"New exotics" from the $\widetilde{U}(12)$-scheme

- Pionic / Kaonic Transitions of Charmed Meson Systems -

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1. Introduction

U~(12)-scheme S. Ishida, M. Ishida, and T.M., PTP104 (2000)

- In recent years, we have proposed the U~(12)-scheme, a relativistic covariant level-classification scheme of hadrons.

- The U~(12) scheme predicts the existence of a new type of `exotic' hadrons, called chiral states, out of the conventional non-relativistic SU(6) classification scheme.

● In this work we concentrate on (open-)charmed mesons, where many experimental progress have been made during past few years.

(Newly discovered charmed mesons)

$$\begin{cases} Ds_0^*(2317) & \text{(BABAR'03, CLEO, Belle, FOCUS)} \\ Ds_1(2460) & \text{(CLEO'03, Belle, BABAR)} \end{cases}$$

$$\begin{cases} D_0^*(2400) & \text{(Belle'04, FOCUS'04)} \\ D_1^*(2430) & \text{(FOUCUS'04)} \end{cases}$$

$Ds_J^*(2632)$ (SELEX'04)

$Ds_1(2700)(J^P = 1^-)$ (Belle'06,BABAR'06)

$Ds_J^*(2860)$ (BABAR'06)

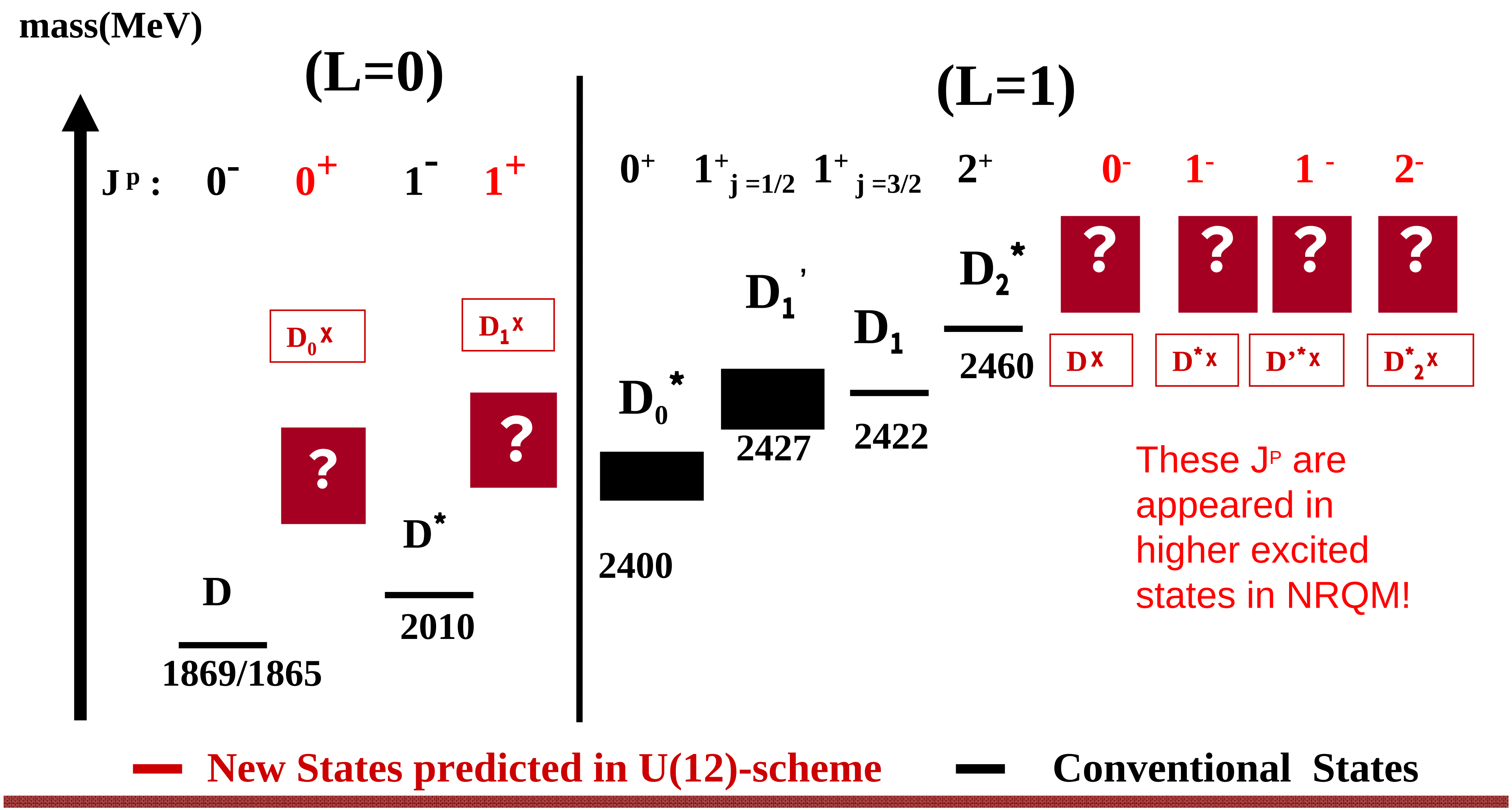
● They offer good candidates of chiral states.

