ISSUES IN DETERMINING V_{us}, m_s (ETC.)

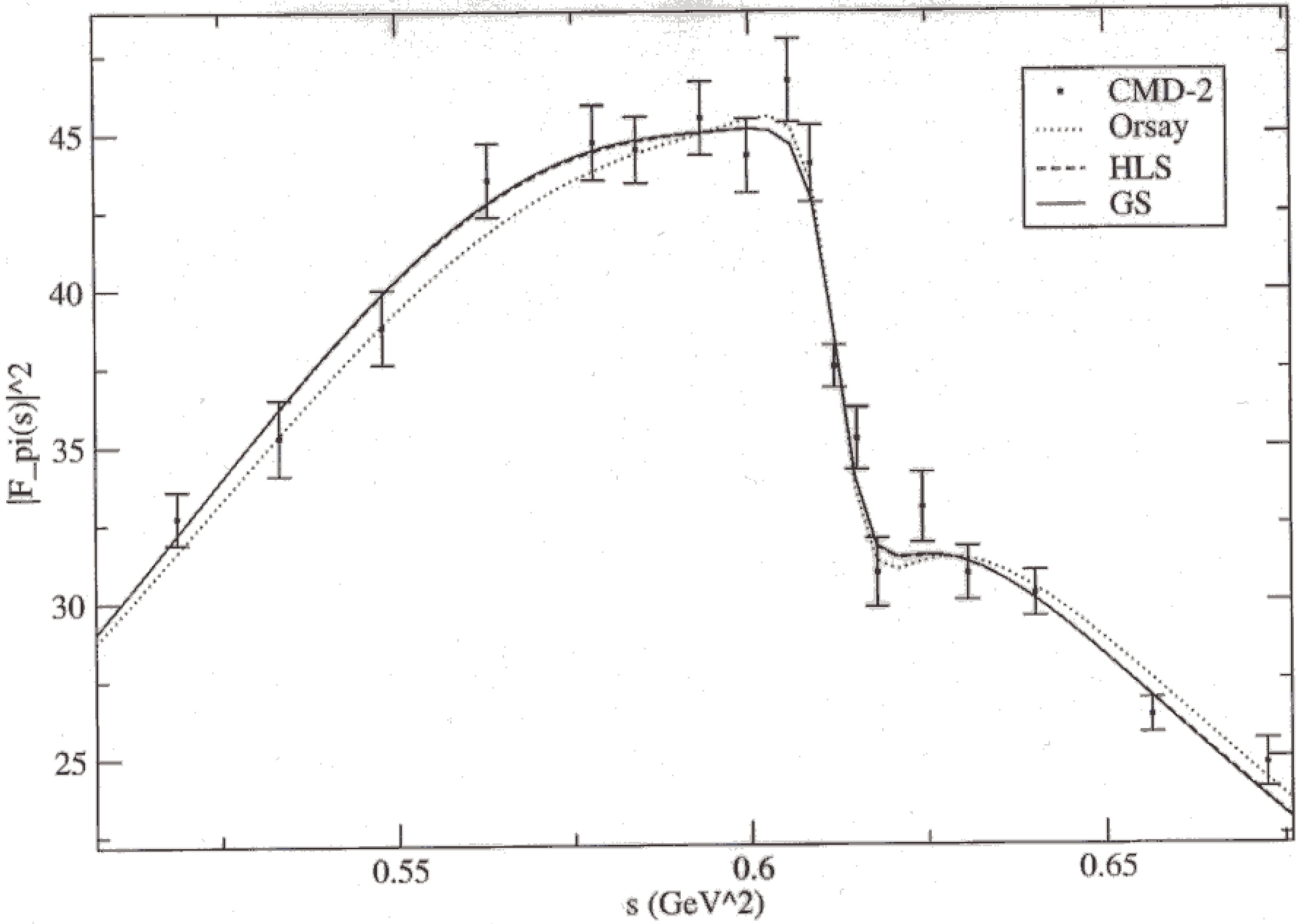
Kim Maltman, TAU'04, September, 2004

OUTLINE

- *Information from the non-strange spectral functions*
  - *a few comments on IB and a_μ^{had}*
  - *ud V-A sum rules, duality violation and the chiral limit $K \to \pi\pi$ EWP ME's*
- *Issues in the determination of V_{us}, m_s*
- *Results for V_{us}, m_s*

