda, ...

# Contents

- Fragmentation Function has been analyzed with  $e^+e^-$  data, SIDIS, and pp data together
- Fragmentation function is important for analysis of high energy scattering with hadron production
  - With DSS kaon FF,  $\Delta_s > 0$  was obtained in the middle  $x$
  - Neutral pion FF is also important for  $\Delta G$ . (especially gluon part)
- Fragmentation function analysis at Belle
  - Collins fragmentation ( published )
  - Normal fragmentation function analysis (on-going)
    - $\sqrt{s} = 10.52$  GeV, pion, kaon



# Quark Transversity and Collins Fragmentation

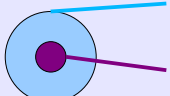
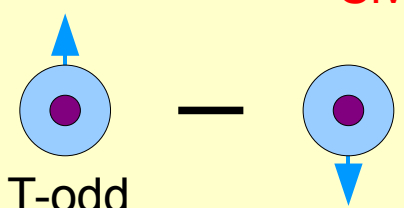
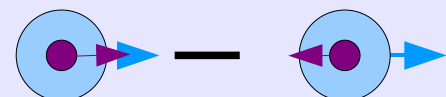

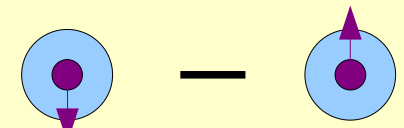
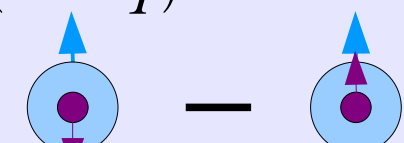
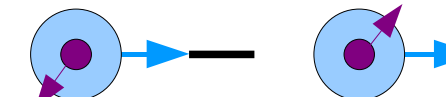

# Transverse momentum dependent parton distribution function

Unpolarized

Long. polarized

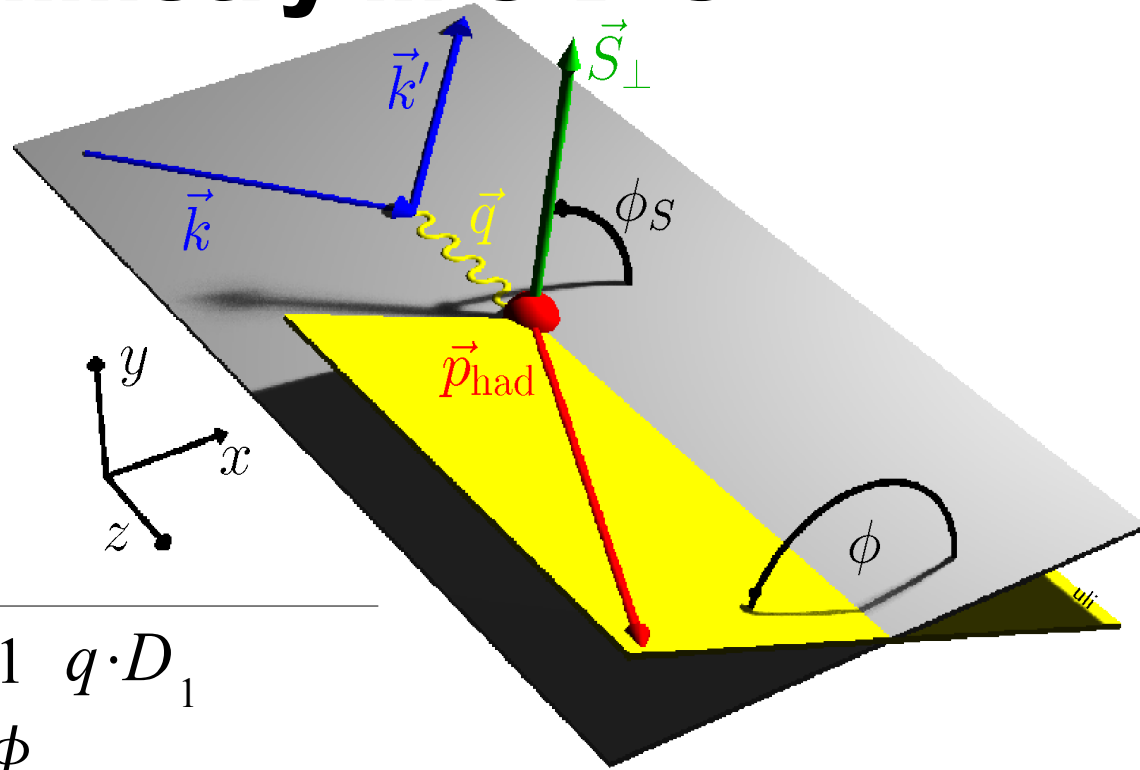
Trans. polarized

Nucleon

Parton	Unpol.	$f_1(=q)$ Number density  Nucleon Parton	These survive after integration over $k_\perp$ .	$f_{1T}^\perp$ <b>Sivers</b> naïve T-odd 
	Long. pol.		$g_{1L}(=\Delta q)$ Helicity 	$g_{1T}$ 
	Trans. polarized	$h_1^\perp$ Boer-Mulders naïve T-odd 	<b>Chira-odd requires chiral-odd partner</b>	$h_{1T}^\perp(=\delta q)$ <b>Transversity</b> 
			$h_{1L}^\perp$ 	$h_{1T}^\perp$ 



# Collins asymmetry in SIDIS



Unpol Target

$$d\sigma_{UU} = 1 \cdot q \cdot D_1 \cos 2\phi$$

Long. Pol. Target

$$d\sigma_{UL} = \sin 2\phi$$

$S_L$

Trans. Pol. Target

$$d\sigma_{UT}$$

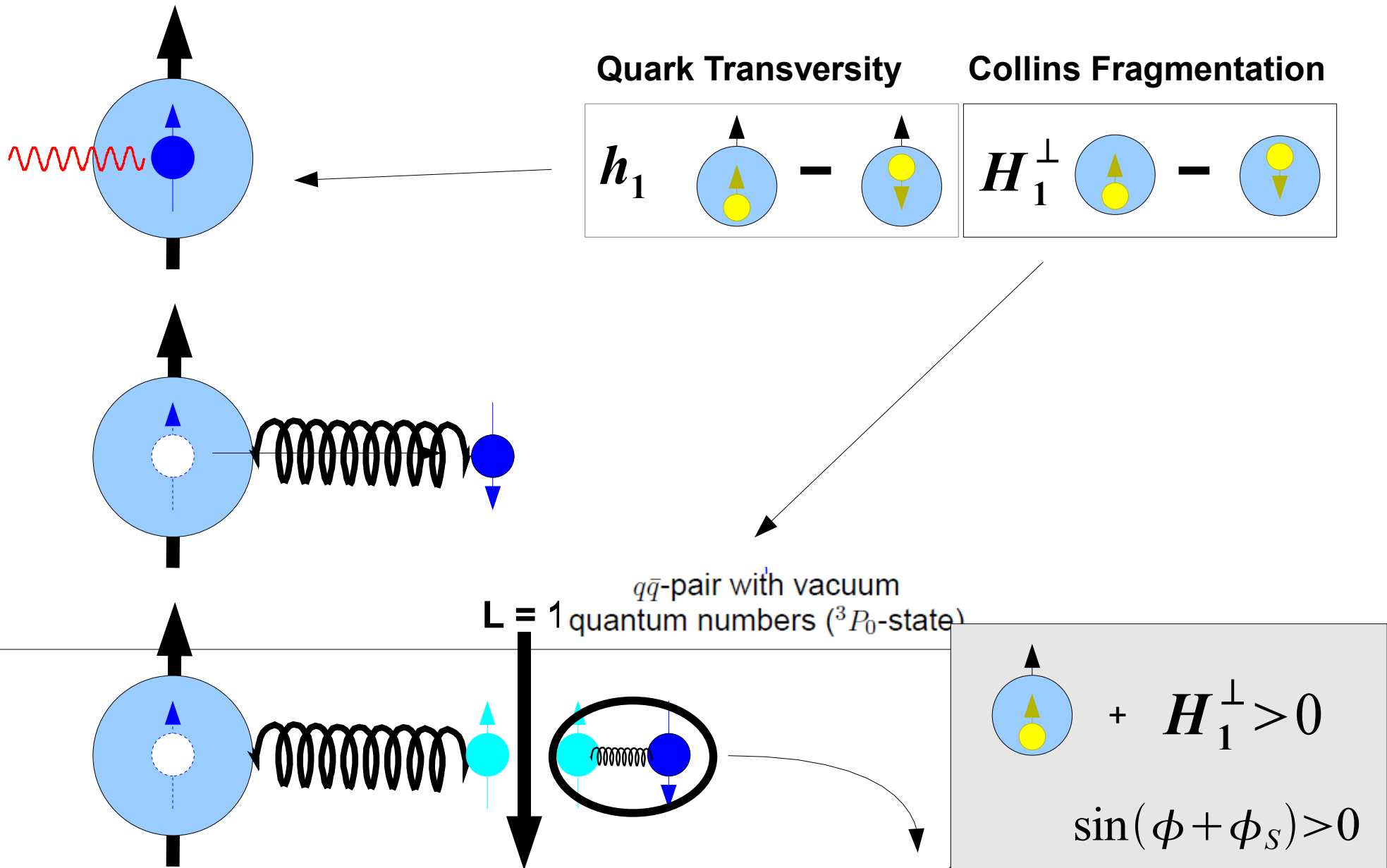
$S_T$

$$\sin(\phi - \phi_S) \mathbf{f}_{1T}^\perp \cdot D_1 \quad \text{Sivers}$$

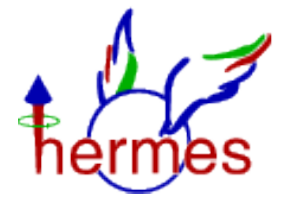
$$\sin(\phi + \phi_S) \mathbf{h}_1 \cdot \mathbf{H}_{1T}^\perp \quad \text{Collins}$$