- Charmed Mesons in Conventional Classification Scheme

	L=0		L=1			
(L-S) $^{2S+1}L_J$	$^1S_0$	$^3S_1$	$^3P_0$	$(^3P_1 \oplus$	$^1P_1)$	$^3P_2$
(S_Q-j_q) $J^P_{j_q}$	$0^-_{1/2}$	$1^-_{1/2}$	$0^+_{1/2}$	$(1^+_{1/2},$	$1^+_{3/2})$	$2^+_{3/2}$
	D	D^*	D_0^*	D_1'	D_1	D_2^*
Mass (in GeV)	1.869/ 1.865	2.010/ 2.007	2.40/ 2.35	2.427	2.420	2.460

taken from PDG 2008

●U(12)×O(3,1) classification scheme



Purpose of this work

- In this work, we study Pionic / Kaonic transition of D and Ds mesons by using the wave function determined from U(12)-scheme.

- Through this analyses, we will examine the possible realization of U(12)$\times$O(3,1) symmetry in open charmed meson sector.

2. Description of Composite Hadrons in the U~(12)-Scheme

We have proposed the U~(12)-scheme, a relativistically covariant level-classification scheme of hadrons.
S. Ishida, M. Ishida and T. M., PTP104(2000)

$$\tilde{U}(12)_{SF} \supset \tilde{U}(4)_S \otimes SU(3)_F$$

(U~$(4)_S$ being the pseudo-unitary homogeneous Lorentz-group)

The U~(12)-scheme has ***a unitary U(12)$_{SF}$ symmetry in the rest frame of hadrons***, embedded in the covariant U~$(12)_{SF}$-representation space.

In the U(12)-scheme, conventional non-relativistic symmetry,

$$SU(6)_{SF} \supset SU(2)_\sigma \times SU(3)_F$$

is extend into

$$U(12)_{SF} \supset \underline{U(4)_S} \times SU(3)_F$$

at the rest frame of hadrons.

$$\underset{4\times4}{\gamma} = \underset{2\times2}{\rho} \otimes \underset{2\times2}{\sigma}$$

ρ - spin

$$\boxed{U(4)_S \supset SU(2)_\sigma \times SU(2)_\rho}$$

$$\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix} \begin{matrix}\sigma\\ \sigma\end{matrix}\ \rho$$

The remarkable point in this scheme is a introduction of the new symmetry SU(2)$_\rho$ for “Confined Quarks”.

●Basic vectors for the new SU(2)$_\rho$ space are given by the Dirac spinors with on-shell 4-velocity of hadrons,

$$\bar{\rho}_3 \equiv -{}^t\rho_3$$

$$q \quad \Big(u_+(v=0), u_-(v=0)\Big) \quad (\rho_3 \; u_\pm(0) = \pm u_\pm(0))$$

$$\bar{q} \quad \Big(v_+(v=0), v_-(v=0)\Big) \quad (\bar{\rho}_3 \; v_\pm(0) = \pm v_\pm(0))$$

Chirality : $\gamma_5 = -\rho_1$

Parity : $\gamma_4 = \rho_3$

●The u_- and v_- with exotic quantum numbers (j^p=$(1/2)^-$) leads to a new type of `exotic' states, called ***chiral states***, which do not appear in the non-relativistic scheme.

