Tau 04

VNIVERSITAT DE VALÈNCIA

Hadronic decays of the Tau lepton: A theoretical point of view

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CSIC

Euridice Network

Summary

- Short overview : Experiment vs theory
- Model building or do we care about QCD?
- $\tau^- \to P^- \nu_\tau$
- $\tau^- \to (2P)^- \nu_\tau$
- $\tau^- \to (3P)^- \nu_\tau$
- $\tau^- \to (> 3P)^- \nu_\tau$
- Conclusions

Summary

- Short overview : Experiment vs theory
- Model building or do we care about QCD?
- $\tau^- \to P^- \nu_\tau$ ← Uninteresting
- $\tau^- \to (2P)^- \nu_\tau$
- $\tau^- \to (3P)^- \nu_\tau$

  Reasonably good control
- $\tau^- \to (> 3P)^- \nu_\tau$

  Poor knowledge
  [Rougé, 1996,1998]
  [Sobie, 1995,1999]

  Theory
- Conclusions

Short overview : Experiment vs Theory

$$\mathcal{M}(\tau \to H\nu_\tau) = \frac{G_F}{\sqrt{2}} V_{CKM}\ \overline{u_{\nu_\tau}}\ \gamma^\mu (1-\gamma_5)\ u_\tau\ H_\mu$$

$m_\tau \approx 1.8$ GeV

Decay in an energy region driven by resonances

$$H_\mu = \left\langle H \left| \left(\mathcal{V}_\mu - \mathcal{A}_\mu\right) e^{i\mathcal{L}_{QCD}} \right| 0 \right\rangle = \sum_i \underbrace{(\ldots\ldots\ldots)^i_\mu}_{\text{Lorentz structure}}\ \overbrace{F_i\left(q^2,\ldots.\right)}^{\text{Form Factor}}$$

Determination of form factors: Present situation

Models

Gounaris & Sakurai (1968)

Kühn & Santamaría (1990)

→ Parameterization of the Form Factor →

Experimental Data

Analysis : TAUOLA

[Was et al, 1990,1993]

→

Masses

Widths

Couplings

Hadronic modes ~ 66%

2P

~ 26 %

- $\pi^-\pi^0, K^-K^0$ / $K^-\pi^0, \overline{K^0}\pi^-$
  - Branching fractions ✓
  - Spectrum ✓
  - ALEPH, CLEO-III, OPAL.....BABAR
- η-modes
  - Branching fractions

3P

~ 20 %

- $\pi\pi\pi$
  - Branching fractions
  - Spectrum
  - Structure functions
  - ✓✓
  - ALEPH, CLEO-III, DELPHI,OPAL
- $KK\pi$ / $K\pi\pi$
  - Branching fractions ✓
  - Spectrum ~
  - ALEPH, CLEO-III, DELPHI,OPAL
- η-modes
  - Branching fractions
- KKK
  - Not yet seen

>3P

~ 7 % { $4\pi, 5\pi$; $K3\pi$ }

- Branching fractions ✓
- Spectrum ~

ALEPH, CLEO-II, OPAL.....BABAR

>3P

~ 7 % { $4\pi, 5\pi$; $K3\pi$ }

Branching fractions	✓	ALEPH, CLEO-II, OPAL.....BABAR
Spectrum	~	

.....waiting for BELLE....

☑

>3P

~ 7 % $\left\{ \begin{array}{l} 4\pi, 5\pi \\ K3\pi \end{array} \right.$

- Branching fractions ✓
- Spectrum ~

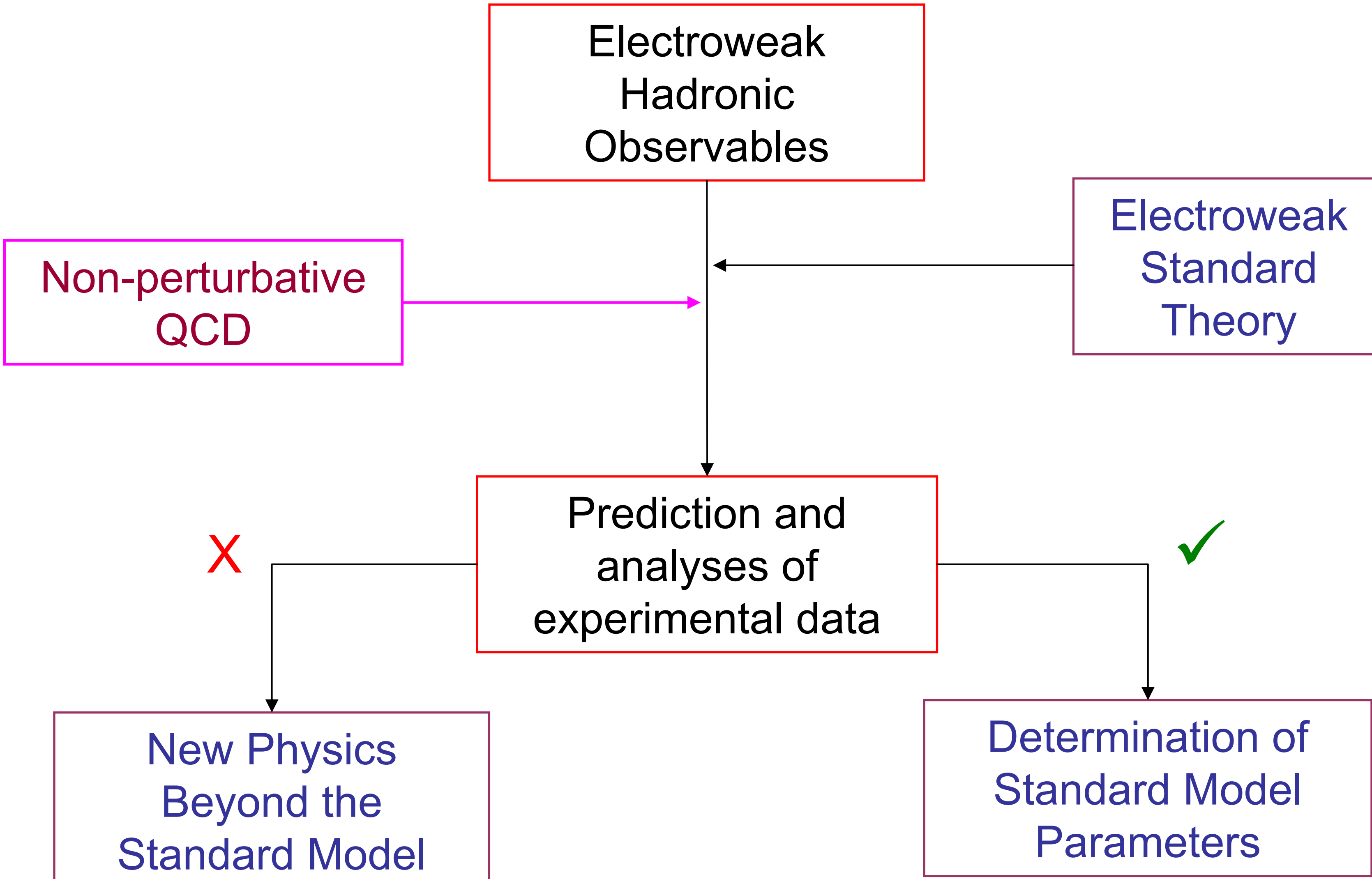
ALEPH, CLEO-II, OPAL.....BABAR

…..waiting for BELLE….