$$\sin(3\phi - \phi_S)$$

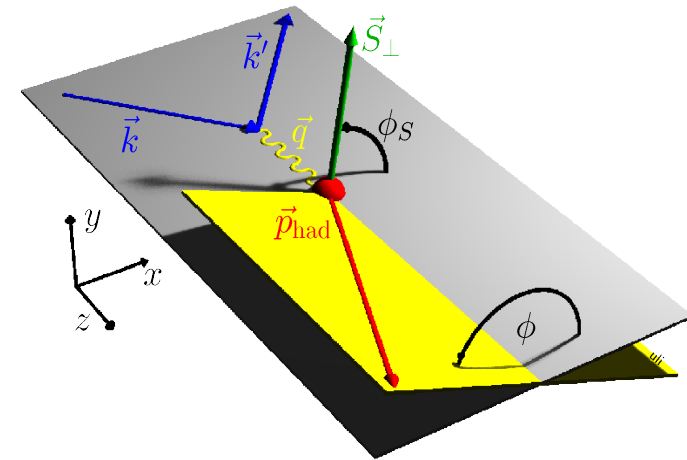
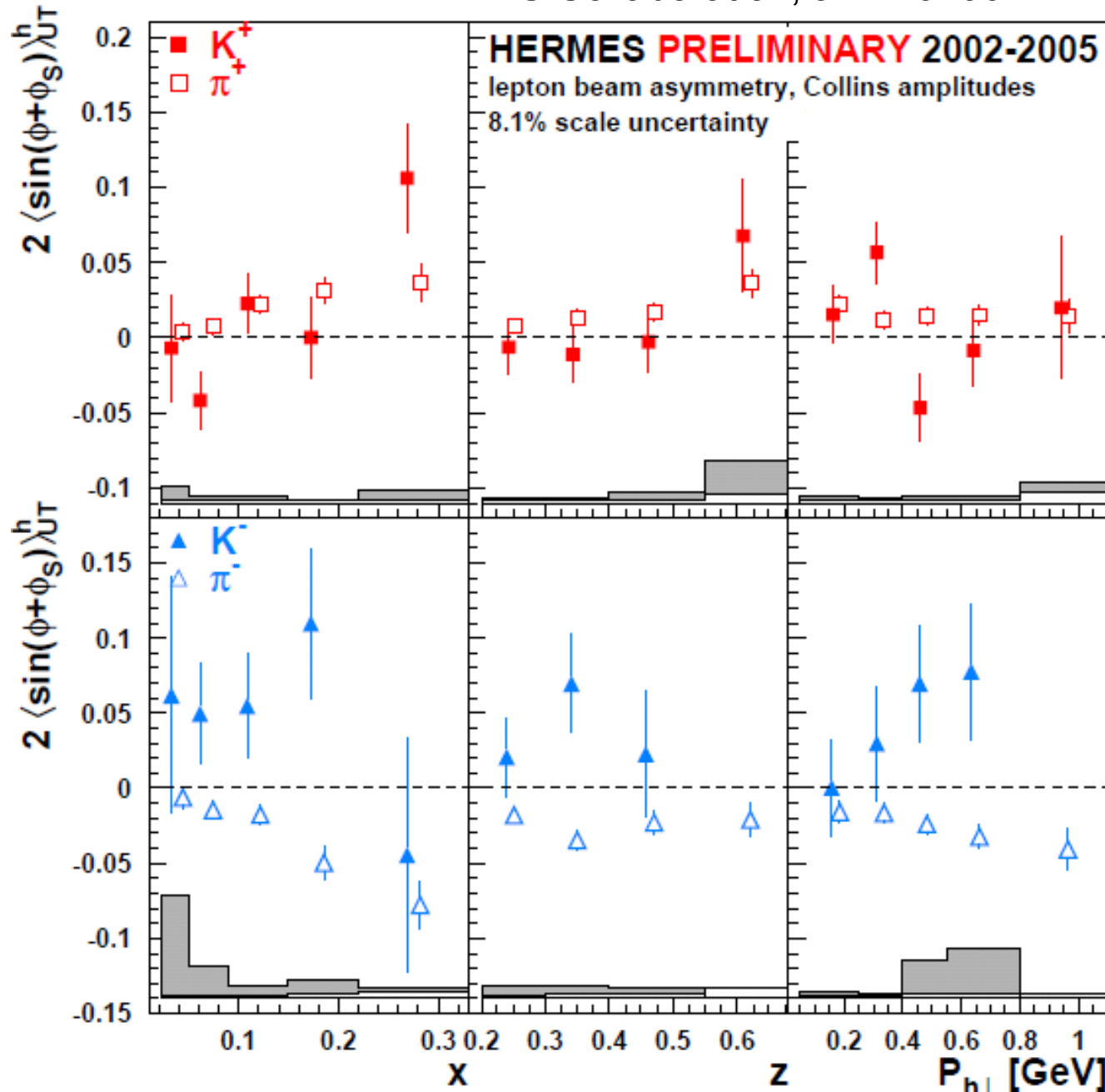
# Collins mechanism