●Kinematical Framework

$\boxed{\text{U}^{\sim}(12) \times \text{O}(3,1) \text{ (boosted LS) WF}}$

Feynman-Kislinger-Ravndal , Sogami, Y. S. Kim et al. , Namiki et al. , Ishida et al.

General WF of qqbar mesons are given by the following Klein-Gordon field with one each upper and lower indices.

C.M. Coordinate **Relative Coordinate** **Bargman-Wigner Spinor WF** **Flavor WF** **Definite Metric Type 4-Dim. Oscillator WF**

$$\Phi(X,x)_A{}^B \sim N e^{+iP_\mu X_\mu}\ (W\underbrace{(v)_{\mu\nu\cdots})\ {}_\alpha{}^\beta\ M_a{}^b}\ f\underbrace{{}^{(L)}_{\mu\nu\cdots}(v,x)}$$

(Here $A=(\alpha,a)$ etc. denotes Dirac spinor / flavor indices)

$$\boxed{\textbf{Spin}} \otimes \boxed{\textbf{Space-Time}}$$

A relativistic extension of conventional NRQM by separately boosting!

A remarkable point is that WFs of hadron are described as the *direct product of spin part and space-time part.*

●U(4) spin part

The expansion basis of qqbar meson spin WF is given by direct product of the respective Dirac spinors corresponding to relevant constituent quarks and anti-quarks. They consist of totally 16 members of bi-Dirac space.

$$\boxed{W(v)_\alpha{}^\beta := u_r(v)_\alpha \bar{v}_{r'}(v)^\beta} \qquad (r, r') = (\rho_3, \bar{\rho}_3)$$

<u>Total 16 comp.</u>

Complete set of bi-Dirac spinor for describing the qqbar mesons

$$W(v) = u_+(v)\bar{v}_+(v) \oplus \boldsymbol{u_-(v)\bar{v}_-(v)} \mapsto \begin{cases} \text{Ps} \times 2 \\ \text{V} \times 2 \end{cases}$$

$$\oplus\ u_-(v)\bar{v}_+(v) \oplus u_+(v)\bar{v}_-(v) \mapsto \begin{cases} \text{S} \times 2 \\ \text{A} \times 2 \end{cases}$$

➡ **In the U~(12)-scheme, the *chiral states* are defined as the states which includes *at least one $\rho_3 = -$ component*.**

What does the (u-, v-) stand for ?

$$\frac{\chi \;:\; \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}}{SU(2)_\sigma} \quad \Rightarrow \quad \frac{u(\mathrm{v}=0)}{U(4)_S} \;:\; \underbrace{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix}}_{\rho_3=+}, \underbrace{\cancel{\begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\end{pmatrix}}}_{\rho_3=-}$$

$$\supset SU(2)_\sigma \otimes \underline{SU(2)_\rho}$$

$$\mathrm{v} = \mathrm{P}/P_0 = 0$$

The (u_+, v_+) corresponds to conventional constituent quark degree of freedom, while (u_-, v_-) represents ***relativistic effect*** ***for confined system***.

We suppose that the u- (v-) is also realized as the physical degrees of freedom in composite hadrons.
This freedom may be dynamically generated, although it seems a very difficult problem that what is the explicit field theoretical representation of u- and v-.

One pion decays of S- and P-wave D(c-nbar)-mesons

Explicit Form of Internal WF

(1) Ground States $(L=0)$

	$(\rho_3, \bar{\rho}_3)$	J^P	Candidate	$W(v)_\alpha^\beta \left(\sim (c_{\rho_3})_\alpha \; (\bar{n}_{\bar{\rho}_3})^\beta\right) \otimes f^{(L=0)}(v,x)$
①	$(+,+)$	0^-	D(1869)	$[\frac{i\gamma_5}{2\sqrt{2}}(1+iv\gamma)]_\alpha{}^\beta \otimes \frac{\beta}{\pi}\exp\left[-\frac{\beta}{2}(x_\mu^2+2(v_\mu x_\mu)^2)\right]$
②		1^-	D*(2010)	$[\frac{i\gamma_\mu}{2\sqrt{2}}(1+iv\gamma)]_\alpha{}^\beta \epsilon_\mu(v) \otimes \frac{\beta}{\pi}\exp\left[-\frac{\beta}{2}(x_\mu^2+2(v_\mu x_\mu)^2)\right]$