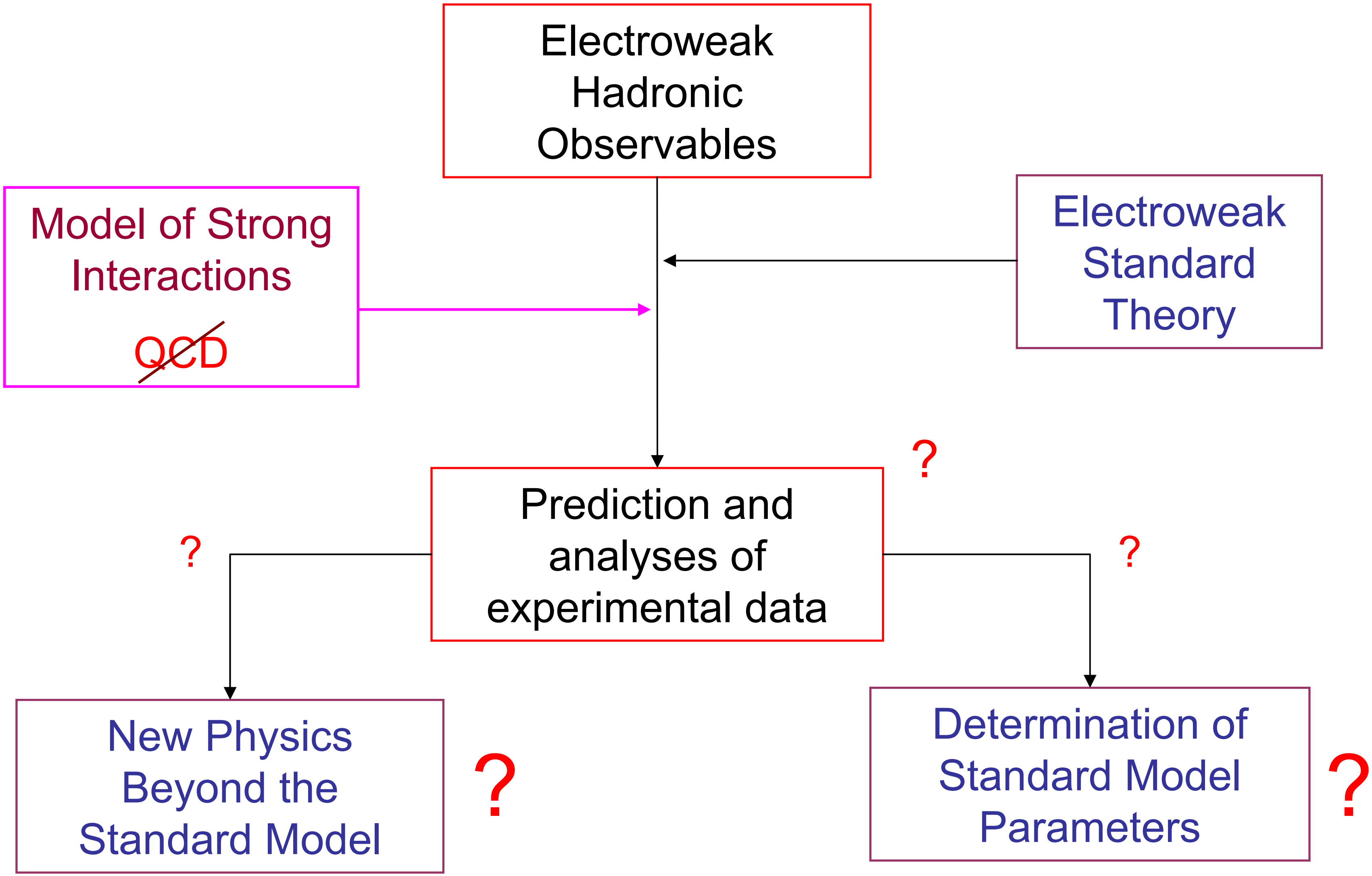
☑

Is Theory at the same good level?

Model building or do we care about QCD?



Model building or do we care about QCD?



Models and parameterizations

- [Gounaris, Sakurai,1968] — Pion form factor (resonance dynamics)
- [Tsai, 1971] — Current Algebra parameterization
- [Fischer, Wess, Wagner, 1980]
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  [Kühn, Santamaría, 1990]
  [Decker, Finkemeier, Kühn, Mirkes, Was…,1990-2000]
  — Kühn & Santamaría model. Parameterization of 2P, 3P
- [Bruch, Khodjamirian, Kühn, 2004] — (KS,GS) modified – dual QCD($N_C \to \infty$)

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- [Beldjoudi, Truong, 1995] — Current Algebra + Dispersion Relations ($\pi\pi$, $K\pi$, $K\eta$, 3π, $K\pi\pi$)

- [Guerrero, Pich, 1997] [Pich, Portolés, 2001] — χPT+ RχT + Dispersion Relations (Pion form factor)

- [Sanz-Cillero, Pich, 2003] [Rosell, Sanz-Cillero, Pich, 2004] — RχT + large-N_C expansion (Pion form factor)

- [Gómez Dumm, Pich, Portolés, 2004]
  - RχT (chiral symmetry)
  - Large-N_C expansion
  - Asymptotic behaviour ruled by QCD
  - (3π) $\longrightarrow$ all 3P

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Kühn & Santamaría Model

[Kühn, Santamaría, 1990]

KS:

- χPT $O(p^2)$ ✓
- Vector meson dominance ✓
- Asymptotic behaviour ruled by QCD ✓
- $$BW_R = \frac{M_R^2}{M_R^2 - s - i\,\sqrt{s}\,\Gamma_R(s)}$$?

Example : Vector form factor of the pion

$$F_V(s) = \frac{BW_\rho\left(\frac{1+\alpha\,BW_\omega}{1+\alpha}\right) + \beta\,BW_{\rho'} + \gamma\,BW_{\rho''} + \ldots}{1+\beta+\gamma+\ldots}$$

Kühn & Santamaría Model

[Kühn, Santamaría, 1990]

~~QCD~~

KS:

- χPT $\mathcal{O}(p^2)$ ✓
- χPT $\mathcal{O}(p^4)$ ☒
- Vector meson dominance ✓
- Asymptotic behaviour ruled by QCD ✓
- $$BW_R = \frac{M_R^2}{M_R^2 - s - i\,\sqrt{s}\,\Gamma_R(s)}$$?

Example : Vector form factor of the pion

$$F_V(s) = \frac{BW_\rho \left(\frac{1 + \alpha\, BW_\omega}{1 + \alpha} \right) + \beta\, BW_{\rho'} + \gamma\, BW_{\rho''} + \ldots.}{1 + \beta + \gamma + \ldots}$$

Gounaris & Sakurai Model

[Gounaris, Sakurai, 1968]

GS

Effective-range : $\delta_1^1\,(\pi\pi \to \pi\pi)$ ✓

VMD
$$\left.\cot\delta_1^1\right|_{s=M_\rho^2} = 0$$
$$\left.\frac{d\delta_1^1}{ds}\right|_{s=M_\rho^2} = \frac{1}{M_\rho \Gamma_\rho}$$
✓

$$BW_R = \frac{M_R^2 + d \cdot M_R \Gamma_R \left(M_R^2\right)}{M_R^2 - s + f(s) - i\,\sqrt{s}\,\Gamma_R(s)}$$
?