Fits to CMD-2 Data

Interference Region

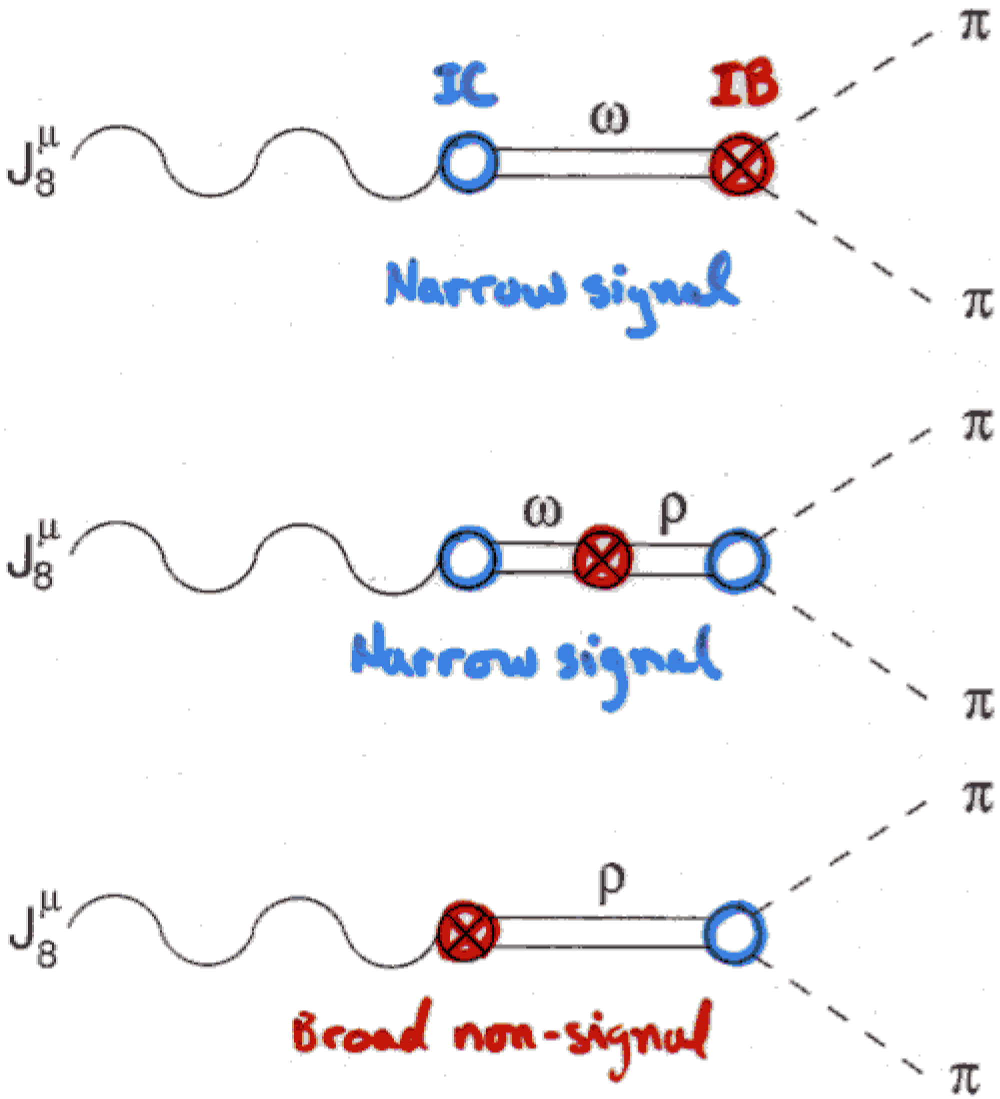


USES OF THE ud V,A SPECTRAL DATA

- IB in $e^+e^- \to \pi^+\pi^-$, the relation to $\tau^- \to \nu_\tau\pi^-\pi^0$, and a_μ^{had}
  - Uncertainty on integrated IB interference signal in $\pi\pi$ larger than integrated signal
    * e.g., using CMD2 data, integrated interference contributions to a_μ (units of 10^{-10}):