$\epsilon_\mu(v)$:polarization vector of mesons $\quad \beta=\sqrt{K\mu} \quad \frac{1}{\mu}=\frac{1}{m_Q}+\frac{1}{m_q}$

(1) Ground States ($L=0$) Cont'd

	$(\rho_3, \bar{\rho}_3)$	J^P	Candidate	$W(v)_\alpha^\beta \left(\sim (c_{\rho_3})_\alpha \ (\bar{n}_{\bar{\rho}_3})^\beta\right) \otimes f^{(L=0)}(v,x)$
③	$(+,-)$	0^+	D_0*(2352)	$[\frac{1}{2\sqrt{2}}(1-iv\gamma)]_\alpha{}^\beta \otimes \frac{\beta}{\pi}\exp\left[-\frac{\beta}{2}(x_\mu^2+2(v_\mu x_\mu)^2)\right]$
④		1^+	D_1(2420) /D'$_1$(2430)	$[\frac{i\gamma_5\gamma_\mu}{2\sqrt{2}}(1+iv\gamma)]_\alpha{}^\beta \epsilon_\mu(v) \otimes \frac{\beta}{\pi}\exp\left[-\frac{\beta}{2}(x_\mu^2+2(v_\mu x_\mu)^2)\right]$

(2) Excited States ($L=1$)

	$(\rho_3, \bar{\rho}_3)$	${}^{2S+1}L_J^P$	Candidate	$W(v)_\alpha^\beta \left(\sim (c_{\rho_3})_\alpha \ (\bar{n}_{\bar{\rho}_3})^\beta\right) \otimes f^{(L=1)}(v,x)$
⑤	$(+,+)$	${}^3P_0^+$	D_0*(2352)	$[\frac{\gamma_\mu\epsilon_{\mu\nu}(v)}{2\sqrt{2}}(1+iv\gamma)]_\alpha{}^\beta \otimes \sqrt{2\beta}\,(x_\nu+(v_\lambda x_\lambda)v_\nu)\, f^{(L=0)}(v,x)$

(2) Excited States ($L=1$) Cont'd

	$(\rho_3, \bar{\rho}_3)$	${}^{2S+1}L_J^P$	Candidate	$W(v)_\alpha^\beta \left(\sim (c_{\rho_3})_\alpha \ (\bar{n}_{\bar{\rho}_3})^\beta\right) \otimes f^{(L=1)}(v,x)$
⑥	$(+,+)$	${}^3P_1^+$	$D_1(2420)$ /$D'_1(2430)$	$[\frac{\gamma_\mu \epsilon_{\mu\nu}(v)}{2\sqrt{2}}(1+iv\gamma)]_\alpha{}^\beta \otimes \sqrt{2\beta}\,(x_\nu + (v_\lambda x_\lambda)v_\nu)\, f^{(L=0)}(v,x)$
⑦		${}^1P_1^+$	$D_1(2420)$ /$D'_1(2430)$	$[\frac{i\gamma_5}{2\sqrt{2}}(1+iv\gamma)]_\alpha{}^\beta \epsilon_\nu(v) \otimes \sqrt{2\beta}\,(x_\nu + (v_\lambda x_\lambda)v_\nu)\, f^{(L=0)}(v,x)$
⑧		${}^3P_2^+$	$D_2^*(2460)$	$[\frac{\gamma_\mu \epsilon_{\mu\nu}(v)}{2\sqrt{2}}(1+iv\gamma)]_\alpha{}^\beta \otimes \sqrt{2\beta}\,(x_\nu + (v_\lambda x_\lambda)v_\nu)\, f^{(L=0)}(v,x)$

3. Possible Assignments for Observed Charmed Mesons in $U(12)_{SF} \times O(3,1)_L$

Here we try to assign some of the observed mesons to the predicted Qqbar multiplets in the $\tilde{U}(12)_{SF}$ classification scheme, resorting to their particle properties, and estimate the masses of missing members.