☑ Vector form factor of the pion : ρ(770) only

Generalized GS

à la KS

$$F_V(s) = \frac{BW_\rho \left(\frac{1+\alpha\ BW_\omega}{1+\alpha}\right) + \beta\ BW_{\rho'} + \gamma\ BW_{\rho''} + \ldots.}{1+\beta+\gamma+\ldots}$$

Form Factors

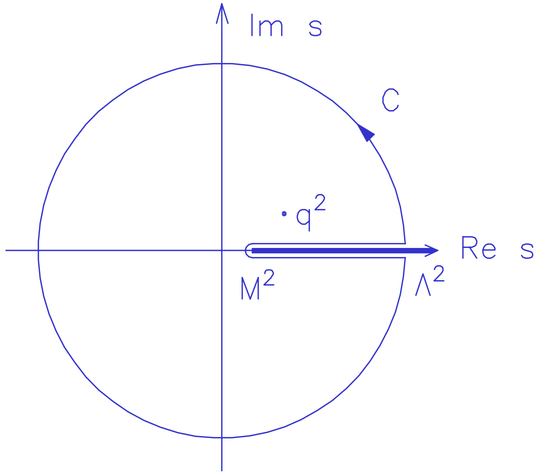
Analytic functions

Dispersion relations

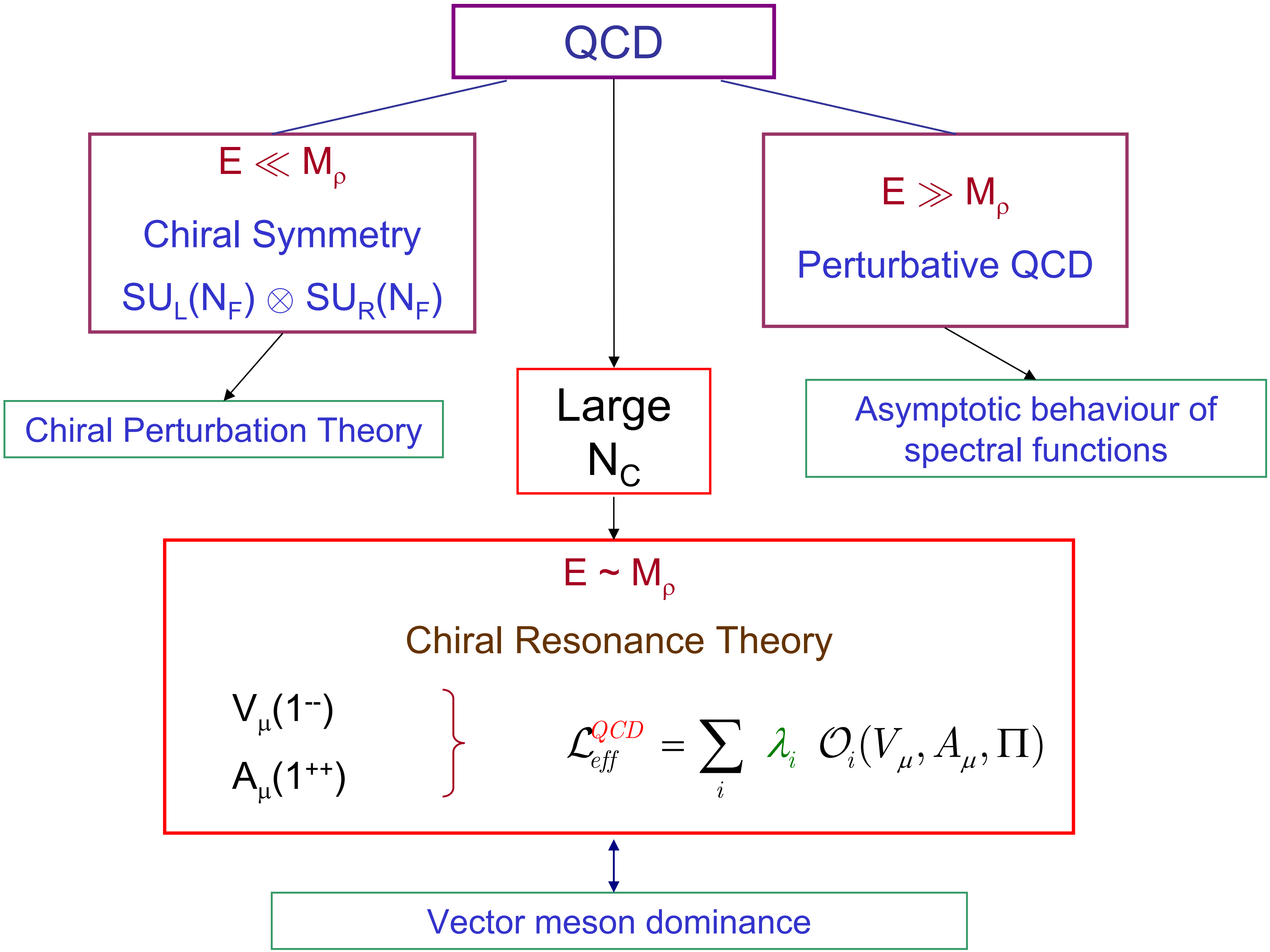
Properties at different energy scales are related

Unitarity

Spectral functions

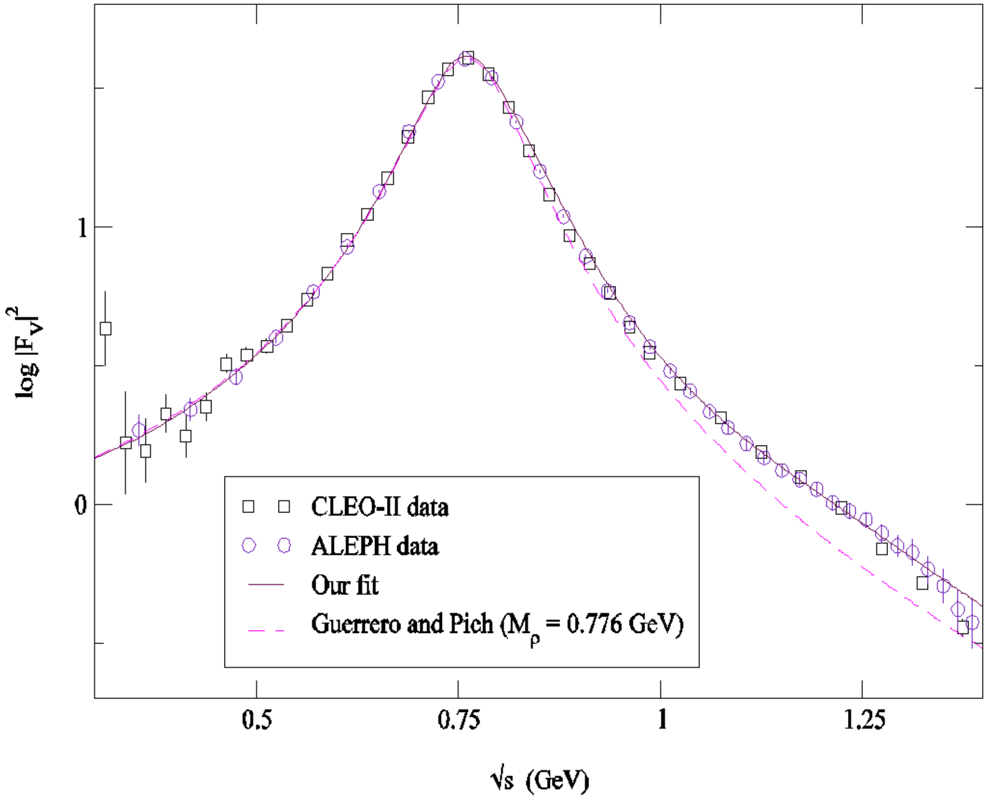


$$\mathrm{Im}\,\Pi_{\mu\nu} \propto \sum_{n}^{\infty} \int d\rho_n \left\langle 0 \middle| V_\mu \middle| n \right\rangle \left\langle n \middle| V_\nu^\dagger \middle| 0 \right\rangle$$


