Extraction of m_s and $|V_{us}|$ from Hadronic Tau Decays

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- Introduction: Theoretical Framework

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- Fixed m_s: Determination of $|V_{us}|$

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Introduction: Theoretical Framework

ALEPH, OPAL and CLEO ⇨ High precision status of

$$R_\tau \equiv \frac{\Gamma\left[\tau^- \to \mathrm{hadrons}\,\nu_\tau(\gamma)\right]}{\Gamma\left[\tau^- \to e^-\nu_e\nu_\tau(\gamma)\right]}$$

[and related observables] further increase at B-factories •

Sizeable correction in the semi-inclusive τ-decay width into Cabibbo-suppressed modes due SU(3) breaking •

Obtain the strange quark mass and $|V_{us}|$!

Advantage: The experimental uncertainty can be systematically reduced !

Introduction: Theoretical Framework

A lot of work !

- M. Davier ;
- S. Chen, A. Höcker, M. Davier ;
- K. Maltman ;
- K. Chetyrkin, J. Kühn, A. Pivovarov ;
- A. Pich, J.P. ;
- S. Chen, M. Davier, E. Gámiz, A. Höcker, A.Pich, J.P. ;
- E. Gámiz, M. Jamin, A. Pich, J.P., F. Schwab •

Very much improvable with expected B-factories accuracy !

Introduction: Theoretical Framework

Two-point correlation functions for vector $V_{ij}^{\mu} \equiv \bar{q}_i \gamma^{\mu} q_j$ and axial-vector $A_{ij}^{\mu} \equiv \bar{q}_i \gamma^{\mu} \gamma_5 q_j$ two-quark currents •

$$\Pi^{\mu\nu}_{V,ij}(q) \equiv i \int \mathrm{d}^4x \, e^{iq\cdot x} \langle 0|T\left([V^{\mu}_{ij}]^{\dagger}(x) V^{\nu}_{ij}(0)\right)|0\rangle$$

$$\Pi^{\mu\nu}_{A,ij}(q) \equiv i \int \mathrm{d}^4x \, e^{iq\cdot x} \langle 0|T\left([A^{\mu}_{ij}]^{\dagger}(x) A^{\nu}_{ij}(0)\right)|0\rangle$$

$$i, j = u, d, s\,;$$

Lorentz decomposition

$$\Pi^{\mu\nu}_{V(A),ij}(q) \equiv \left[q^{\mu}q^{\nu} - q^2 g^{\mu\nu}\right] \Pi^{T}_{V(A),ij}(q^2) + q^{\mu}q^{\nu} \Pi^{L}_{V(A),ij}(q^2)\,;$$

Im $\Pi^{J}_{V(A),ij}(q^2)$ are proportional to the corresponding spectral functions •

Introduction: Theoretical Framework

Using the analytic properties of $\Pi^J(s)$

$$R_\tau \equiv -i\pi \oint_{|s|=M_\tau^2} \frac{\mathrm{d}s}{s} \underbrace{\left[1-\frac{s}{M_\tau^2}\right]^3 \left\{3\left[1+\frac{s}{M_\tau^2}\right]\right.}_{\text{phase space factors: order three zero in real axis } \surd} \left. D^{L+T}(s) + 4 D^L(s)\right\} ;$$

$$D^{L+T}(s) \equiv -s \frac{\mathrm{d}}{\mathrm{d}s}\left[\Pi^{L+T}(s)\right] ; \qquad D^L(s) \equiv \frac{s}{M_\tau^2}\frac{\mathrm{d}}{\mathrm{d}s}\left[s\,\Pi^L(s)\right]$$

Large enough Euclidean Q^2 $\Rightarrow$ $\Pi^{L+T}(Q^2)$ and $\Pi^L(Q^2)$ organised in series of dimensional operators using OPE

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Moreover, we can decompose R_τ into

$$R_\tau \equiv R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

according to the quark content

$$\Pi^J(s) \equiv |V_{ud}|^2 \{\Pi^J_{V,ud}(s) + \Pi^J_{A,ud}(s)\} + |V_{us}|^2 \{\Pi^J_{V,us}(s) + \Pi^J_{A,us}(s)\}$$

★ Additional information obtained from the moments

$$R_\tau^{kl} \equiv \int_0^1 \mathrm{d}z \; (1-z)^k \, z^l \; \frac{\mathrm{d}R_\tau}{\mathrm{d}z} \equiv R^{kl}_{\tau,V+A} + R^{kl}_{\tau,S}$$