$$M_n^2 = M_0^2 + n\Omega, \;\; n = L + 2N$$

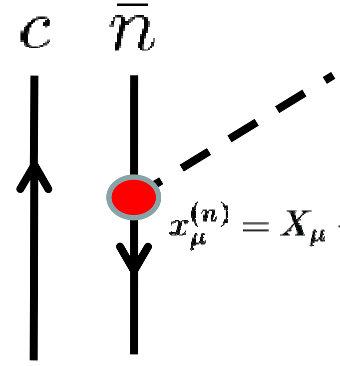
Possible assignments for the observed Ds mesons

(Input values are underlined)

Name	L=0 (~2GeV)			L=1 (~2.5 GeV)								
P	$^1S_0$	0-	Ds(1968)				$^1P_1$	1+	Ds1(2460)			
S		0+	**Ds_0 (2317)**					1-	**D*s1(~2740)**			
V	$^3S_1$	1-	**Ds^* (2112)**	$^3P_0$	0+	**Ds_0 (~2480)**	$^3P_1$	1+	Ds1(2536)	$^3P_2$	2+	**Ds_2(2573)**
A		1+	**Ds_1 (2460)**		0-	**Ds (????)**		1-	**D*s1(????)**		2-	**D_{s2} (~2860)**

Predicted Ds1(~2740) could be identified to recent observed Ds1(2690)?

4. Pionic and Kaonic Transitions



$$x_\mu^{(n)} = X_\mu + \frac{m_c}{m_n + m_c} x_\mu$$

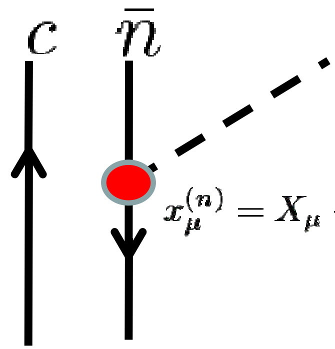
- Treating the decays in the U~(12) scheme

Pionic / Kaonic transitions of HL mesons are described by the following transition matrix element ;

$$T = \langle D_{\mathrm{final}}(v')\pi(q)|D_{\mathrm{initial}}(v)\rangle = \int d^4x f^*(v',x)\langle W(v)^{(+)} V(v,v',q,x) \bar{W}(v')^{(-)}\rangle f(v,x) e^{+i\frac{m_c}{m_n+m_c} q_\nu x_\nu}$$

$$\times \quad (2\pi)^4 \delta^4(Mv - M'v' - q)$$

Here we assume the emitted Ps-mesons as a local field. Interaction vertex V is assumed as



$$x_\mu^{(n)} = X_\mu + \frac{m_c}{m_n + m_c} x_\mu$$

$$g_p \langle \bar{\Phi}(x_1, x_2)(i\gamma_5)\pi(x_1)\Phi(x_1, x_2)\rangle$$

We also introduce an extra-term to reproduce effect of spin-orbit coupling,

$$g\frac{d}{2m_1}\langle\bar{\Phi}(x_1,x_2)[-i\gamma_5(\overrightarrow{\partial_1}+\overleftarrow{\partial_1})_\mu+\gamma_5\sigma_{\mu\nu}((\overrightarrow{\partial_1}-\overleftarrow{\partial_1})_\nu)]\Phi(x_1,x_2)\rangle B_\mu(x_1)$$

$$\left(\quad gB_\mu(x_1) = \frac{g_A}{\sqrt{2}f_\pi}\frac{\partial}{\partial x_1}\pi(x_1) \quad \right)$$

(Feynman- Kislinger- Ravndal (1971))

Space-Time Overlapping

Space-Time Overlapping gives Lorentz invariant form factor.

Example1: S-wave G.S. → S-wave G.S.

$$I_G(v,v',q) = \int d^4x f^{(L=0)}(v,x) f^{(L=0)}(v',x) e^{iq_\mu x_\mu \frac{m_c}{m_n+m_c}}$$

$$= (\frac{2MM'}{M^2+M'^2-m_{\pi^2}}) \exp\left(-\frac{m_n}{m_c}\frac{(M^2-M'^2)^2-m_\pi^2(M^2+M'^2)}{2\Omega(M^2+M'^2-m_\pi^2)}\right)$$

$\boxed{\to 1}$

Example 2: P-wave Ex.S. → S-wave G.S.

$$I_P(v,v',q)_\nu = \int d^4x f_\nu^{(L=1)}(v,x) f^{(L=0)}(v',x) e^{iq_\mu x_\mu \frac{m_c}{m_n+m_c}}$$

$$= i\sqrt{\frac{m_c}{\Omega m_n}}\frac{M^2+M'^2-m_\pi^2}{2MM'}(M' v_\nu - M v'_\nu) I_G(v,v',q)$$

etc.