Model	δa_μ	χ^2/dof
GS	1.1 ± 0.5	0.94
HLS	-1.8 ± 1.0	0.97
Simple Orsay	-2.0 ± 1.1	1.5

    * $\Rightarrow \sim 3 \times 10^{-10}$ uncertainty in δa_μ

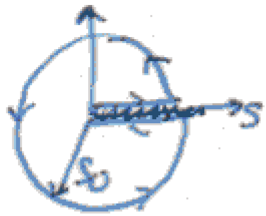


J_8^μ
IC
ω
IB
π
π
Narrow signal
J_8^μ
ω
ρ
π
π
Narrow signal
J_8^μ
ρ
π
π
Broad non-signal

- IB in $e^+e^- \to \pi^+\pi^-$, in principle, includes *broad* ρ contribution due to IB J_8^μ-ρ coupling [FIG]
  - resulting interference signal broad, NOT narrow like $\omega \to \pi^+\pi^-$
  - $\Rightarrow$ NOT experimentally determinable
  - sum rule for mixed-isospin 38 correlator with narrow expt'l interference contribution as input $\Rightarrow$ contribution to $\boxed{\delta a_\mu = (3.6 \pm 6.9) \times 10^{-10}}$
  - unlikely to be able to significantly improve error situation
- *CONCLUSION: theory error on IB correction appears (perhaps significantly) underestimated*

- Sum rules for the ud V-A correlator difference
  - $D = 6$ part of OPE yields chiral limit $K \to \pi\pi$ Q_8 EWP ME (one of two dominant contributions to ϵ'/ϵ in SM)
  - hybrid dispersive sum rules for chiral limit Q_7 ME
  - interesting for study of pattern of duality violation in QCD
  - improved V/A separation for $\bar{K}Kn\pi$ states, improved 4π data highly desirable
  - see KM, Cirigliano, Donoghue, Golowich PLB 522 (2001) 245, PLB 555 (2003) 71 (re EWP ME's); KM, VC, EG PRD 68 (2003) 054013 (re duality violation, EWP ME's) and related Refs. therein for details

ISSUES IN DETERMINING m_s, V_{us}



- Basic FESR rel'n ($w(s)$ analytic)

$$\int_{s_{th}}^{s_0} ds\, w(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\Pi(s)$$

DATA OPE

  - sizeable duality violation in ud V,A FESR's, *even at* $s_0 \sim m_\tau^2$ UNLESS $w(s_0) = 0$ ("pinched" weights)

  - $y \equiv s/s_0$, $w(s) \to w(y)$ for pinching

  - $a_k y^k$ in $w(y)$, $\Pi_{OPE} = \sum_D c_D/Q^D \Rightarrow$ integrated OPE contribution $\sim \frac{a_k c_{2k+2}}{s_0^k}\delta_{D,2k+2}$

  - different $D \Rightarrow$ different s_0 dependence (separation of different D/check on assumed absence of higher D)

- For $s_0 = m_\tau^2$, $w_T(y) = (1-y)^2(1+2y)$, $w_L(y) = -2y(1-y)^2$,

$$R^{V/A;ij} \equiv \frac{\Gamma[\tau^- \to \nu_\tau \, \text{hadrons}_{V/A;ij}(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]} ,$$

$$\begin{aligned} R^{V/A;ij} / \left[12\pi^2 |V_{ij}|^2 S_{EW}\right] \\ = \int_0^1 dy \left[w_T(y)\,\rho^{(0+1)} + w_L(y)\,\rho^{(0)}\right] \\ = \frac{i}{2\pi} \oint_{|y|=1} dy \left[w_T(y)\,\Pi^{(0+1)} + w_L(y)\,\Pi^{(0)}\right] \end{aligned}$$

(+ analogous OPE/spectral integral rel'ns for $s_0 \neq m_\tau^2$, different weights)