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$$R_\tau^{kl} \equiv N_c\, S_{\rm EW} \left(|V_{ud}|^2 + |V_{us}|^2\right) \left[1 + \delta^{kl(0)}\right]$$
$$+ \sum_{D\geq 2} \left[|V_{ud}|^2\, \delta_{ud}^{kl(D)} + |V_{us}^2|\, \delta_{us}^{kl(D)}\right]$$

$\delta_{ud}^{kl(D)}$ and $\delta_{us}^{kl(D)}$ ⇦ dimension D-operators •

The most important being $D = 2$ $[m_s^2]$ and $D = 4$ $[m_s\langle\bar{q}q\rangle]$ •

The flavour SU(3)-breaking quantity

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = N_c\, S_{\rm EW} \sum_{D\geq 2}\left[\delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)}\right]$$

enhances the sensitivity to the strange quark mass •

Introduction: Theoretical Framework

- $\delta_{ij}^{kl(2)}$ known to $\underline{\mathcal{O}(a^3)}$ for $J = L$ and $\underline{\mathcal{O}(a^2)}$ for $J = L + T$

★ Chetyrkin; Gorishny, Kataev, Larin, Sugurladze; Chetyrkin, Kühn; Becchi, Narison, de Rafael; Bernreuther, Wetzel •

- Extensive analysis by Pich & J.P.

- Perturbative $L + T$ series converges very well $\surd$
- Perturbative L series behaves very badly !

In following applications, $\delta_{ij}^{kl(4)}$ fully included while $\delta_{ij}^{kl(6)}$ estimated to be of order or smaller than error of $D = 4$ •

Fixed m_s: Determination of $|V_{us}|$

QCD Sum Rules, Lattice QCD and Tau Hadronic Data:

$m_s[2\mathrm{GeV}] = 95 \pm 20$ MeV ➾ δR_τ^{kl} predicted from theory !

Bad QCD behaviour of $J = L$ component in δR_τ^{kl}

➡ Theory uncertainty much reduced using

phenomenology for scalar/pseudoscalar correlators $\surd$

Dominant pseudoscalar us spectral function

$$s^2 \frac{1}{\pi} \mathrm{Im}\, \Pi^L_{us,A} = 2 f_K^2 m_K^4 \delta(s - m_K^2) + 2 f^2_{K(1460)} M^4_{K(1460)} BW(s)\,;$$

Normalized Breit-Wigner:Kambor,Maltman

Fixed m_s: Determination of $|V_{us}|$

Scalar spectral functions from M. Jamin, J.A. Oller, A.Pich $\surd$

Comparison of these spectral functions with QCD

	$R^{00,L}_{us,A}$	$R^{00,L}_{us,V}$	$R^{00,L}_{ud,A} \times 10^3$
OPE	$-\ 0.144 \pm 0.024$	-0.028 ± 0.021	-7.79 ± 0.14
Pheno.	$-\ 0.135 \pm 0.003$	-0.028 ± 0.004	-7.77 ± 0.08

★ Perturbative QCD for $J = L + T$ converges very well and OPE included up to $D = 6$ $\surd$

➡ $\delta R^{kl,L}_\tau$ from phenomenology while $\delta R^{kl,L+T}_\tau$ from QCD •

Fixed m_s: Determination of $|V_{us}|$

Smallest theory uncertainty for $(k,l) = (0,0)$

$$\delta R^{00}_{\tau,th} = (0.162 \pm 0.013) + (6.1 \pm 0.6) m_s^2 - (7.8 \pm 0.8) m_s^4 = 0.218 \pm 0.026$$

(Coefficients are MS-bar at 2 GeV)

$$|V_{us}|^2 = \frac{R^{kl}_{\tau,S}}{\frac{R^{kl}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{kl}_{\tau,th}}$$

Using OPAL update:

- ★ OPAL and CLEO: New branching fraction $B(\tau^- \to K^- \pi^+ \pi^- \nu)$
- ★ $R_{\tau,V+A} = 3.469 \pm 0.014$ and $R_{\tau,S} = 0.1677 \pm 0.0050$

Fixed m_s: Determination of $|V_{us}|$

$|V_{us}| = 0.2208 \pm 0.0033_{\text{exp}} \pm 0.0009_{\text{th}} = 0.2208 \pm 0.0034$ •

(using PDG value $|V_{ud}| = 0.9738 \pm 0.0005$)

Uncertainty becomes experimental issue !