# Collins Asymmetry at HERMES

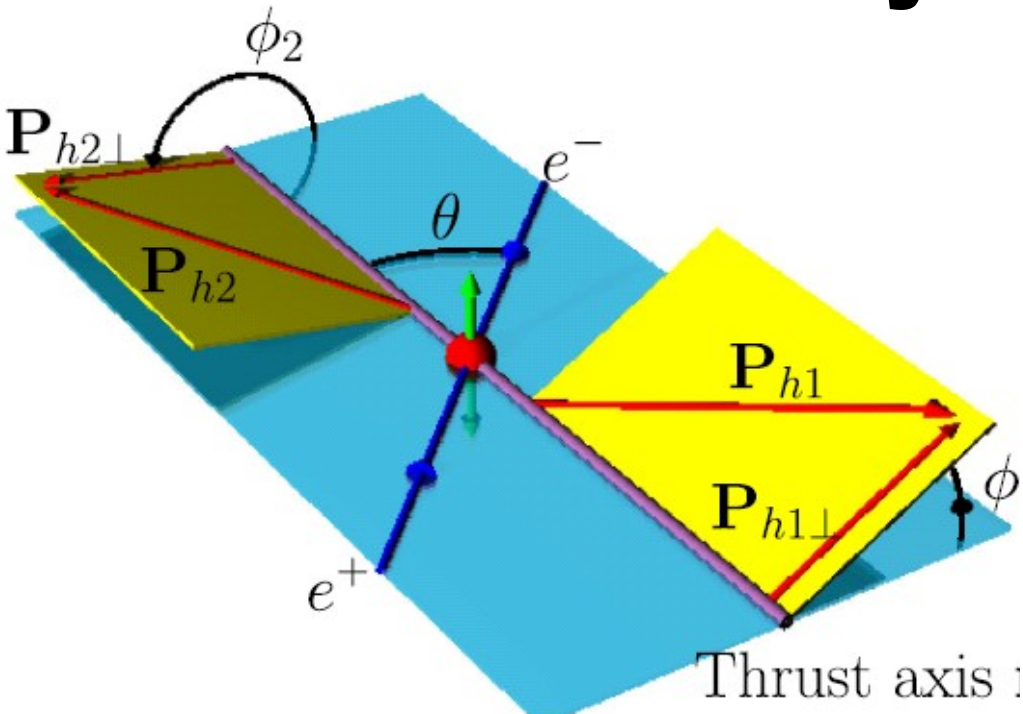


HERMES Collaboration, arXiv:0706.2242

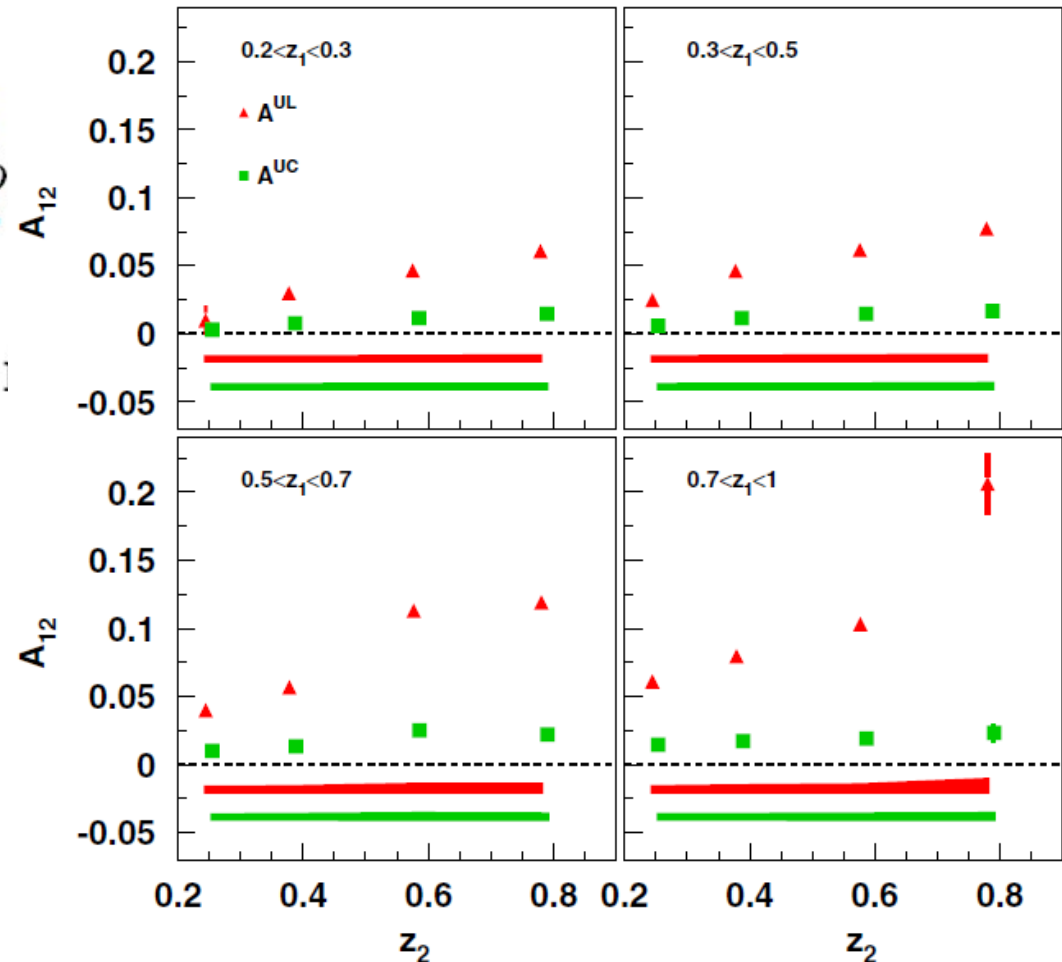
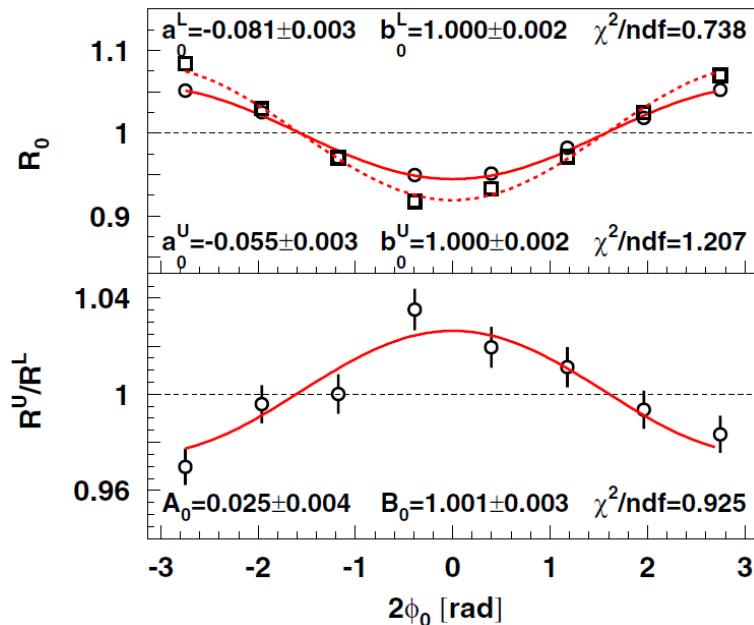


# Collins asymmetry at Belle

Belle collaboration, Phys. Rev. D78 (2008) 032011



$$R_{\alpha}^i / R_{\alpha}^j = A_{\alpha}^{ij} \cos(\beta_{\alpha}) + B_{\alpha}^{ij}$$