Results

Here we use the following parameters;

- Coupling const. : $g_A = 0.7$, g_p=11 GeV
- Regge Slope Inverse : $\boldsymbol{\Omega} = M(D_2^*/Ds_2^*)^2 - M(D^*/Ds^*)^2$

(1-1) $D^*(1^-) \rightarrow D(0^-)\ \pi$ (P-wave decay)

Process	Our Results	Experiments (PDG 2008)
$D^{*+} \rightarrow D^0\ \pi^+$	71.9 keV	65 ± 15 keV
$D^{*+} \rightarrow D^+\ \pi^0$	34.2 keV	29 ± 7.2 keV
$D^{*0} \rightarrow D^0\ \pi^0$	57.2 keV	< 1.3 MeV

(1-2) $D_s^*(1^-) \rightarrow D_s(0^-)\ \pi$ (P-wave decay)

Process	Our Results	Experiments (PDG 2008)
$D_s^{*+} \rightarrow D_s\ \pi$	0.01 keV	< 0.11 MeV

Here we used isospin violating factor; 0.65×10^{-4}
(Cho and Wise, PRD49(1994))

(2-1) $D_2^*(2460;{}^3P_2) \to D(0^-)\ \pi$ and $D^*(1^-)\ \pi$ (D-wave decay)

Process	Our Results	Experiments (PDG 2008)
$D_2^* \to D\ \pi$	38 MeV	
$D_2^* \to D^*\ \pi$	18 MeV	
$D_2^{*(\text{total})}$	56 MeV	43 ± 4 MeV 37 ± 6 MeV

(2-2) $Ds_2^*(2573;{}^3P_2) \rightarrow D(0^-)\,K$ and $D^*(1^-)\,K$ (D-wave decay)

Process	Our Results	Experiments (PDG 2008)
$Ds_2^* \rightarrow D\,K$	18.6 MeV	
$Ds_2^* \rightarrow D^*\,K$	1.5 MeV	
$Ds_2^{*(\text{total})}$	20.1 MeV	20 ± 5 MeV

(3-1) $D_0^*(2350;{}^3P_0)\to D\,\pi$ (S-wave decay)
and $D_0^{*\chi}(2350;{}^1S_0) \to D\,\pi$ (S-wave decay)

Process	Our Results	Experiments (PDG 2008)
$D_0^*({}^3P_0)\to D\,\pi$	216 MeV	261 ± 50 MeV
$D_0^{\chi *}\to D\,\pi$	528MeV	

Interfering two resonances ?!

(3-2) $Ds_0^*(\sim 2480^*; {}^3P_0) \rightarrow D\ K$ (S-wave decay)
and $Ds_0^{*\chi}(2317; {}^1S_0) \rightarrow D\ \pi$ (S-wave decay)

Process	Our Results	Experiments (PDG 2008)
$Ds_0^*(\sim 2480; {}^3P_0) \rightarrow D\ K$	244 MeV	**????**
$D_0^{\chi *} \rightarrow D\ \pi$	75 keV	< 3.8 MeV

* Godfrey and Isgur ('85)

A broad state above DK threshold !?

Here we also used isospin violating factor; 0.65×10^{-4}
(Cho and Wise, PRD49(1994))

(5) $Ds_1^{*}\chi$ (~2700;$^1P_1$)→D K (p-wave decay) and $Ds_1^{*}\chi$(~2700;$^1P_1$) → D* K (p-wave decay)

Process	Our Results		Experiments (PDG 2008)
$Ds_1^{*}\chi$ (~2700;$^1P_1$)→D K	347 MeV	416 MeV	$108 \pm 23^{+36}_{-31}$ MeV
$Ds_1^{*}\chi$(~2700;$^1P_1$) → D* K	69 MeV		Brodzicka et al., (Belle Collaboration 2007)