QCD
$E \ll M_\rho$
Chiral Symmetry
$SU_L(N_F) \otimes SU_R(N_F)$
Chiral Perturbation Theory
Large N_C
$E \gg M_\rho$
Perturbative QCD
Asymptotic behaviour of spectral functions
$E \sim M_\rho$
Chiral Resonance Theory
$V_\mu(1^{--})$
$A_\mu(1^{++})$
$\mathcal{L}_{eff}^{QCD} = \sum_i \lambda_i \, \mathcal{O}_i(V_\mu, A_\mu, \Pi)$
Vector meson dominance

$\tau^- \to \pi^-\pi^0\nu_\tau$: Vector form factor of the pion

$$\left\langle \pi^-(p)\ \pi^0(p')\middle| V_\mu^- \middle| 0 \right\rangle = \sqrt{2}\,(p-p')_\mu\, F_V(s) \ , \qquad s = (p+p')^2$$



[Guerrero, Pich, 1997]

χPT + Omnès solution + VMD

Excellent description of the ρ(770) up to 1 GeV

[Pich, Portolés, 2001]

RχT + Omnès solution

Extends the description up to 1.3 GeV

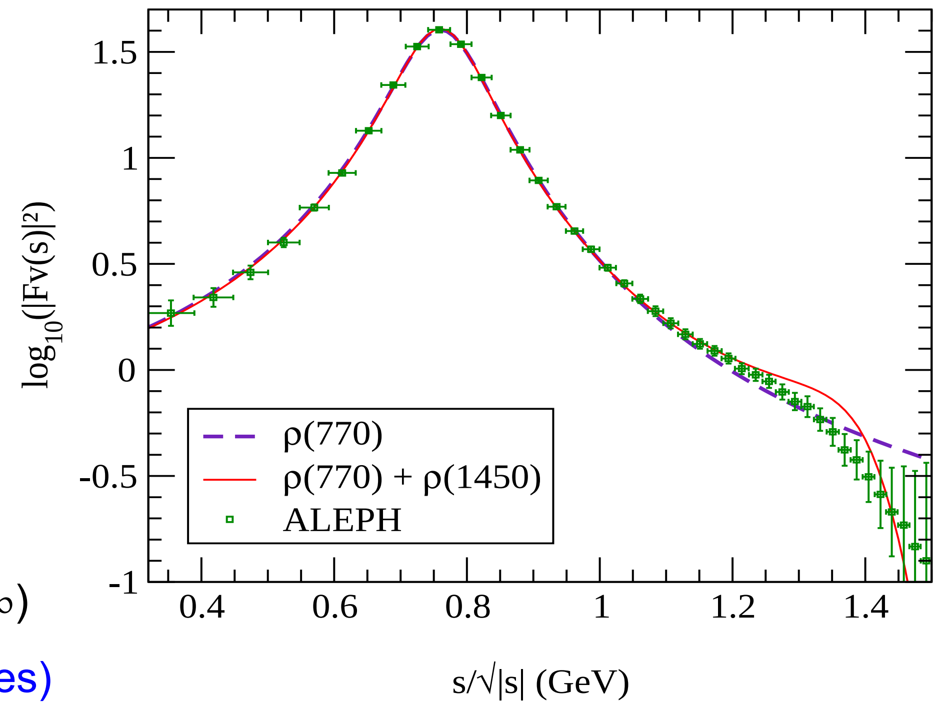
(Includes information on ρ(1450) through the ππ elastic phase-shift)

→ M_ρ = (775.9±0.5) MeV

[Sanz-Cillero, Pich, 2003]

RχT + Dyson-Schwinger-like resummation

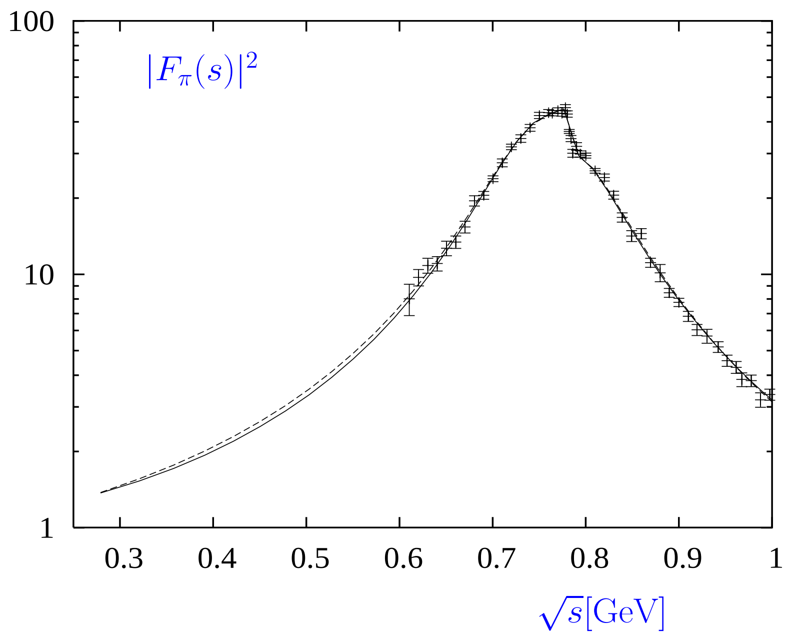
Explicit inclusion of ρ(770) and ρ(1450)

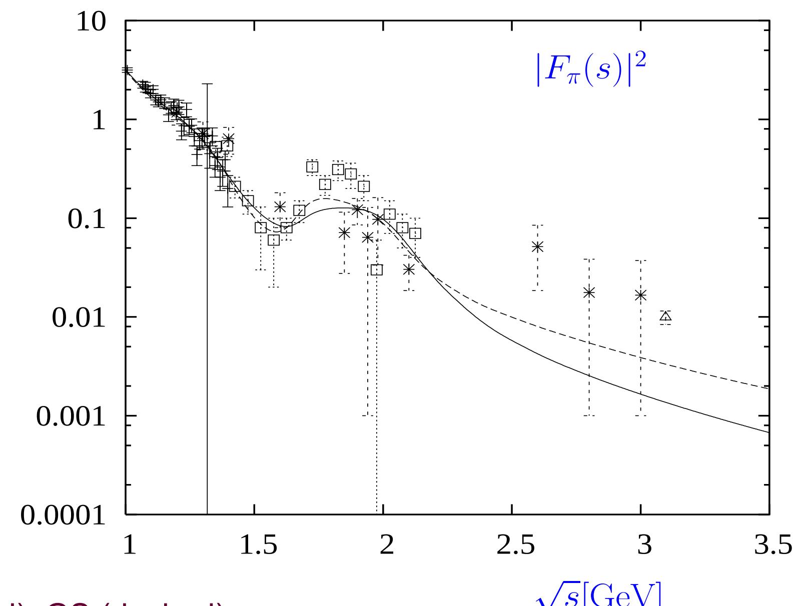


[Bruch, Khodjamirian, Kühn, 2004]

KS and GS models in a dual-QCD($N_C \to \infty$)

Modelization (infinite number of resonances)





KS (solid), GS (dashed)

Isospin breaking corrections

[Cirigliano, Ecker, Neufeld, 2001,2002]

Radiative corrections ($\tau^- \to \pi^- \pi^0 \nu_\tau \gamma$)

Bigger correction comes from S_{EW}

Strong cancellation between FF and kinematical corrections to $(g\text{-}2)_\mu$

Warning: Definition of Masses !