PDG 2004: $|V_{us}| = 0.2200 \pm 0.0026$
Kl3 (E865,KTeV,KLOE):
Jamin et al, $|V_{us}| = 0.2229 \pm 0.0026$ to Leutwyler-Roos 0.2259 ± 0.0022
f_K/f_π Marciano, MILC: $V_{us} = 0.2219 \pm 0.0026$
Unitarity: $|V_{us}| = 0.2265 \pm 0.0022$

Remark: If experimental $B(\tau \to K\nu) = (0.686 \pm 0.023)\%$ is replaced by more precise theoretical value $(0.715 \pm 0.004)\%$ based on $K_{\mu 2}$ decay ⇒ $|V_{us}| = (0.2219 \pm 0.0034)$ •

Fixed $|V_{us}|$: Determination of m_s

Using OPAL data with $|V_{us}| = 0.2208 \pm 0.0034$ and $\delta R^{kl,L}_{\tau,\mathrm{phen}}$

$$\Rightarrow \quad \delta R^{kl,L+T}_{\tau} = \delta R^{kl}_{\tau} - \delta R^{kl,L}_{\tau,\mathrm{phen}} \quad \bullet$$

$$m_s^2(M_\tau^2) \simeq \frac{M_\tau^2}{1-\varepsilon_d^2} \frac{1}{\Delta_{kl}^{L+T(2)}(a_\tau)} \left[\frac{\delta R^{kl,L+T}_{\tau}}{18 S_{EW}} + \frac{8}{3}\pi^2 \frac{\delta O_4(M_\tau^2)}{M_\tau^4} Q^{L+T}_{kl}(a_\tau) \right]$$

known in perturbative QCD: very good convergence $\surd$

$$\delta O_4(M_\tau^2) \equiv \langle m_s \bar{s}s - m_d \bar{d}d \rangle \simeq -[1.5 \pm 0.4] 10^{-3}\ \mathrm{GeV}^4$$

and $\varepsilon_d \equiv m_d/m_s$ •

Fixed $|V_{us}|$: Determination of m_s

Moments $(0,0)$ and $(1,0)$ dominated by experimental uncertainty, we only use

	$(2,0)$	$(3,0)$	$(4,0)$
$m_s(M_\tau)$ MeV	93.2^{+34}_{-44}	86.3^{+25}_{-30}	79.2^{+21}_{-23}

Weighted average

$$m_s(M_\tau) = [84 \pm 23] \text{ MeV}$$
$$m_s(2\text{GeV}) = [81 \pm 22] \text{ MeV} \bullet$$

★ Larger OPAL $B(\tau^- \to K^-\pi^+\pi^-\nu)$ $\surd$

⇨ reduced the strong ALEPH (k,0)-moment dependence in m_s $\surd$

⇨ smaller value for m_s (115± 20→ 85 $\pm$ 20) MeV !

Combined Fit to $|V_{us}|$ and m_s

Ultimate procedure

- simultaneous fit to $|V_{us}|$ and m_s for a set of moments

First step, neglect the correlations and use the five OPAL moments R_τ^{00} to R_τ^{40}

Fit $|V_{us}| = 0.2196$ and $m_s(2\mathrm{GeV})$=76 MeV

Compatible with previous results $\surd$

Combined Fit to $|V_{us}|$ and m_s

★ Rather strong correlations ➾ expected uncertaintities similar to individual ones •

★ Moment-dependence of m_s is reduced in the fit •

Full analysis including correlations is under way !

Results and Conclusions

High precision tau hadronic (Cabibbo-suppressed) data from ALEPH, OPAL at LEP and CLEO at CESR provide already competitive results on $|V_{us}|$ and m_s •

Using OPAL spectral functions:

- $|V_{us}| = 0.2208 \pm 0.0034$
- $m_s(2\mathrm{GeV}) = [81 \pm 20]$ MeV

Combined fit to determine both $|V_{us}|$ and m_s ready soon !

Results and Conclusions

Open questions:

- Moment dependence of m_s very much reduced after OPAL and CLEO new $B(\tau^- \to K^- \pi^+ \pi^- \nu)$
  - ⇨ what happens with $K\pi\pi\pi$?

  Origin of remaining moment dependence ?
- ALEPH data, (S. Chen et al)

  m_s determination fulfils quark-hadron duality and OPE ,
  - ⇨ what happens with $|V_{us}|$? (see K. Maltman's talk)
- Low experimental $B(\tau \to K\nu)$ compared to theoretical prediction based on $K_{\mu 2}$ decays ?

Results and Conclusions

Previous issues: theoretical or experimental origin?

⇨ need more accurate measurements combined with theoretical analyses !

★ With expected B-factories accuracy, τ hadronic decays have the potential to provide one of the most accurate measurements for $|V_{us}|$ and m_s !