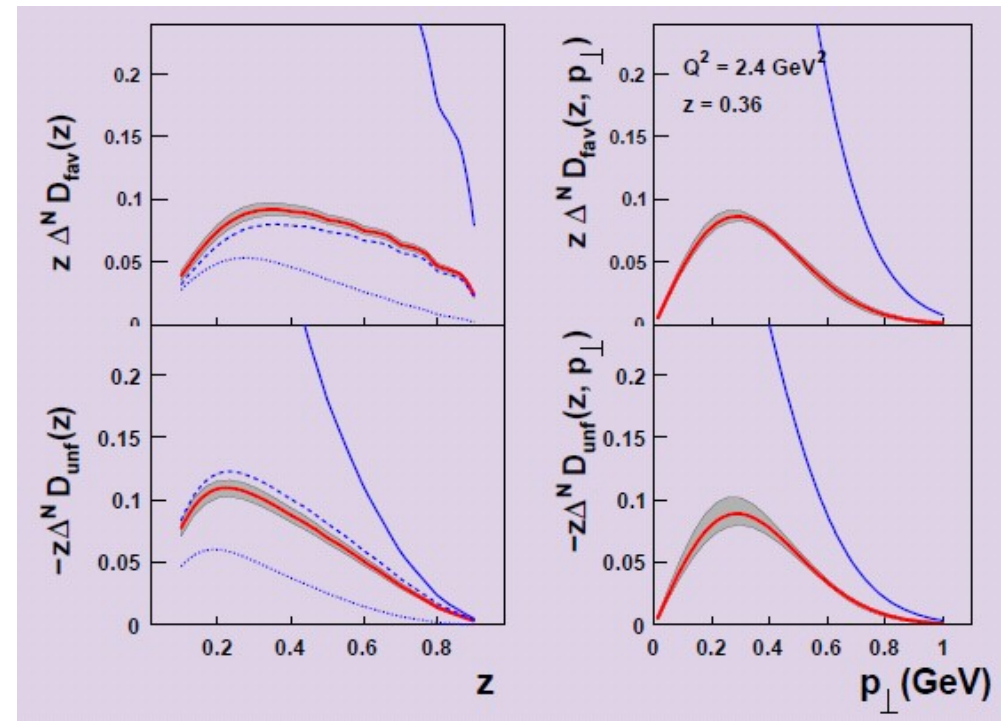
# Extracted transversity and Collins FF

Alexei Prokudin @ SPIN2008

Quark transversity



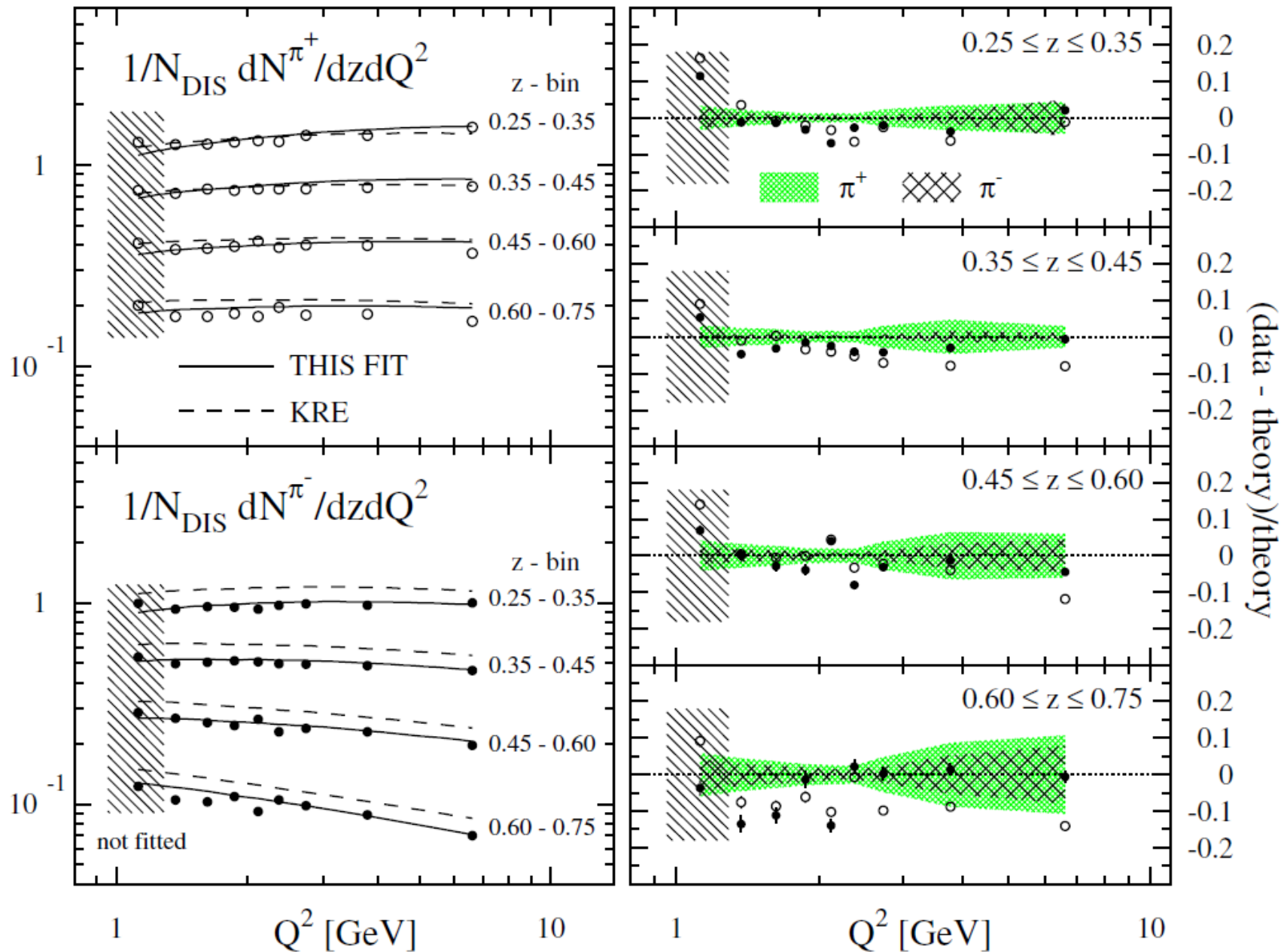
Collins fragmentation function



*END*

# HERMES Multiplicity: pion

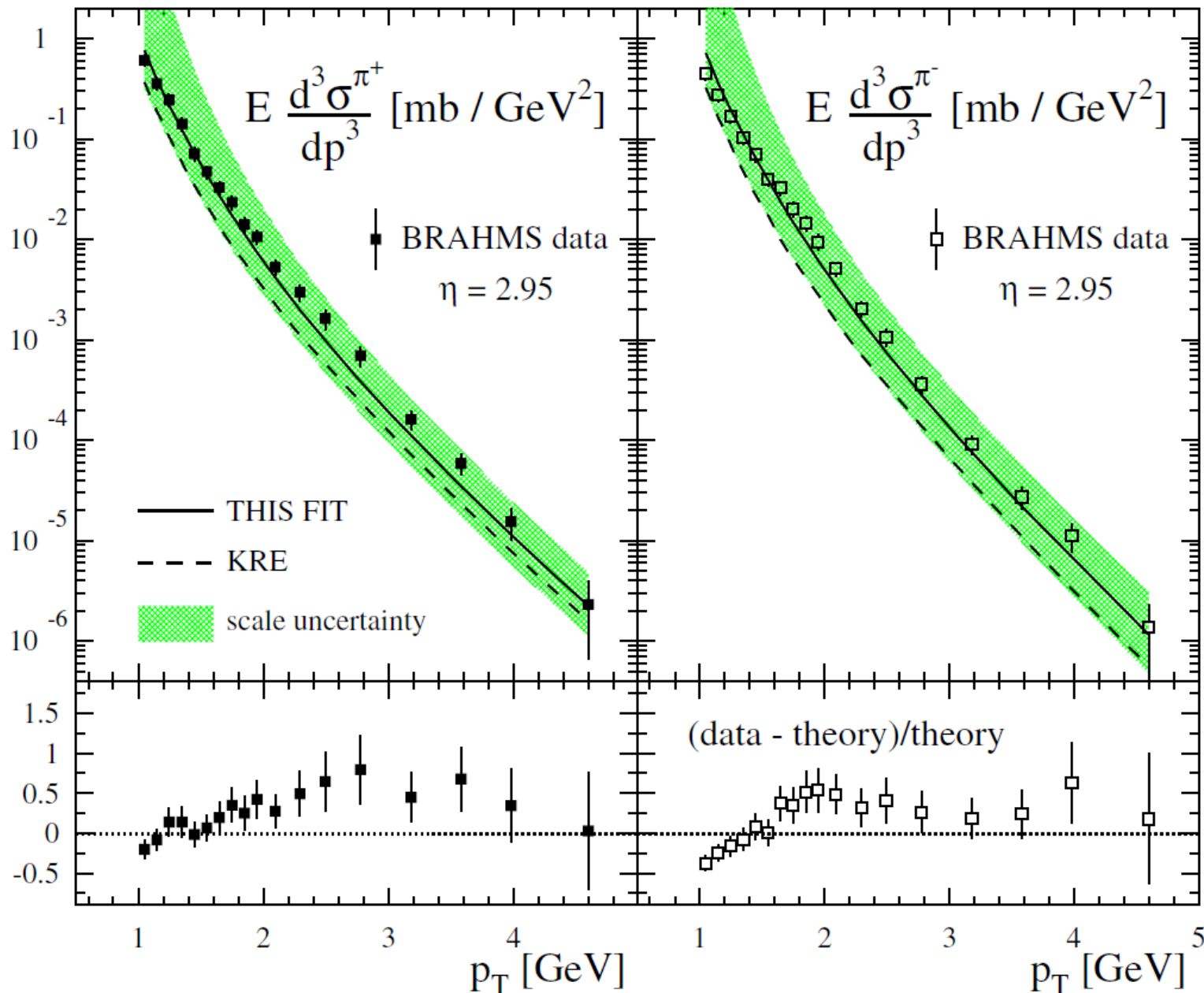
DSS, Phys. Rev. D75, 114010 (2007)





# Charged pion production in p-p

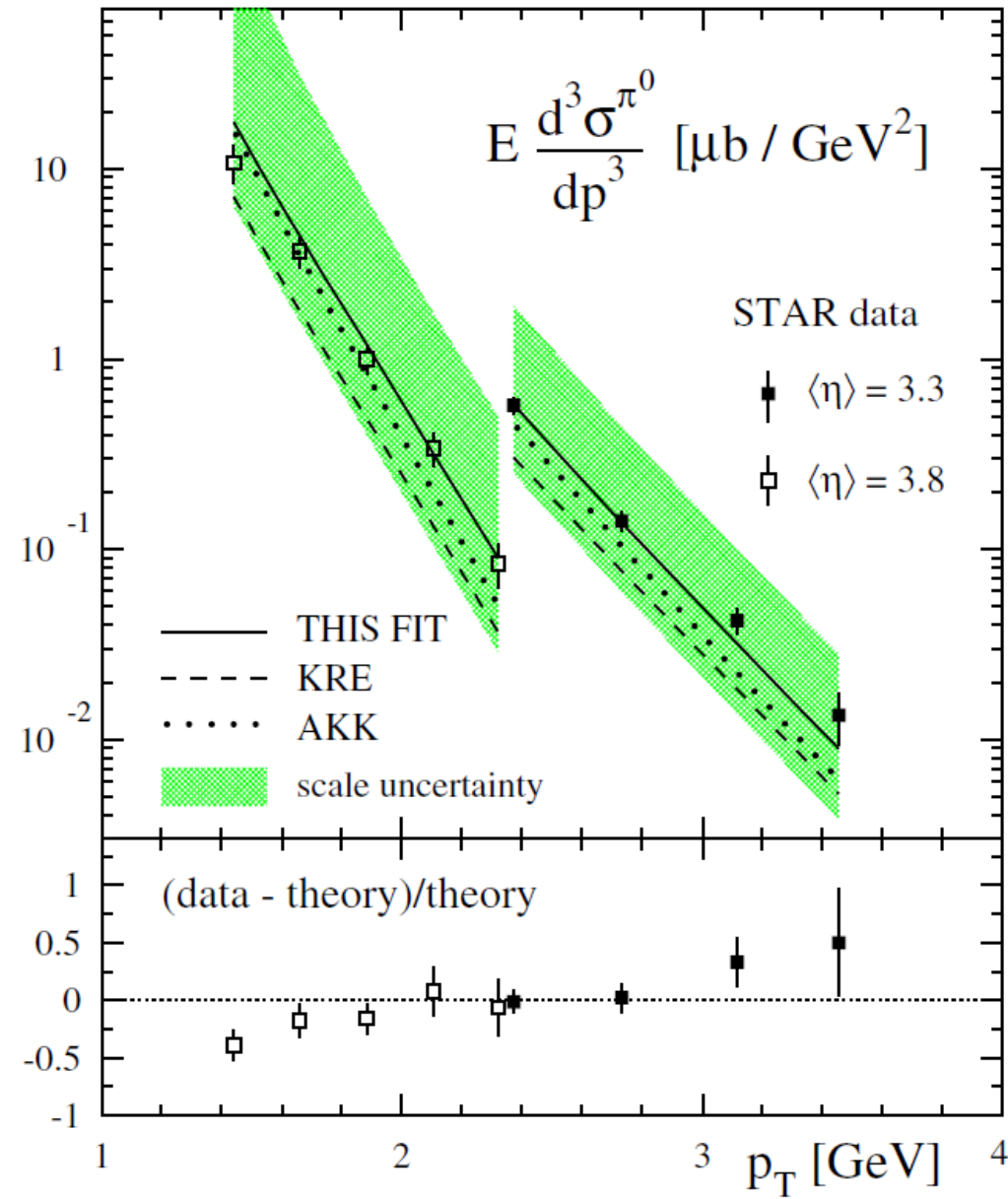
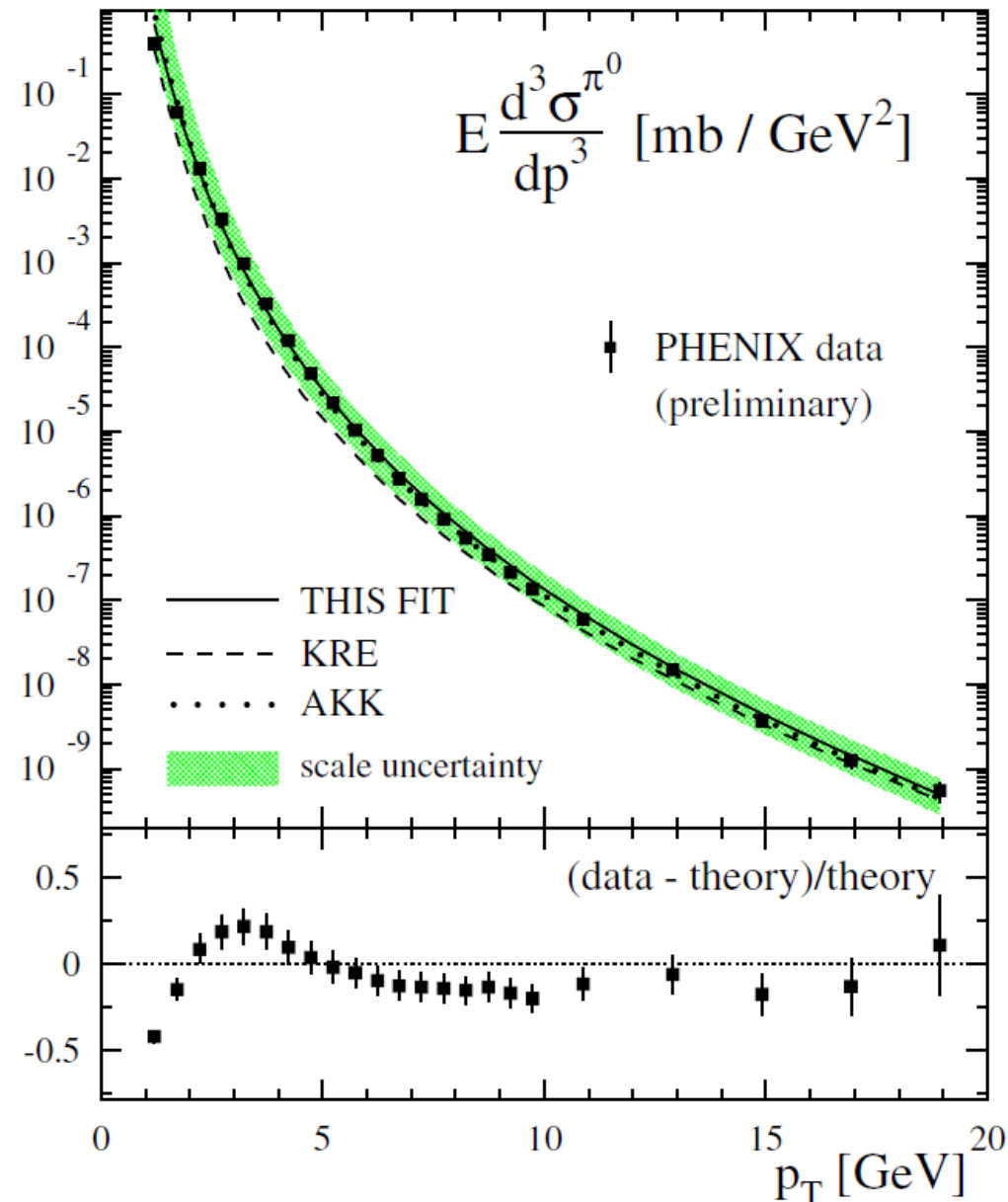
DSS, Phys. Rev. D75, 114010 (2007)