Reference	$M_{\rho^\pm} - M_{\rho^0}$ (MeV)
[Ghozzi, Jegerlehner,2004]	2.57 ± 0.83
[De Trocóniz, Yndurain, 2002]	1.20 ± 0.78
[Pich, Portolés, 2003]	-1.90 ± 0.86
[Bijnens, Gosdzinsky, 1996] (THEORY)	$-0.7 < \Delta M_\rho < 0.4$

$\tau^- \to (\pi\pi\pi)^- \nu_\tau$: Axial-vector form factors

$$\langle \pi^-_{p_1} \pi^-_{p_2} \pi^+_{p_3} | (V^-_\mu - A^-_\mu)\, e^{i L_{QCD}} | 0 \rangle = \left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} \right) \Big[F_1^A(Q^2, s_1, s_2)\, (p_1 - p_3)^\nu + F_1^A(Q^2, s_2, s_1)\, (p_2 - p_3)^\nu \Big]$$

$Q = p_1 + p_2 + p_3$

$s_i = (Q - p_i)^2$

$$+ \cancel{F_2^A} Q_\mu + i\, \cancel{F_3^V} \varepsilon_{\mu\alpha\beta\gamma} p_1^\alpha p_2^\beta p_3^\gamma$$

$m_\pi = 0$ $\quad SU(2)_I$

Kühn & Santamaría Model

$$F_1^A(Q^2, s_1, s_2) = N|_{\chi O(p^2)}\ BW_{a_1}(Q^2) \frac{BW_\rho(s_1) + \alpha\ BW_{\rho'}(s_1) + \beta\ BW_{\rho''}(s_1)}{1 + \alpha + \beta}$$

$\tau^- \to (\pi\pi\pi)^- \nu_\tau$ in the Resonance Effective Theory

[Gómez Dumm, Pich, Portolés, 2004]

- Chiral Resonance Theory (RχT)

  [Ecker et al, 1989] + ……

- Large-N_C …. but

  - One octet of resonances only
  - Off-shell widths for $\rho(770)$ and $a_1(1260)$

- Asymptotic behaviour of form factors ruled by QCD

$\tau^- \to (\pi\pi\pi)^- \nu_\tau$ in the Resonance Effective Theory

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- Chiral Resonance Theory (RχT)

  [Ecker et al, 1989] + ...

- Large-N_C .... but

  [Gómez Dumm, Pich, Portolés, 2000]

  - One octet of resonances only
  - Off-shell widths for $\rho(770)$ and $a_1(1260)$ ← Not happy

- Asymptotic behaviour of form factors ruled by QCD

Chiral Resonance Theory + Large-N_C

Tensor formalism for spin=1 mesons

$$\mathcal{L}_{R\chi T} = \mathcal{L}_{\chi}^{(2)} + \mathcal{L}_{V} + \mathcal{L}_{A} + \mathcal{L}_{VAP}$$

$$\mathcal{L}_{\chi}^{(2)} = \frac{\mathbf{F}^2}{4} \left\langle u_{\mu} u^{\mu} + \chi_{+} \right\rangle$$

$$\mathcal{L}_{V} = \frac{F_V}{2\sqrt{2}} \left\langle V_{\mu\nu} f_{+}^{\mu\nu} \right\rangle + \frac{iG_V}{\sqrt{2}} \left\langle V_{\mu\nu} u^{\mu} u^{\nu} \right\rangle$$

$$\mathcal{L}_{A} = \frac{F_A}{2\sqrt{2}} \left\langle A_{\mu\nu} f_{-}^{\mu\nu} \right\rangle$$

$$\mathcal{L}_{VAP} = \sum_{i}^{5} \lambda_i \, \mathcal{O}\left(V_{\mu}, A_{\mu}, \Pi\right) = \lambda_1 \left\langle \left[V_{\mu\nu}, A_{\mu\nu} \right] \chi_{-} \right\rangle + \ldots$$

5 unknown couplings

Asymptotic behaviour of $\mathrm{Im}\Pi_{\mu\nu}{}^{A}$

[Floratos, Narison, de Rafael,1979]

$$\mathrm{Im}\,\Pi^{A}_{\mu\nu} = \frac{1}{2}\sum_{N}^{\infty}\int d\rho_{N}\,\delta^{(4)}\left(q-p_{N}\right)\langle 0|\,A_{\mu}\,|N\rangle\langle N|\,A^{\dagger}_{\nu}\,|0\rangle \xrightarrow[(QCD,\ q^{2}\to\infty)]{} \text{Constant}$$

☑ Asymptotic behaviour of Form Factors (QCD)

$$f_{i}\left(\lambda_{k}\right) = 0 \qquad , \qquad i = 1,2$$

3 unknown couplings

☑ Feynman diagrams : 1 only coupling, λ_0

4 parameters

$$\mathcal{L}_{VAP} \rightarrow \lambda_0$$

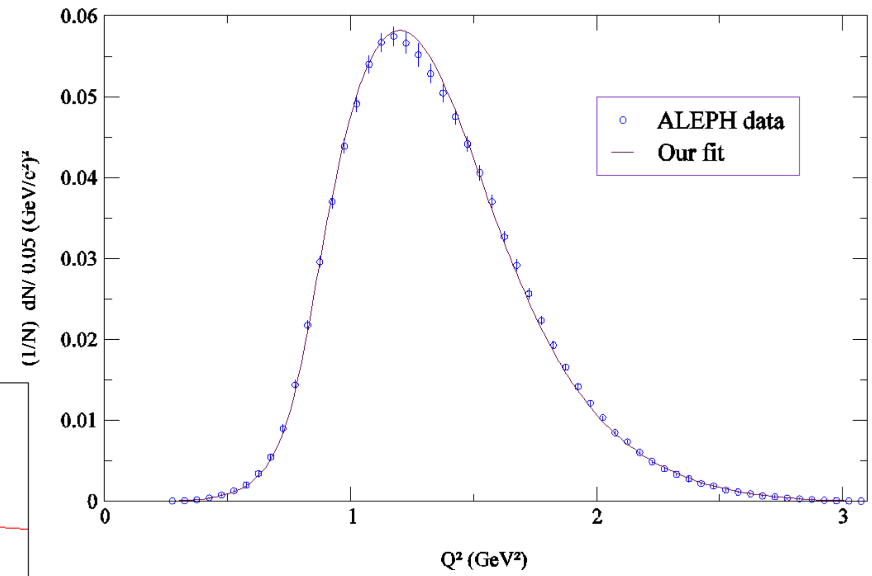
$$a_1(1260) \rightarrow M_{a_1}\ ,\ \Gamma_{a_1}\left(M^2_{a_1}\right)\ ,\ \alpha$$