- "(k, l) spectral weights": $w_{T,L}(y)$ simultaneously $\to (1-y)^k y^l w_{T,L}(y)$ (allow full experimental spectral distribution to be rescaled bin-by-bin without L/T separation)

- Flavor-breaking differences, e.g.,

$$\frac{R^{V+A;ud}}{V_{ud}^2} - \frac{R^{V+A;us}}{V_{us}^2}$$

(+ generalizations to different s_0, weights)

  - leading OPE $D = 2$ term $\propto m_s^2$
  - us spectral integral scaled by wrong V_{us}^2 leaves residual $D = 0$ OPE contribution
  - for weights used in literature, different s_0 dependence, strong ud-us cancellation (to few → several % level) ⇒ strong lever arm for determining V_{us} [Pich, Prades et al., JHEP 0301: 060; hep-ph/0408044] *

PRACTICAL ISSUES/CONSIDERATIONS

- Longitudinal OPE contribution problem
- Integrated $D = 2$ OPE series convergence (and choice of weight)
- Degree of ud-us spectral integral cancellation (and choice of weight)
- Convergence with D (higher D OPE contributions/higher degree weight coefficients – and choice of weight)

LONGITUDINAL OPE CONTRIBUTIONS

- non-π, K pole ("resonance") spectral terms chirally suppressed c.f. pole terms

- integrated longitudinal $D = 2$ OPE series VERY badly converging (terms growing in size to order $O(a^3)$, even at highest possible scale ($s_0 = m_\tau^2$))

- *truncations at fixed integrated order BADLY violate constraints of spectral positivity in scalar, PS resonance regions [KM, Kambor PRD 64 (2001) 093014]*

- $\Rightarrow$ NO CHOICE BUT TO subtract longitudinal spectral contributions and work with $(J) = (0 + 1)$ sum rules

- Subtraction of L part of the experimental distribution/determination of $\rho^{(0+1)}_{V+A;us}$:

  - dominant (π, K pole term) parts of subtraction very accurately known

  - theoretical determination of resonance part [L us, V: Jamin, Oller, Pich NPB 622 (2002) 279; L us, A: Kambor, KM PRD 65 (2002) 074013]

  - results ⇒ longitudinal *resonance* subtraction effect small, even if errors ~ 50%, negligible c.f. other sources

  - compatibility with SR's for us scalar, PS channels, m_s [JOP: EPJC 24, 237; K+M: PRD 65 (2002) 074013, PLB 571, 332]

  - *NOTE:* Necessity of L subtraction removes "(k,l) spectral weight" advantage

- Expt: please quote upper bounds (or value) for $B(\tau \to \nu K_0^*(1430))$ as check on L subtraction!!

THE INTEGRATED $D = 2$ OPE SERIES

- Order-by-order (in a) convergence of integrated $D = 2$, $(V + A)^{(0+1)}_{ud-us}$ OPE series not good for many weights, even after "contour improvement"

- $(0 + 1)$ OPE coefficients known to $O(a^2)$ [Chetyrkin, Kwiatkowski, ZPC 59 (1993) 525], estimated to $O(a^3, a^4)$ [Baikov, Chetyrkin, Kuhn, PRD 67 (2003) 074026]

- $O(a^3, a^4)$ coefficients calibrated against known N_f-dependent $O(a^3, a^4)$ terms

- Some apparent improved convergence actually due to "accidental" cancellation (cancellation in contour integral at one order which does not persist to higher order)

- To monitor: both order-by-order behavoir AND correlator vs. Adler function form

- TABLE

- Some commonly-used weights (e.g., (0,0) spectral weight) do NOT have good convergence behavior

- Some non-spectral weights exist having improved behavior [e.g. w_{20}, w_{10} of KM, Kambor PRD 62 (2000) 093023]

- Some spectral weights are "more equal than others"