# Neutral pion production in p-p

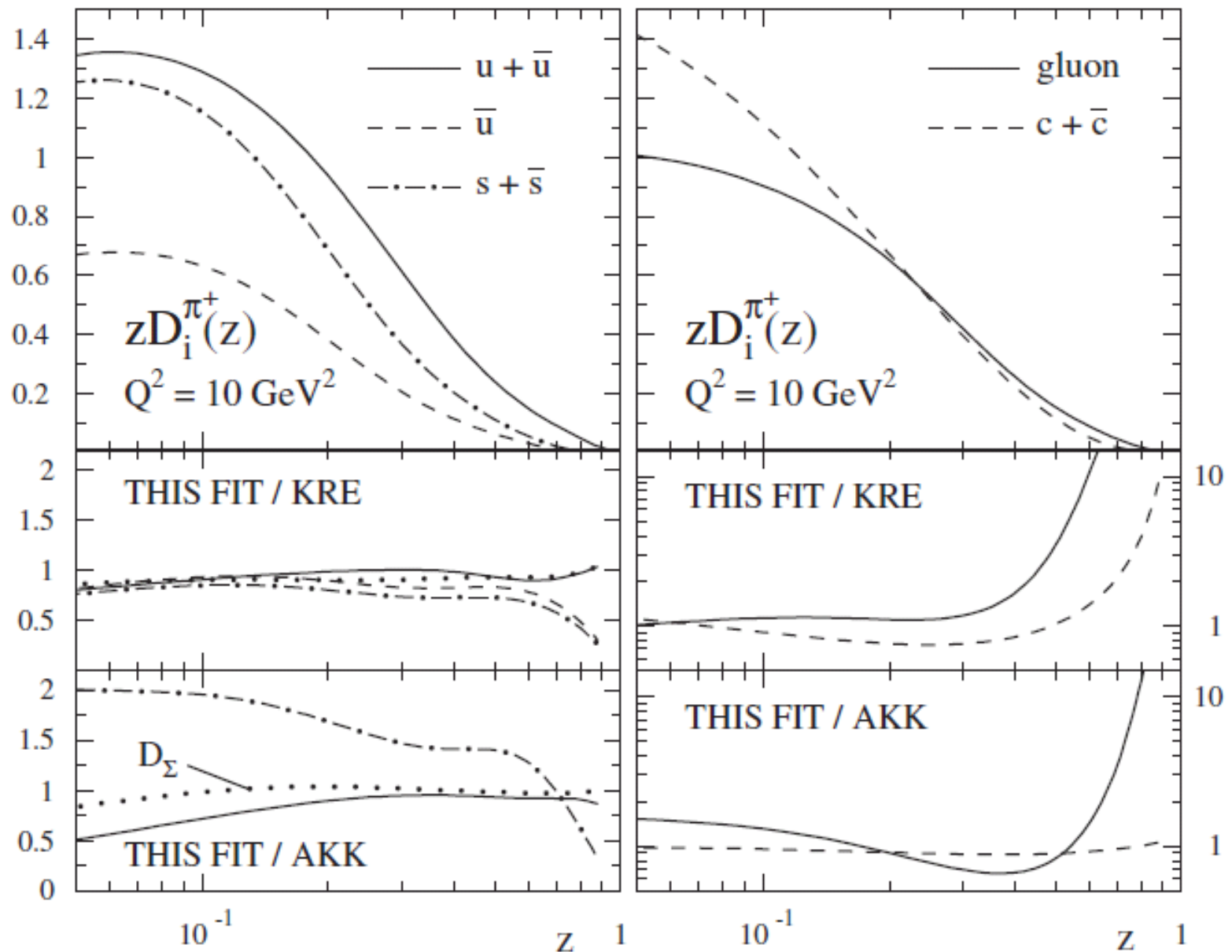
DSS, Phys. Rev. D75, 114010 (2007)

neutral pions by assuming  $D_i^{\pi^0} = [D_i^{\pi^+} + D_i^{\pi^-}]/2$ .