Procedure and results

Fit to the spectrum and BR

[ALEPH, 1998] $\tau^- \rightarrow \pi^-\pi^-\pi^+\nu_\tau$

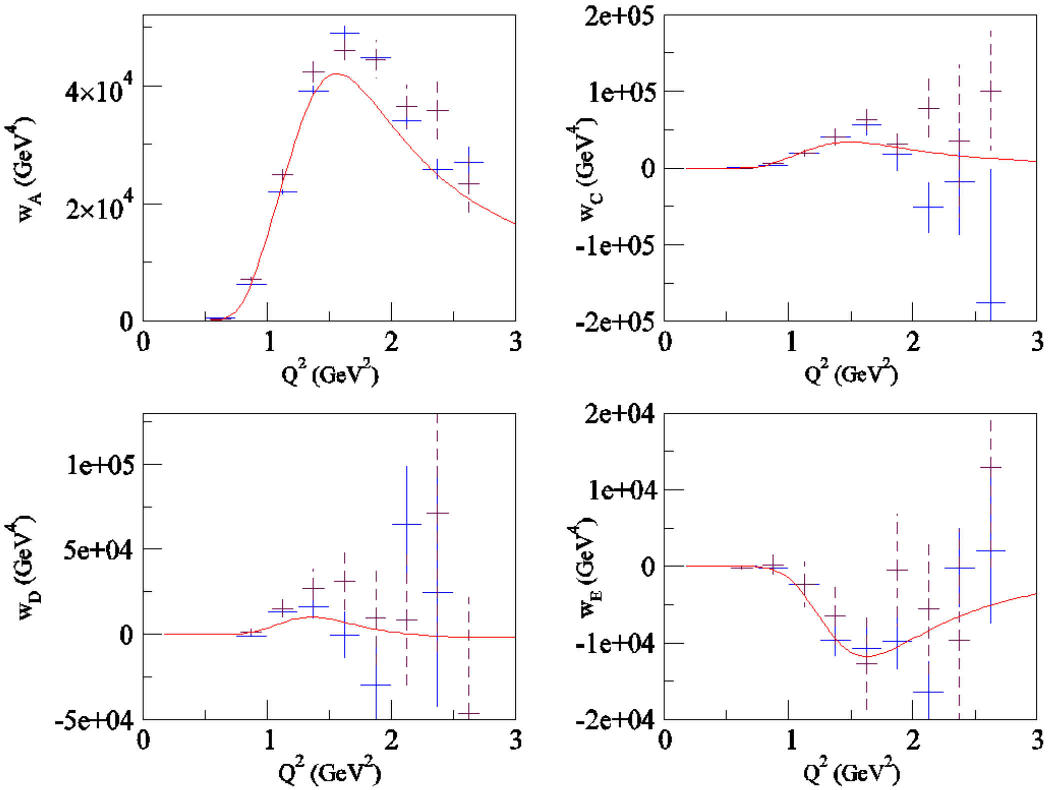
input



output

$\tau^- \rightarrow \pi^-\pi^0\pi^0\nu_\tau$

Structure Functions



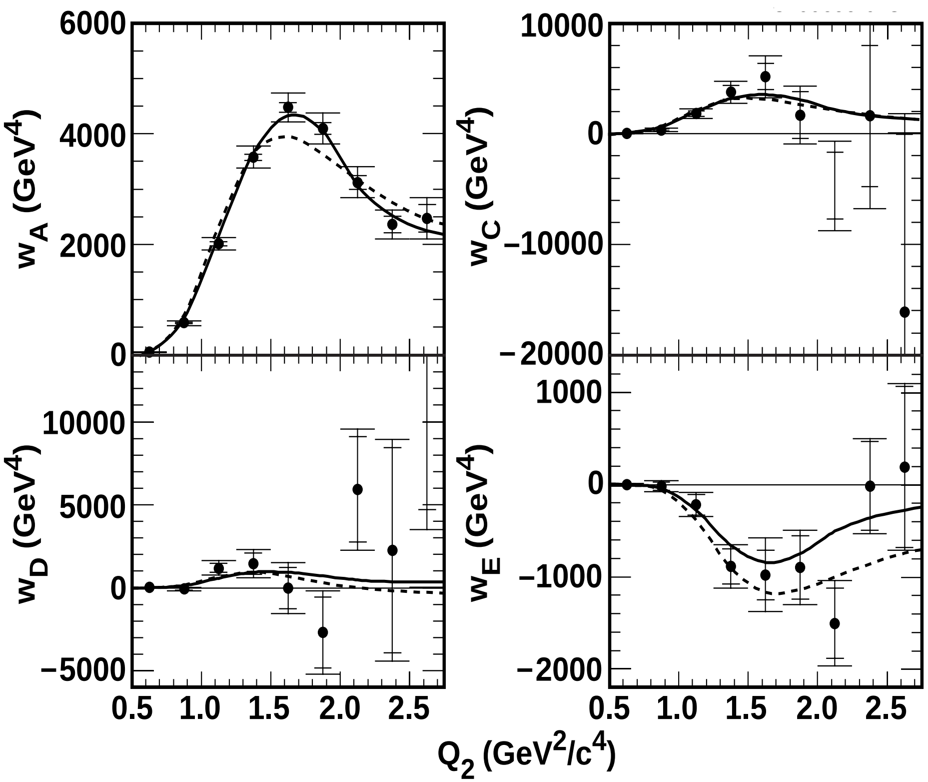
[CLEO-II,2000,(solid)] [OPAL,1997,(dashed)]

M_{a1} = (1.204 ± 0.007) GeV

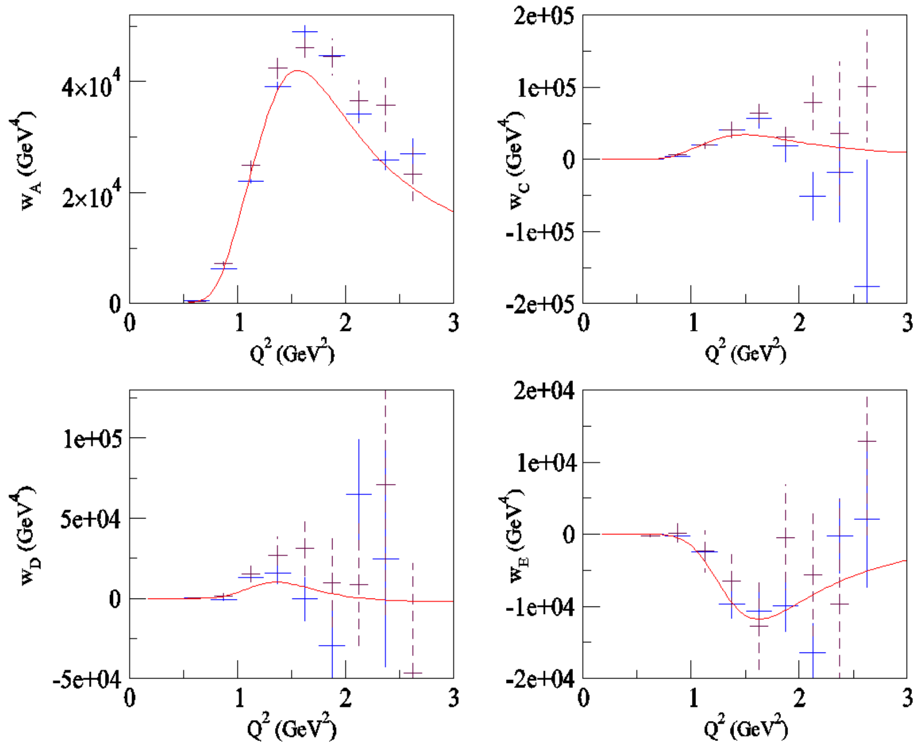
$\Gamma_{a1}(M_{a1})$ = (0.48 ± 0.02) GeV

Comparison of predictions for the Structure Functions

[CLEO-II, 2000]



[Gómez Dumm, Pich, Portolés, 2004]



Solid line : CLEO fit (KS inspired)