(6) $D_{s1}^{*\chi}(\sim 2700; {}^3P_1) \rightarrow D\,K$ (p-wave decay) and $D_{s1}^{*\chi}(\sim 2700; {}^3P_1) \rightarrow D^*K$ (p-wave decay)

Process	Our Results		Experiments (PDG 2008)
$D_{s1}^{*\chi}(\sim 2700; {}^3P_1) \rightarrow D\,K$	126 MeV	373 MeV	$108 \pm 23^{+36}_{-31}$ MeV Brodzicka et al., (Belle Collaboration 2007)
$D_{s1}^{*\chi}(\sim 2700; {}^3P_1) \rightarrow D^*K$	247 MeV		

It seems that both $^3P_1$ and $^1P_1$ chiral states are too broad to assign to the experimental $D_{s1}(2700)$. On the one hand, it could be conventional 2S or 1D state, the other hand, there is a possibility that it could be the mixed sates of $^3P_1$ and $^1P_1$ chiral states!

5. Summary

- **The U~(12) scheme predicts the existence of a new type of `exotics', called chiral states, out of the conventional non-relativistic SU(6) classification scheme.**

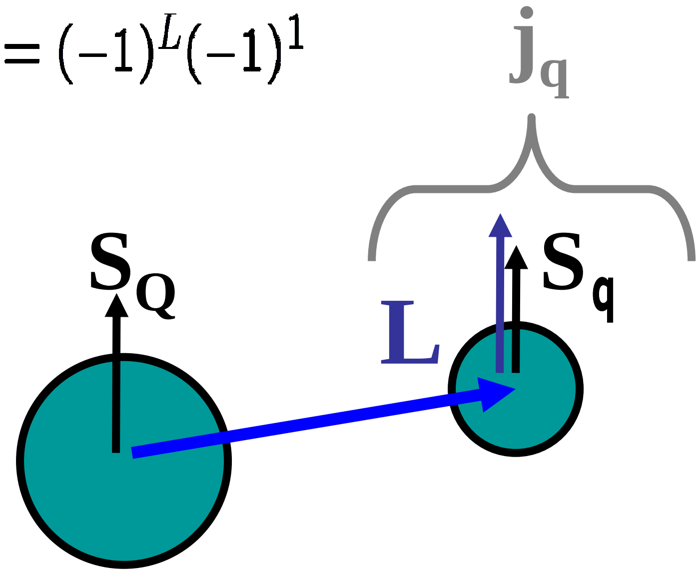
- **To check the validity of the scheme, we study strong decays of HL mesons by applying the U~(12) scheme. Results for conventional states are consistent with the data. In addition, we have predicted the Pionic and Konic decay widths of chiral states. Resultant broad widths obtained in this work are not contradict with experiments (except for $D_{s1}(2700)$).**

- **We show a possible assignments for observed meson from the viewpoint of the $U(12)_{SF} \times O(3,1)_L$. The existence broad chiral states should be checked in experiment, carefully.**

Back Up Slides

Heavy Light Meson System: Conventional Classification
− L-S v.s. S_Q-j_q −

$P = (-1)^L(-1)^1$



$$J = \boxed{S_Q + S_q} + L$$

$$= \boxed{S} + L$$

$$= S_Q + \boxed{j_q}$$

Heavy quark symmetry (HQS)

→ Level classification becomes possible using S_Q-j_q

Channel	Γ(EXP)	Γ(BEH)	Γ(ZZ)	Γ(CS)	Γ(God)
$D^*(1^-)^+ \to D(0^-)^0\pi^+$	65 ± 15 keV	65.1 keV	77 keV	25 keV	-
$D^*(1^-)^+ \to D(0^-)^+\pi^0$	29 ± 7.2 keV	30.1 keV	35 keV	11 keV	-
$D^*(1^-)^0 \to D(0^-)^0\pi^0$	< 1.3 MeV	43.6 keV	58 keV	16 keV	-
$D_0^*(0^+)(2352) \to D(0^-)\pi$	261 ± 50 MeV	~ 1372 MeV ?	248 MeV	283 MeV$^{**)}$	277 MeV
$D_1(1^+)(2420) \to D^*(1^-)\pi$	20.4 ± 1.7 MeV	-	21.6 MeV	22 MeV	25 MeV
$D_1'(1^+)(2430) \to D^*(1^-)\pi$	$384^{+107+74}_{-75-74}$ MeV	~ 1372 MeV?	220 MeV	272 MeV	244 MeV
$D_2^*(2^+)(2460) \to D(0^-)\pi$	-	-	39 MeV	35 MeV	37 MeV
$D_2^*(2^+)(2460) \to D^*(1^-)\pi$	-	-	19 MeV	20 MeV	18 MeV
$D_2^*(2^+)(2460) \to D(0^-)\eta$	-	-	0.1 MeV	0.08 MeV	-
$D_2^*(2^+)(2460)^+$ total	37 ± 6 MeV	-	59 MeV	55 MeV	55 MeV