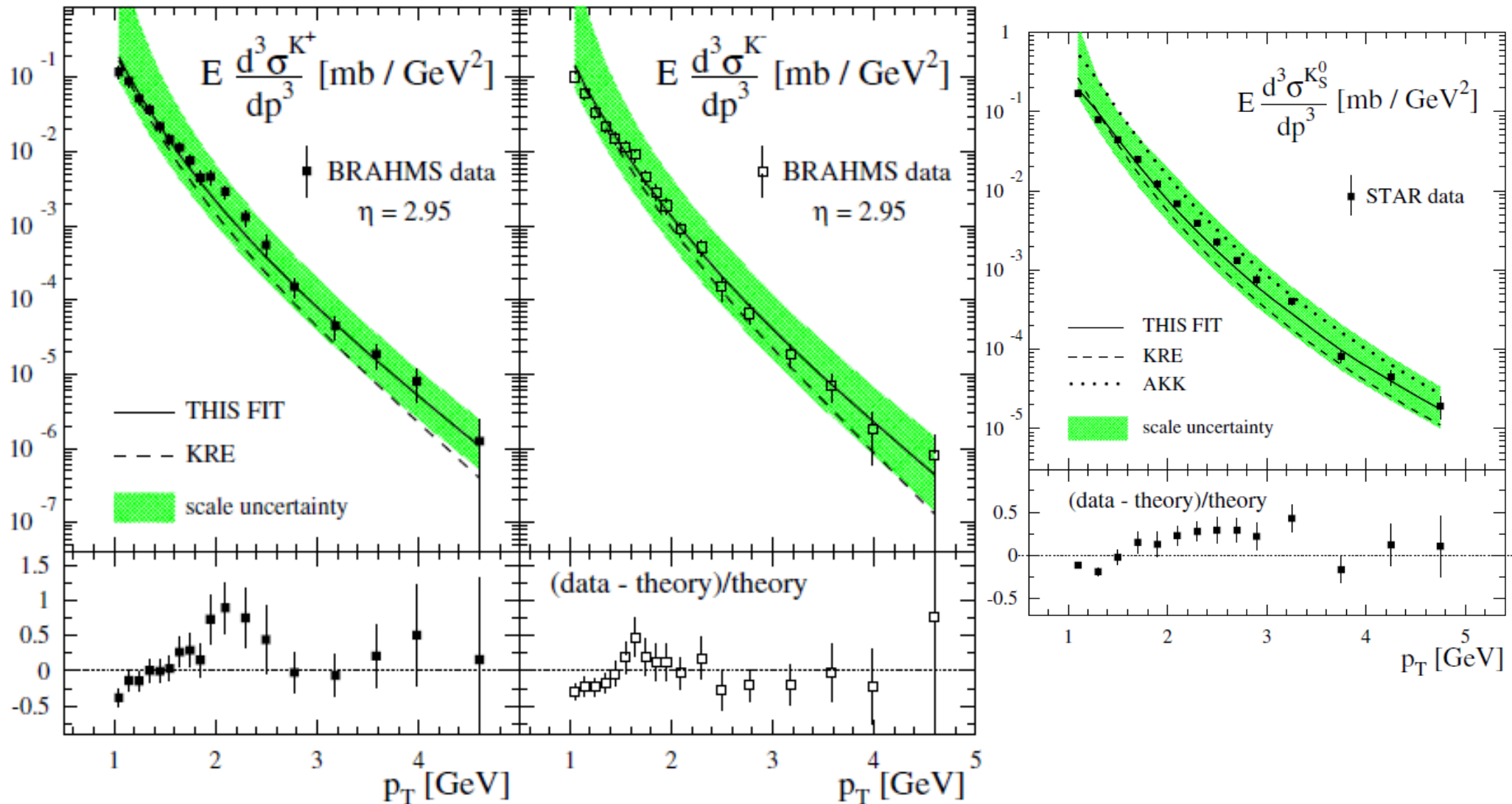
# Extracted pion fragmentation function

DSS, Phys. Rev. D75, 114010 (2007)



# Kaon production in pp scattering

DSS, Phys. Rev. D75, 114010 (2007)



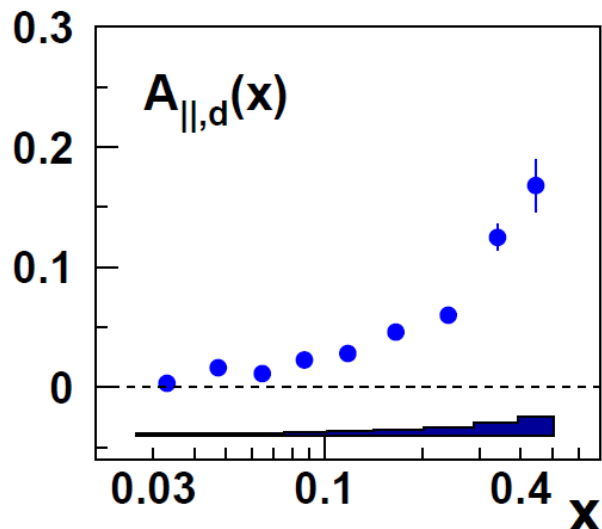


# Helicity distribution function



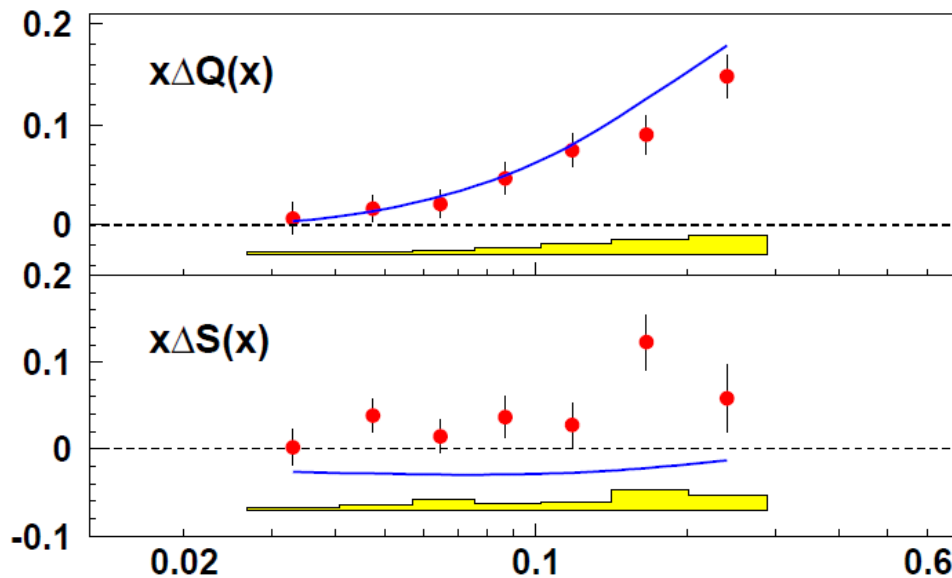
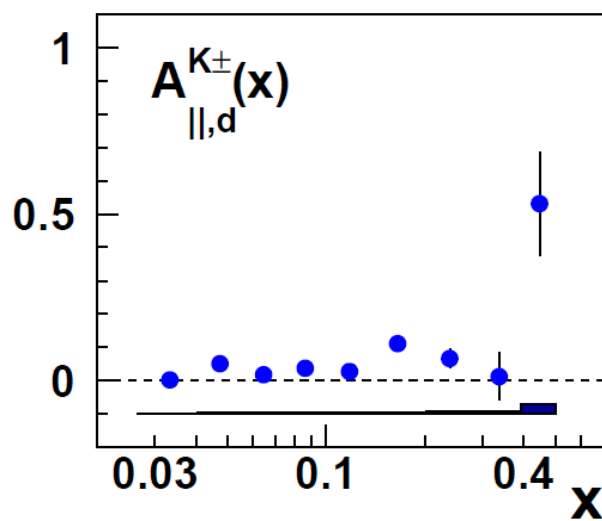
## LO extraction

HERMES Collaboration, Phys. Lett. B666 (2008) 446-450

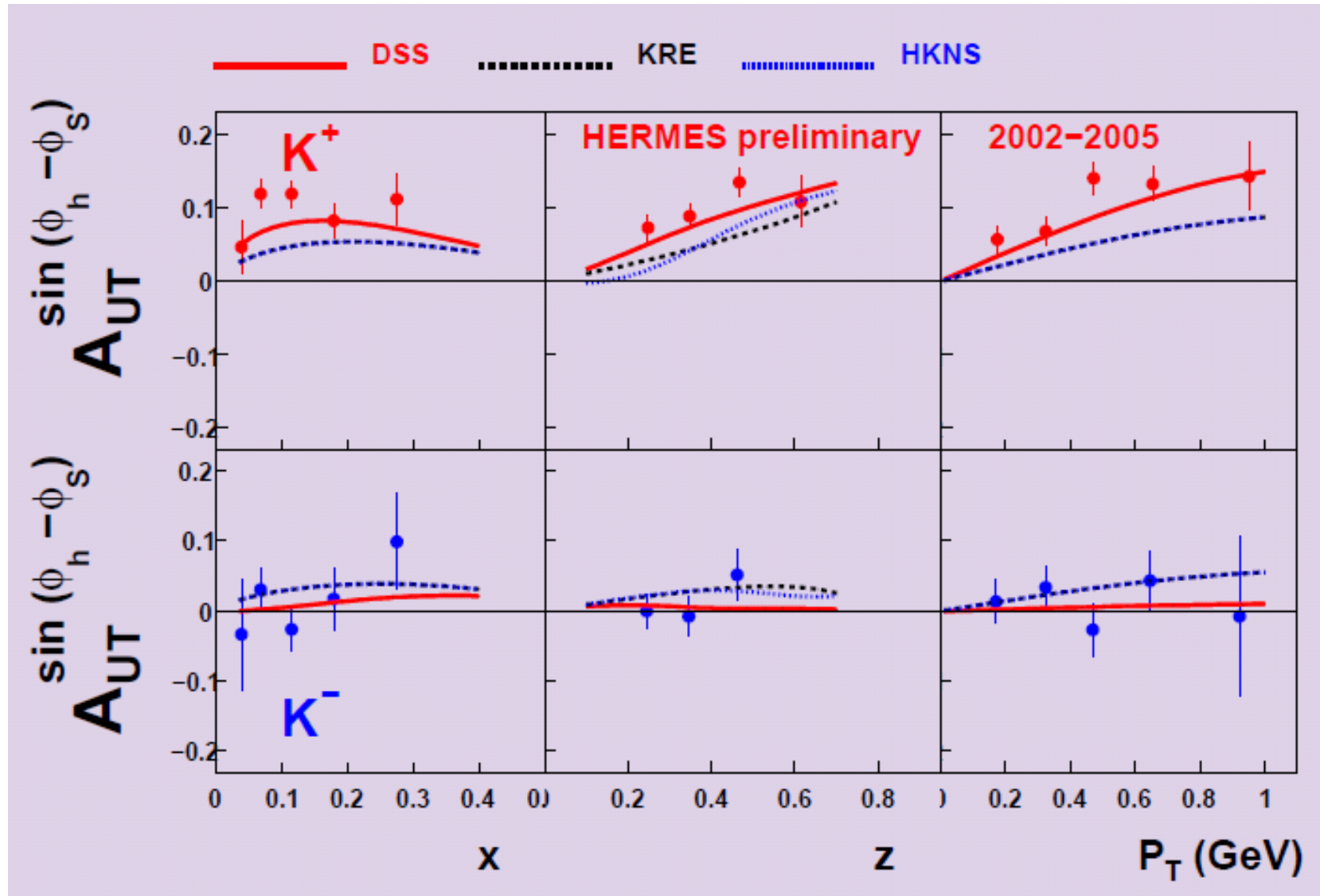


$$A_{||,d}(x) \frac{d^2 N^{DIS}(x)}{dx dQ^2} = \mathcal{K}_{LL}(x, Q^2) [5\Delta Q(x) + 2\Delta S(x)]$$

$$A_{||,d}^{K^\pm}(x) \frac{d^2 N^K(x)}{dx dQ^2} = \mathcal{K}_{LL}(x, Q^2) \times \left[ \Delta Q(x) \int \mathcal{D}_Q^K(z) dz + \Delta S(x) \int \mathcal{D}_S^K(z) dz \right]$$