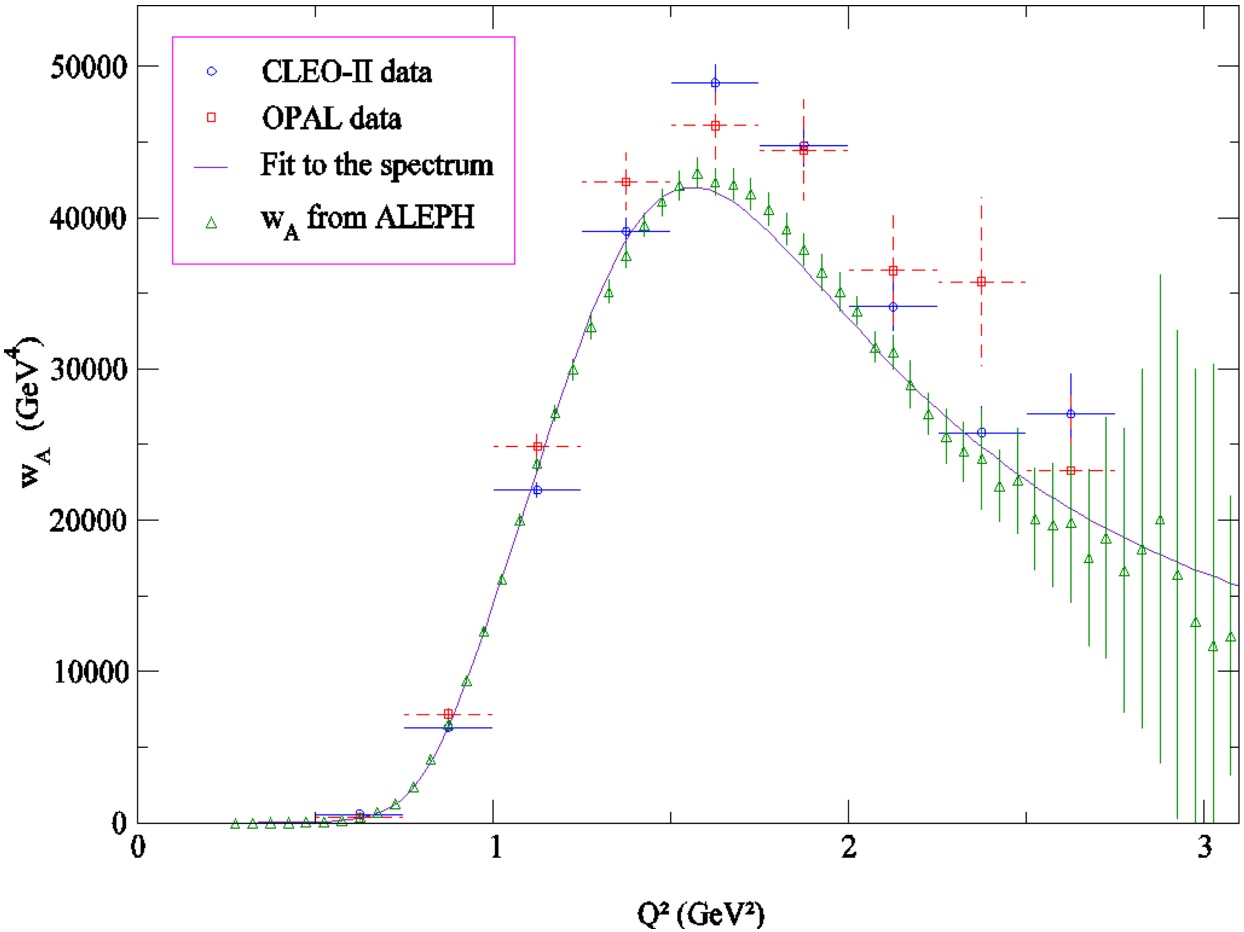
Dashed line : Kühn & Santamaría model

Solid data : CLEO-II (2000)

Dashed data : OPAL (1997)

Note : Both plots have different normalization

ALEPH vs (CLEO,OPAL) : slight discrepancy



$$\frac{d\Gamma}{dQ^2} \propto w_A\left(Q^2\right)$$

ALEPH data

$$\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$$

CLEO-II & OPAL data

$$\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$$

[ALEPH,1998] [OPAL, 1997] [CLEO-II, 2000]

Does it matter?

From first principles YES

In practice .... too

[CLEO-II, Phys. Rev. D61 (1999) 012002]

Kühn & Santamaría + form factors (finite size)

"The most significant result is the observation [in $\tau^- \to \pi^- \pi^0 \pi^0 \nu_\tau$] of large contributions to the substructure from intermediate states involving the isoscalar mesons σ, $f_0(1370)$, and $f_2(1270)$".

[Gómez Dumm, Pich, Portolés, 2004]

Good description with $\rho(770)$ and $a_1(1260)$ only.

$$\tau^- \to K^+ K^- \pi^- \nu_\tau$$

[Liu (CLEO),2003]

"The comparison between the data and MC (all=backgrounds+signal) shows that the decay is not well modelled in korb. The modelling of the substructure needs improvement."

[CLEO-III,2004]

$$F_3^V = -\frac{1}{2\sqrt{2}\pi^2 F^3} \sqrt{R_B} \frac{BW_\omega + \alpha BW_{K^*}}{1+\alpha} \frac{BW_\rho + \lambda BW_{\rho'} + \delta BW_{\rho''}}{1+\lambda+\delta}$$

Wess-Zumino	Analysis of data
$\sqrt{R_B} = 1$	$\sqrt{R_B} = 1.80 \pm 0.53$

$$\tau^- \to K^+ K^- \pi^- \nu_\tau$$

[Liu (CLEO),2003]

"The comparison between the data and MC (all=backgrounds+signal) shows that the decay is not well modelled in korb. The modelling of the substructure needs improvement."

Not QCD

[CLEO-III,2004]

$$F_3^V = -\frac{1}{2\sqrt{2}\pi^2 F^3}\sqrt{R_B}\,\frac{BW_\omega + \alpha BW_{K^*}}{1+\alpha}\,\frac{BW_\rho + \lambda BW_{\rho'} + \delta BW_{\rho''}}{1+\lambda+\delta}$$

Wess-Zumino

$$\sqrt{R_B} = 1$$

Analysis of data

$$\sqrt{R_B} = 1.80 \pm 0.53$$

Conclusions

✓ Good experimental status : Our experimentalist colleagues have done their job with very good marks.

❑ Theory slowly improving : Not every parameterization of form factors is allowed, in fact only one : QCD.

⇨ Kühn & Santamaría Parameterization (TAUOLA) ~~QCD~~

⇨ Resonance Chiral Approach + Large-N_C is an Effective Field Theory approach to QCD ☑

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⇨ Resonance Chiral Approach + Large-N_C is an Effective Field Theory approach to QCD ☑

We will need a NEW TAUOLA to analyse the hadronic decays of the Tau lepton

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Experiment

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$\tau^- \to (2h^-, 3h^-)\,\nu_\tau$: Structure Functions

[Kühn, Mirkes, 1992]

$$d\Gamma = \frac{G_F^2}{4M_\tau} \left|V_{CKM}\right|^2 L_{\mu\nu} H^\mu H^{\nu *} dPS^{(4)}, \qquad L_{\mu\nu} H^\mu H^{\nu *} = \sum_X L_X W_X$$

$H^\mu \to$ / $H^{\nu *} \downarrow$	$J^P = 1^+$	$J^P = 1^-$	$J = 0$
$J^P = 1^+$	W_A, W_C, W_D, W_E		
$J^P = 1^-$	W_F, W_G, W_H, W_I	W_B	
$J = 0$	W_{SB}, W_{SC}, W_{SD}, W_{SE}	W_{SF}, W_{SG}	W_{SA}