CONVERGENCE BEHAVIOR OF $w(y)$-WEIGHTED $D = 2$ OPE INTEGRALS

(0,0) *SPECTRAL WEIGHT CASE* (0+1)

s_0 [GeV^2] →	1.95	1.95	2.55	2.55	3.15	3.15
Order↓	ADLER	CORR	ADLER	CORR	ADLER	CORR
$O(a^0)$	.00174	.00187	.00169	.00189	.00165	.00187
$O(a^1)$	.00044	.00015	.00047	.00024	.00047	.00027
$O(a^2)$	-.00004	-.00036	.00009	-.00012	.00014	-.00002
$O(a^3)$	-.00056	-.00106	-.00027	-.00056	-.00013	-.00033
$O(a^4)$	-.00123	-.00187	-.00068	-.00104	-.00042	-.00066
Sum to $O(a^2)$	.00215	.00167	.00225	.00200	.00225	.00212
Sum to $O(a^3)$	.00159	.00061	.00198	.00144	.00212	.00179
Sum to $O(a^4)$	.00035	-.00126	.00130	.00040	.00170	.00113

$s_0 = 3.15$ GeV^2 $(1,0),\cdots,(4,0)$ *SPECTRAL WEIGHT CASES*

Weight →	(1,0)	(1,0)	(2,0)	(2,0)	(3,0)	(3,0)	(4,0)	(4,0)
Order↓	ADLER	CORR	ADLER	CORR	ADLER	CORR	ADLER	CORR
$O(a^0)$	.00199	.00242	.00231	.00294	.00260	.00347	.00289	.00401
$O(a^1)$	.00073	.00051	.00099	.00076	.00125	.00103	.00152	.00133
$O(a^2)$	.00042	.00024	.00072	.00055	.00106	.00092	.00144	.00136
$O(a^3)$	.00014	-.00009	.00049	.00026	.00091	.00073	.00142	.00136
$O(a^4)$	-.00019	-.00052	.00016	-.00023	.00065	.00026	.00131	.00100
Sum to $O(a^2)$	.00314	.00317	.00401	.00425	.00491	.00542	.00586	.00671
Sum to $O(a^3)$	.00329	.00308	.00450	.00451	.00582	.00615	.00728	.00806
Sum to $O(a^4)$	.00309	.00256	.00466	.00428	.00647	.00641	.00859	.00906

$s_0 = 3.15$ GeV^2 w_{10}, w_{20}, $w_{3/4}(y)$ *CASES*

Weight →	w_{10}	w_{10}	w_{20}	w_{20}	$w_{3/4}$	$w_{3/4}$
Order↓	ADLER	CORR	ADLER	CORR	ADLER	CORR
$O(a^0)$	.00199	.00247	.00225	.00288	.00205	.00258
$O(a^1)$	.00076	.00057	.00096	.00076	.00082	.00064
$O(a^2)$	.00051	.00041	.00073	.00062	.00059	.00050
$O(a^3)$	.00035	.00027	.00057	.00049	.00046	.00040
$O(a^4)$	.00021	.00011	.00042	.00030	.00034	.00025
Sum to $O(a^2)$	.00326	.00345	.00393	.00426	.00346	.00372
Sum to $O(a^3)$	.00361	.00372	.00450	.00475	.00391	.00411
Sum to $O(a^4)$	.00381	.00383	.00492	.00505	.00425	.00436

ud-us SPECTRAL CANCELLATION ISSUES

- ud-us cancellation typically MUCH closer than naive $\sim$ 30% SU(3)-breaking expectations

- e.g., for $s_0 \simeq m_\tau^2$, PDG04 central unitarity-non-constrained values $V_{ud} = 0.9738$, $V_{us} = 0.2200$, $(V+A)^{(0+1)}_{ud-us}$ cancellation to

Weight	Cancellation
$(0,0)$	0.6%
$(1,0)$	4.5%
$(2,0)$	8.3%
$(3,0)$	12%
$(4,0)$	16%
w_{20}	7.5%
w_{10}	3.7%

- strong cancellation magnifies impact of experimental errors

- premium on choosing weight to reduce close cancellation

- premium on reducing experimental error (e.g., using accurately-known $K_{\ell 2}$-based SM prediction for $B[\tau \to \nu_\tau K]$ rather than higher-error experimental determination)

ISSUES RE $D > 6$ OPE CONTRIBUTIONS

- large coefficient of y^k in $w(y) \Rightarrow$ enhanced integrated $D = 2k+2$ OPE contribution (scaling as $1/s_0^k$)