# FF choice in Sivers function analysis





# Data used in helicity distribution

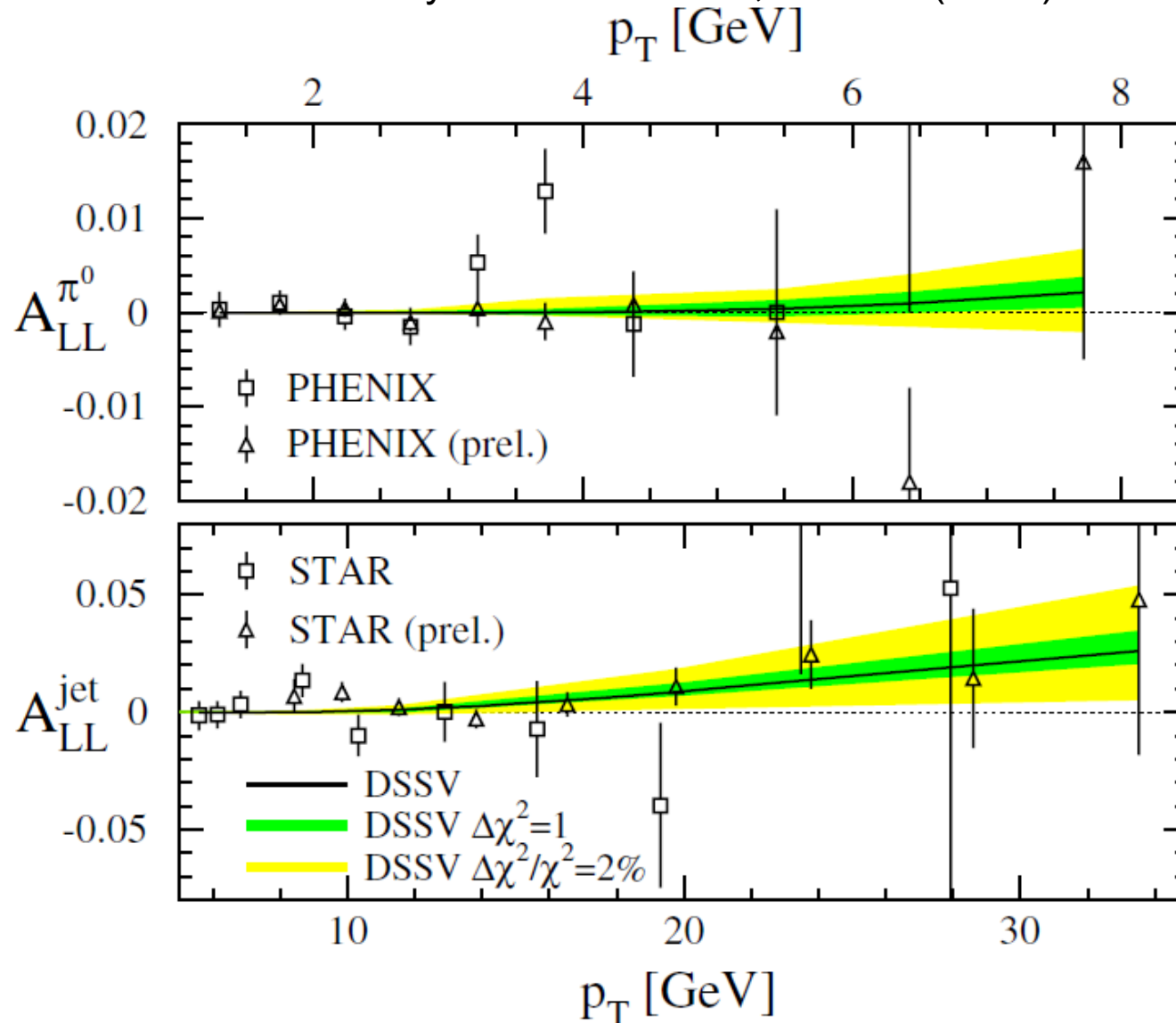
D. de Florian, R. Sassot, M. Stratmann, and W. Vogelsang,  
Phys. Rev. Lett. 101, 072001 (2008)

Experiment	Data fitted	$\chi^2$
<i>DIS</i> : EMC, SMC, COMPASS, E142, E143, E154, E155, HALL-A, CLAS, HERMES	234	186
<i>SIDIS</i> $\pi^\pm$ , $K^\pm$ , $h^\pm$ : SMC, HERMES, COMPASS	189	166.5
$p$ - $p$ 200 GeV, $\pi^0$ : PHENIX (in part prel.)	20	21.3
$p$ - $p$ 62 GeV, $\pi^0$ : PHENIX (prel.)	5	3.1
$p$ - $p$ 200 GeV, jet: STAR (in part prel.)	19	15.7
TOTAL:	467	392.6



# Double spin asymmetry in p-p

D. de Florian, R. Sassot, M. Stratmann, and W. Vogelsang,  
Phys. Rev. Lett. 101, 072001 (2008)

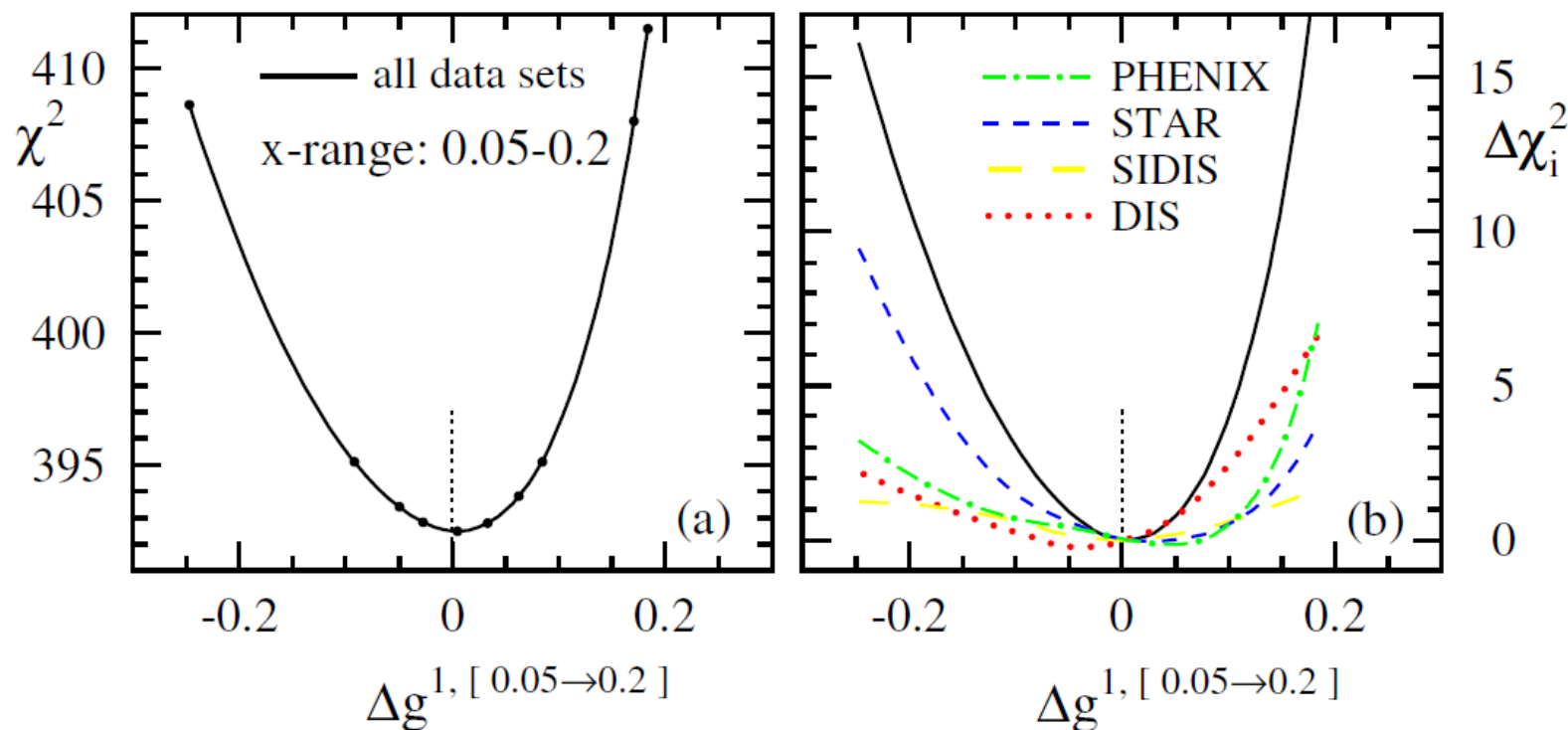


neutral pions by assuming  $D_i^{\pi^0} = [D_i^{\pi^+} + D_i^{\pi^-}]/2$ .