Columns $J^P = 1^-$ and $J = 0$: $h_1\, h_2$

Columns $J^P = 1^+$, $J^P = 1^-$ and $J = 0$: $h_1\, h_2\, h_3$

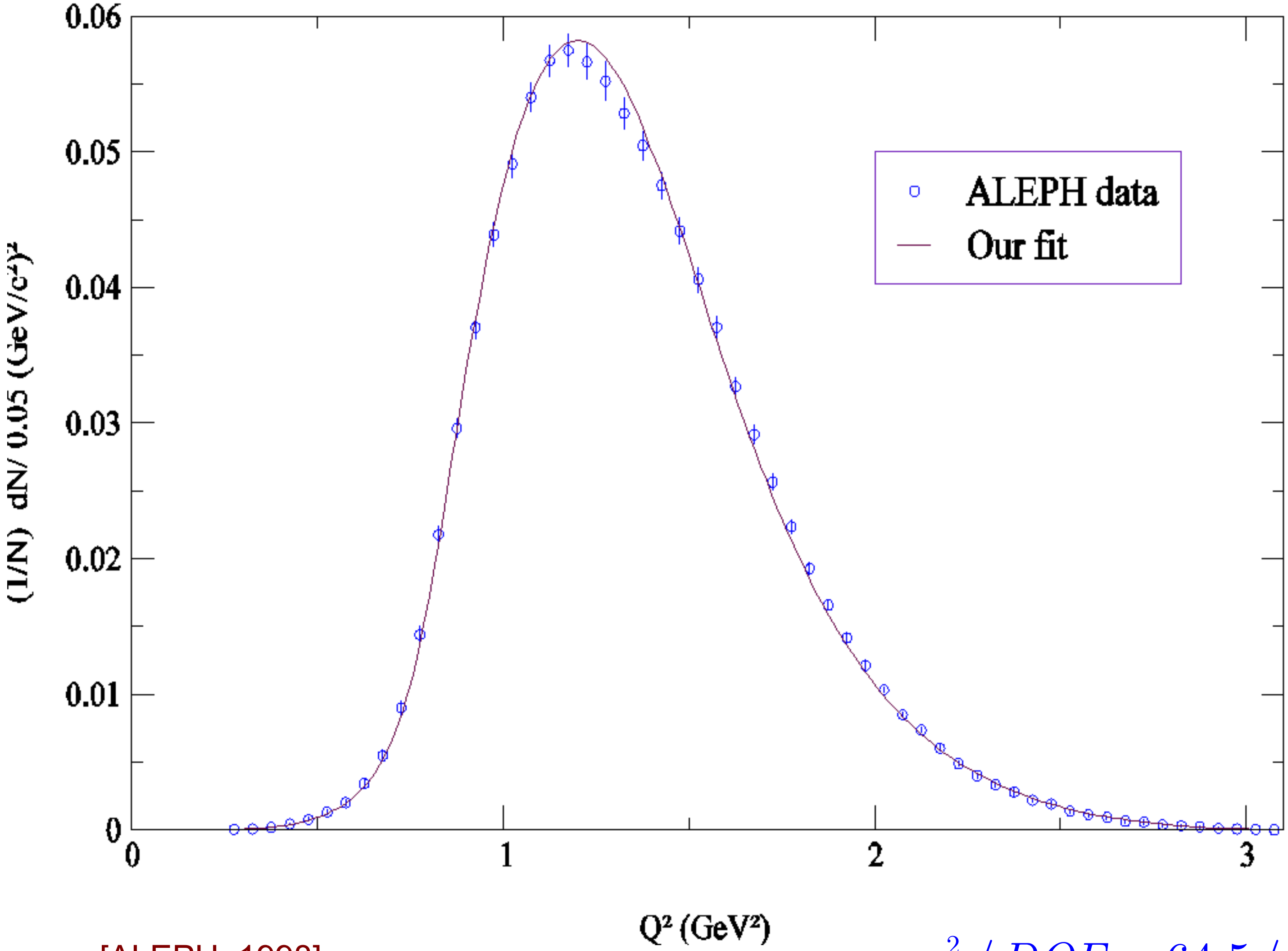
$$\rho_0\left(Q^2\right) = \frac{1}{2(4\pi)^4 Q^4} \int ds_1 ds_2 \; W_{SA}$$

$$\rho_1\left(Q^2\right) = \frac{1}{6(4\pi)^4 Q^4} \int ds_1 ds_2 \left(W_A + W_B\right)$$

$$w_{A,C} = \int ds_1 ds_2 W_{A,C}$$

$$w_{D,E} = \int ds_1 ds_2 \; sign(s_1 - s_2) \; W_{D,E}$$

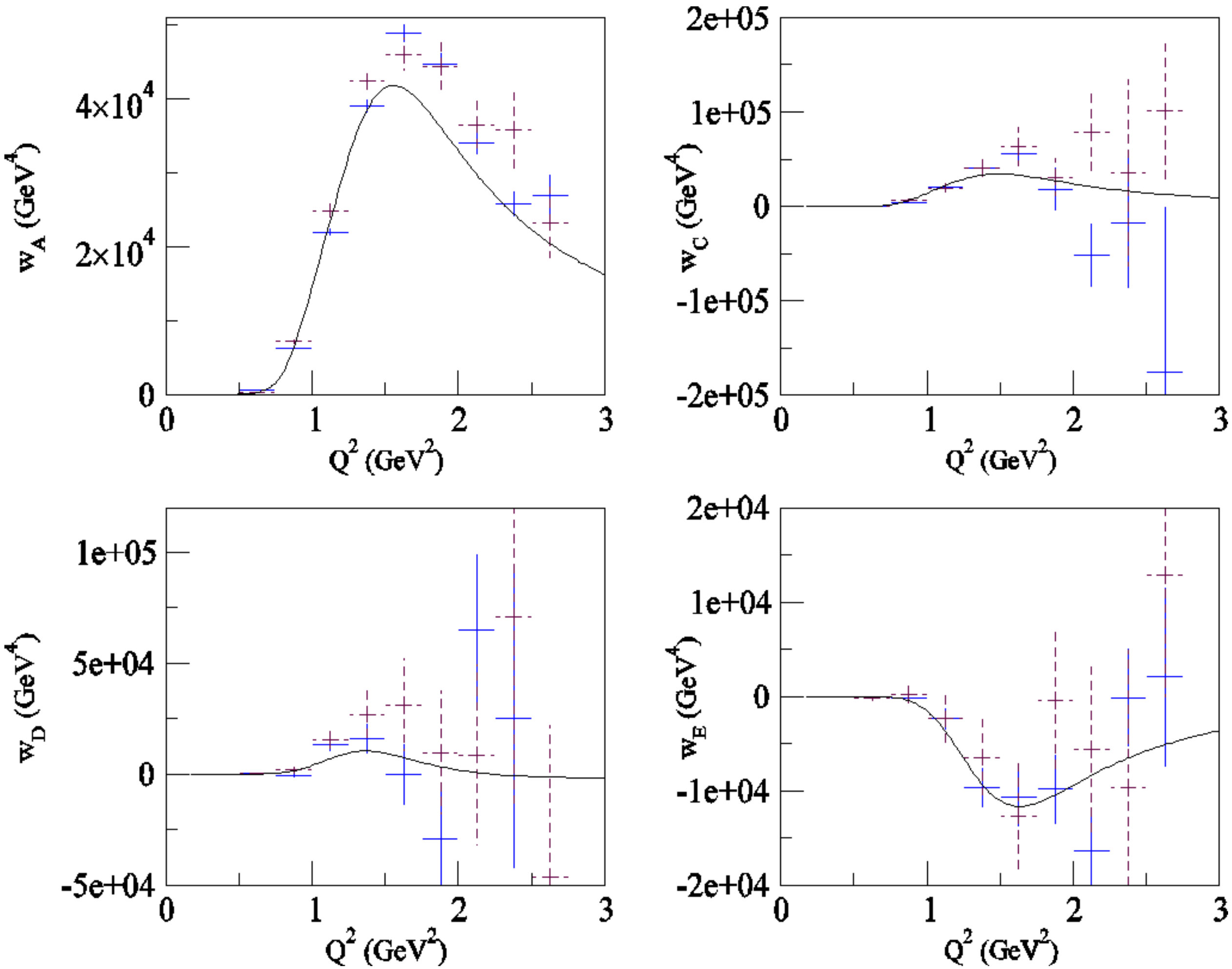
Fit to ALEPH data



[ALEPH, 1998]

$$\chi^2 / DOF = 64.5 / 52$$

Comparison of the predicted Structure Functions with data



[OPAL, 1997 (dashed)] [CLEO-II, 2000 (solid)]