(1) Space-time part : 4-dimentional oscillator function

- Basic equation of motion

$$[\sum_{i=1}^{2}\frac{1}{2m_i}\frac{\partial^2}{\partial x_{i\mu}^2}-U(x_1,x_2)]\Phi(x_1,x_2)_A{}^B=0$$

[H. Yukawa, Phys. Rev. **91** (1953), 415.]

(Potential) pure conf. limit

$$U=U_{conf}+U_{pert}\longrightarrow U(x_1,x_2)=\frac{1}{2}K(x_{1\mu}-x_{2\mu})^2$$

- CM and Relative coordinates

$$\boxed{X_\mu\equiv\frac{(m_1x_{1\mu}+m_2x_{2\mu})}{m_1+m_2},\quad x_\mu\equiv x_{1\mu}-x_{2\mu}}$$

$$[\frac{\partial^2}{\partial X_\mu^2}-\mathcal{M}(x)^2]\Phi(X,x)_{\alpha,a}{}^{\beta,b}=0\qquad\left(d=2(m_1+m_2),\quad \mu=\frac{m_1m_2}{m_1+m_2}\right)$$

$$\mathcal{M}^2(x)=dH(x)=d[-\frac{1}{2\mu}\frac{\partial^2}{\partial x_\mu^2}+U(x)]$$

- Plane Wave Expansion $\left(\mathcal{M}^2\Phi_n^{(+)}{}_A{}^B(x,P_n)=M_n^2\Phi_n^{(+)}{}_A{}^B(x,P_n)\right)$

$$\Phi_A{}^B(x_1,x_2)=\sum_{P_n,n}(e^{iP_n\cdot X}\underline{\Phi_n^{(+)}{}_A{}^B(x,P_n)}+e^{-iP_n\cdot X}\underline{\Phi_n^{(-)}{}_A{}^B(x,P_n)})$$

2-nd quantized!

References

BEH Heavy Quark Chiral Effective Theory : W. A. Bardeen, E. J. Eichten, and C. T. Hill, PRD68 (2003)

ZZ Chiral Quark Model : X. H. Zhong and Q. Zhao, PRD78 (2008)

CS $^3P_0$ Model : F. E. Close and E. S. Swanson, PRD72 (2005)

God Quark Model :S. Godfrey, PRD72 (2005)

PE Chiral Quark Model : M. D. Pierro and E. Eichten, PRD64 (2001)

Remark for decay width of j= (1/2) and j= (3/2)

HQS limit

$\frac{q}{m} \sim 0.15$

$$\left\{\begin{array}{l} {}^{3/2}P_2 \rightarrow {}^{1/2}S_{1,0} + \pi \\ {}^{3/2}P_1 \rightarrow {}^{1/2}S_1 + \pi \end{array}\right. \quad ; \textbf{ D-wave decay}$$

$$\left\{\begin{array}{l} {}^{1/2}P_1 \rightarrow {}^{1/2}S_1 + \pi \\ {}^{1/2}P_0 \rightarrow {}^{1/2}S_0 + \pi \end{array}\right. \quad ; \textbf{ S-wave decay}$$

$$\boxed{\Gamma \sim \frac{q^{2L+1}}{[M]^{2L}}} \Rightarrow \Gamma(3/2) \ll \Gamma(1/2)$$

Experimental values

	D_0^*	D_1^*	D_1	D_2^*
j_q	**1/2**	**1/2**	**3/2**	**3/2**
Γ(MeV)	**260-280**	**384**	**20-25**	**37-43**