# Gluon polarization in polarized proton-proton scattering

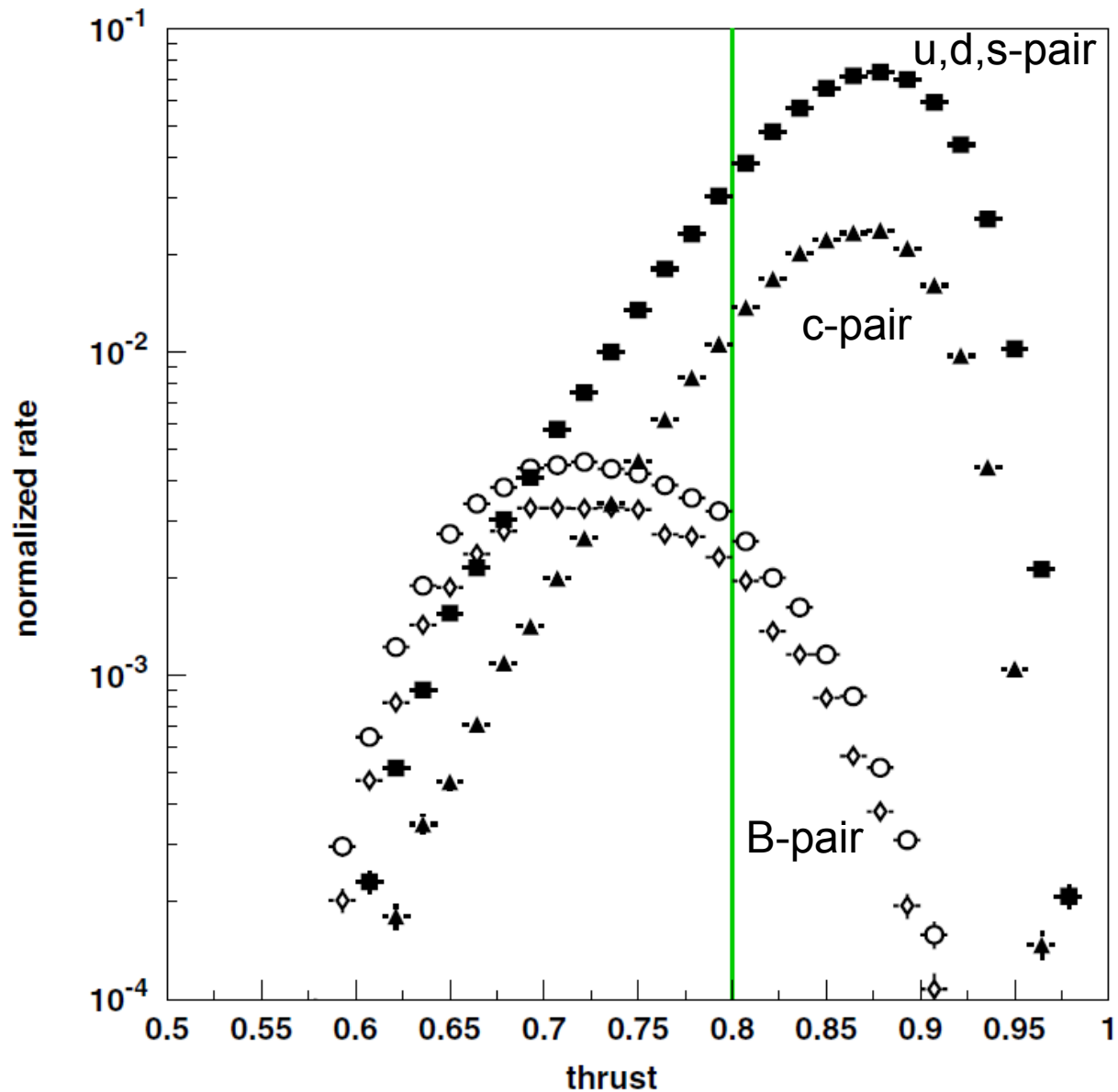
DSSV results....

$$E_H \frac{d^3\sigma}{dp_H^3} = \sum_{a,b,c} f_a \otimes f_b \otimes d\hat{\sigma}_{ab}^c \otimes D_c^H,$$



# Quark flavor sensitivity

Belle collaboration, Phys. Rev. D78 (2008) 032011



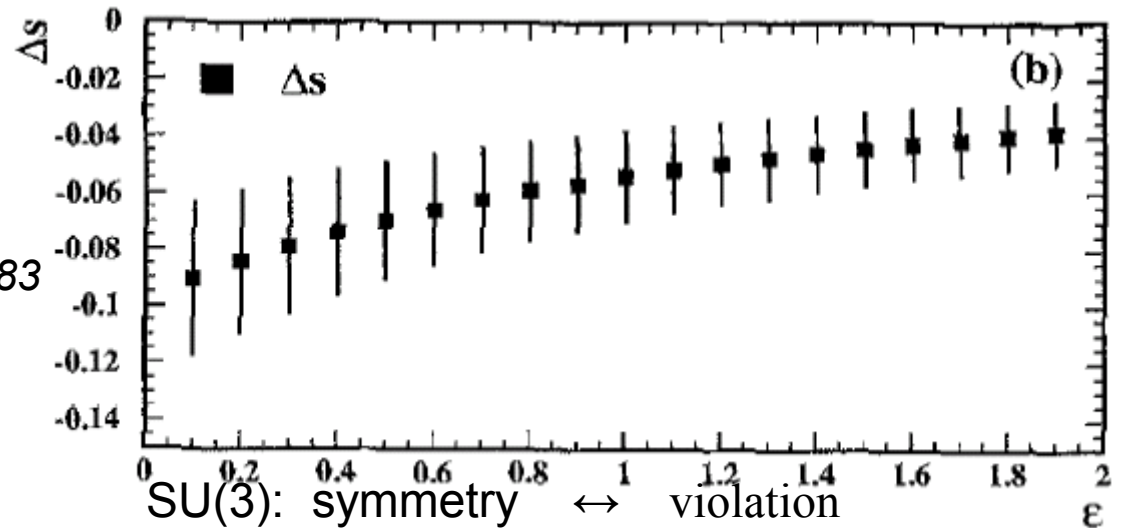
# SU(3) violation and impact on $\Delta s$

*J. Lichtenstadt and H. J. Lipkin, Phys. Lett. B353 (1995) 119*

$$\Delta s = 3 \Gamma_1^p - \frac{3}{2} D \left( \frac{F}{D} + \frac{1}{9} \right)$$

*E. Leader et. al., Phys. Lett. B488 (2000) 283*

$a_8$	$-(\Delta s + \Delta \bar{s})$
0.40	$0.02 \pm 0.01$
3F - D	$0.06 \pm 0.01$
0.86	$0.15 \pm 0.02$



$\chi$ QM calculation:

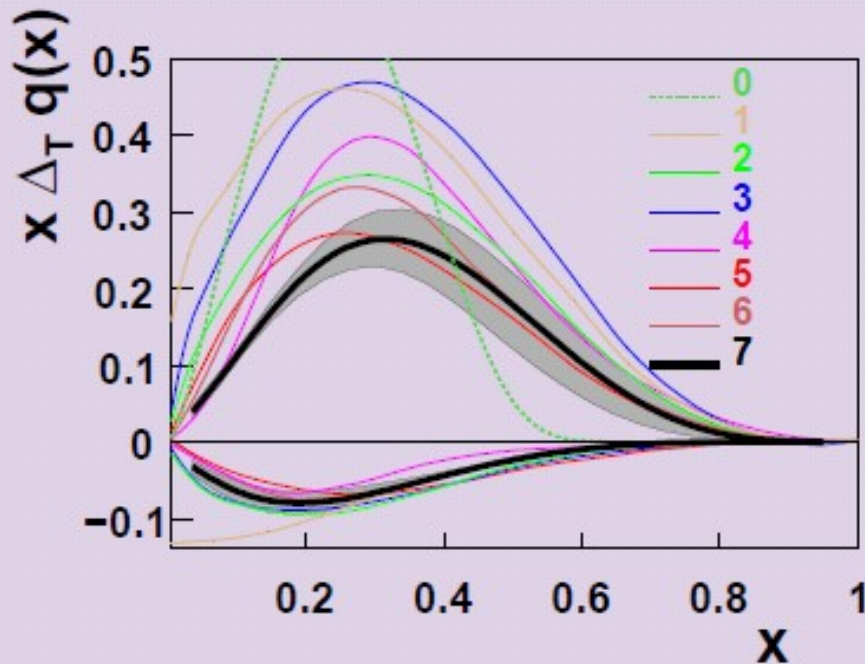
(X. Song et. al, Phys. Rev. D55 (1997) 2624-2629)

SU(3) symmetry:  $\Delta s = -0.1$

SU(3) breaking:  $\Delta s = -0.05$

# Extracted Transversity and models

New extraction is close to most models.



- ① Barone, Calarco, Drago PLB 390 287 (97)
- ② Soffer et al. PRD 65 (02)
- ③ Korotkov et al. EPJC 18 (01)
- ④ Schweitzer et al. PRD 64 (01)
- ⑤ Wakamatsu, PLB B653 (07)
- ⑥ Pasquini et al., PRD 72 (05)
- ⑦ Cloet, Bentz and Thomas PLB 659 (08)
- ⑧ This analysis.

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