- large coefficients in $w_{(N,0)} \equiv \bar{w}_N$ spectral weights for large N

$$
\begin{aligned}
\bar{w}_0(y) &= 1 - 3y^2 + 2y^3 \\
\bar{w}_1(y) &= 1 - y - 3y^2 + 5y^3 - 2y^4 \\
\bar{w}_2(y) &= 1 - 2y - 2y^2 + 8y^3 - 7y^4 + 2y^5 \\
\bar{w}_3(y) &= 1 - 3y + 10y^3 - 15y^4 + 9y^5 - 2y^6 \\
\bar{w}_4(y) &= 1 - 4y + 3y^2 + 10y^3 - 25y^4 + 24y^5 \\
&\quad - 11y^6 + 2y^7
\end{aligned}
$$

- e.g. of potential problem: IF neglected $D > 8$ OPE coefficients $\sim 30\%$ corresponding ud $V - A$ coefficients [KM, VC, EG PRD 69: 054013], weight coefficients ~ 25 dangerous, even for $s_0 \sim m_\tau^2$ (though ~ 1 weight coefficients likely OK) [even signif. if $\times \frac{1}{4}$ more, as for $D=6$ VSA estimate]

- can use s_0 dependence to check for presence of neglected higher D contributions

- *check of stability wrt s_0 ONLY way to test plausibility of neglect of $D > 6$ contributions* (especially important for $w(y)$ with large coefficients)

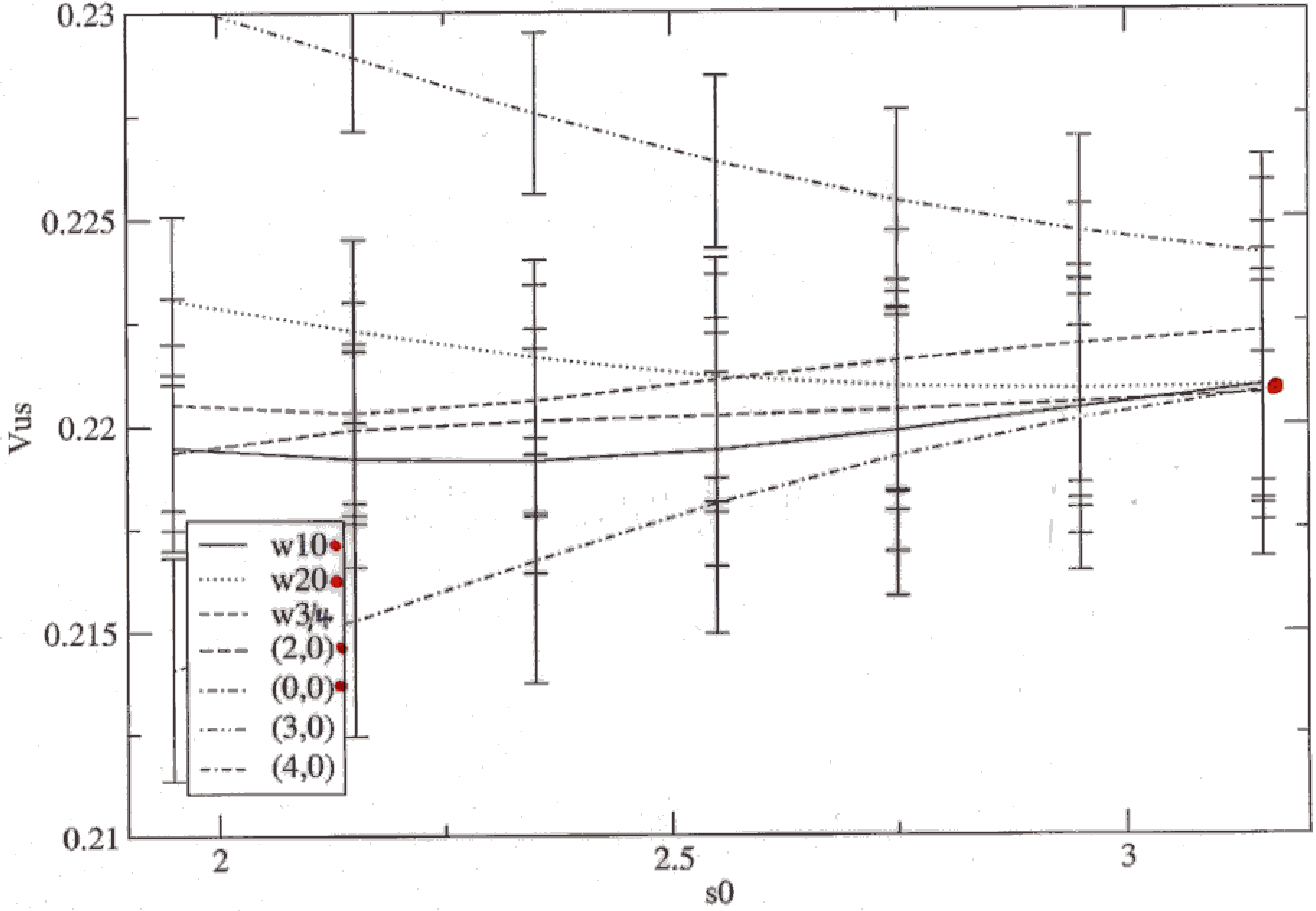
RESULTS FOR V_{us}, m_s

Based on:

- $w(y)$ with good $D = 2$ OPE convergence
- current WA strangeness branching fractions, including 2004 OPAL update
- for non-spectral $w(y)$ (pending access to OPAL distributions): ALEPH99 us distribution rescaled to current BR's
- JOP L us V; K+M L us A subtractions; $m_s(2\ GeV) = 95 \pm 20$ MeV
- $s_0 = (2.55, \cdots, 3.15\ \mathrm{GeV}^2$ analysis window

|Vus| with fixed ms(1 GeV) = 131 MeV [$m_s(2\,\mathrm{GeV})=95$]

Current rescalings ("exp. errors" only)



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1. $|V_{us}|$ RESULTS FOR $s_0 = m_\tau^2$

Weight	$\|V_{us}\| \pm \delta V_{us}^{exp}$
(2,0)	0.2212 ± 0.0027
w_{20}	0.2214 ± 0.0026
w_{10}	0.2214 ± 0.0031
(0,0)	0.2212 ± 0.0038
(3,0)	0.2242 ± 0.0024

c.f. MILC update of Marciano $\Gamma(\pi_{\mu 2})/\Gamma(K_{\mu 2})$ analysis:

0.2219 ± .0026

[Figure for s_0-stability, weight-to-weight consistency]

2. $s_0 = m_\tau^2$ combined $|V_{us}|$, $m_s(2\,GeV)$ fit with $w_{10}, w_{20}, w_{(2,0)}$

Central values: $|V_{us}| = 0.2214$
$m_s(2\,GeV) = 96$ MeV

3. CAUTION!

e.g. $\Gamma(\tau^- \to \nu_\tau K^- \pi^+ \pi^-) = \begin{cases} .214 \pm .037 \pm .029\% \text{ (ALEPH)} \\ .384 \pm .014 \pm .038\% \text{ (CLEO)} \\ .415 \pm .059 \pm .031\% \text{ (OPAL)} \end{cases}$

WA (OPAL'04) : .330 ± .28% (used above)

If → .40% (OPAL+CLEO "average")

$|V_{us}| \to \begin{cases} 0.2231 & w_{(2,0)} \\ 0.2234 & w_{20} \\ 0.2238 & w_{10} \end{cases}$

[Total S=−1 B ~ 3% so .07% shift is ~ 2% total ⇒ induced ~ 1% shift in V_{us}]

Need **accurate** strange decay data

THE (NEAR) FUTURE FOR m_s, $|V_{us}|$

- alternate weight choices much better adapted to extraction of $|V_{us}|$ than those discussed above [with C. Wolfe]

- possible alternate weight choices with reduced ud-us integrated spectral cancellation, but still good OPE D=2 convergence and small coefficients for D>6 contributions [re m_s]

- much reduced us spectral errors from B-factory data eagerly awaited!

- (Public access to covariances needed to find optimized weight choices)