# Muon *g-2*: a theoretical review

**Tau04**
**Nara, September 2004**

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# Outline

QED: present and future (T. Kinoshita)

Electroweak loops

Hadronic effects

* vacuum polarization

(J. Kühn, S. Eidelman, D. Leone, M. Davier, B. Schwarz, K. Hagiwara)

* light-by-light scattering

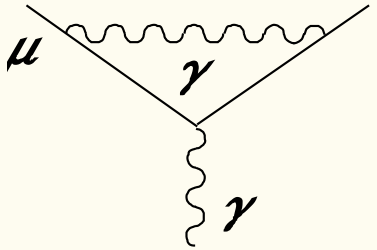
Summary and outlook

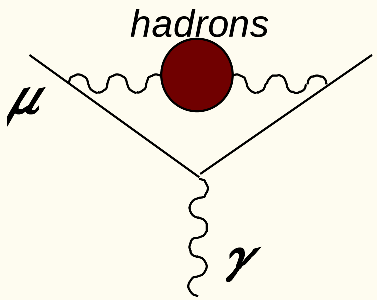
# Muon *g-2*: Standard Model update

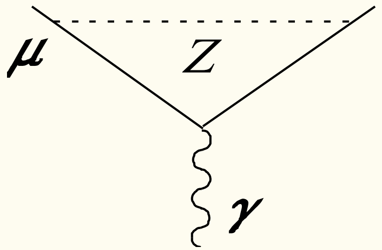
Units: $10^{-11}$

| | | | |
|---|---|---|---|
| QED | | 116 584 719 (1) | hep-ph/0402206 |
| Hadronic | | | |
| | LO | 6 963 (72) | hep-ph/0308213 |
| | NLO | – 98 (1) | hep-ph/0312250 |
| | LBL | 120 (40) | tentative, see hep-ph/0312226 |
| Electroweak | | 154 (3) | hep-ph/0212229 |
| Total SM | | 116 591 858 (82) | |

Experiment – SM Theory = 222 (102) (2.2σ deviation)

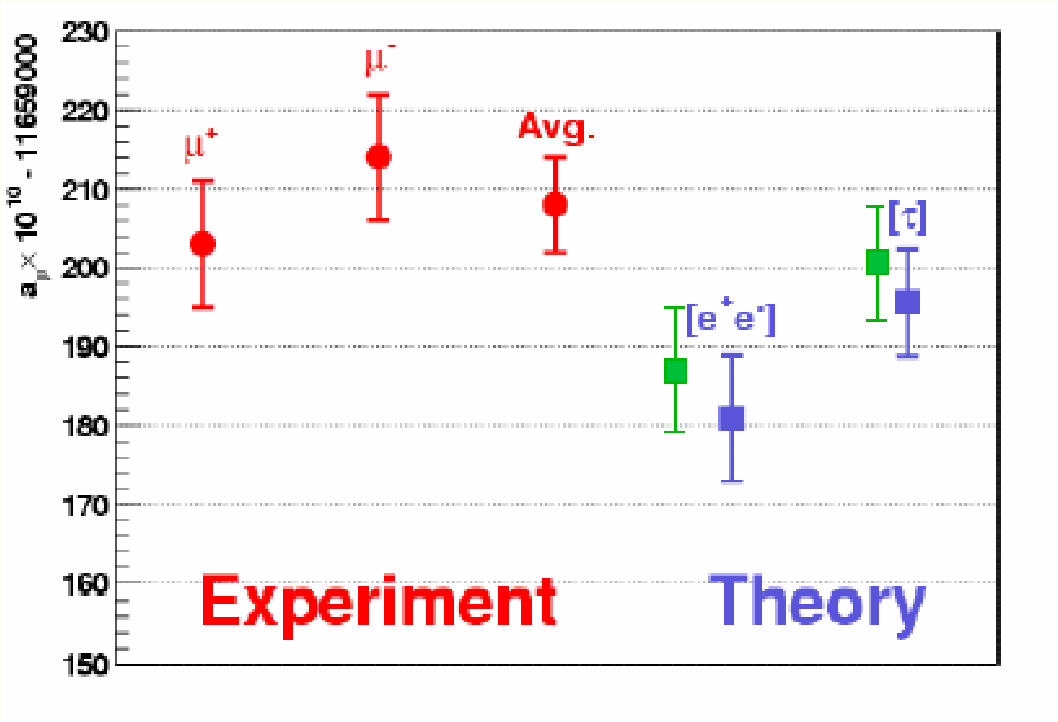


# Muon *g-2*: new data

**Brookhaven, January 2004: $\mu^-$ measurement.**




from A. Vainshtein

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 222 \pm 102 \cdot 10^{-11}$$

$$\rightarrow 2.2\sigma$$

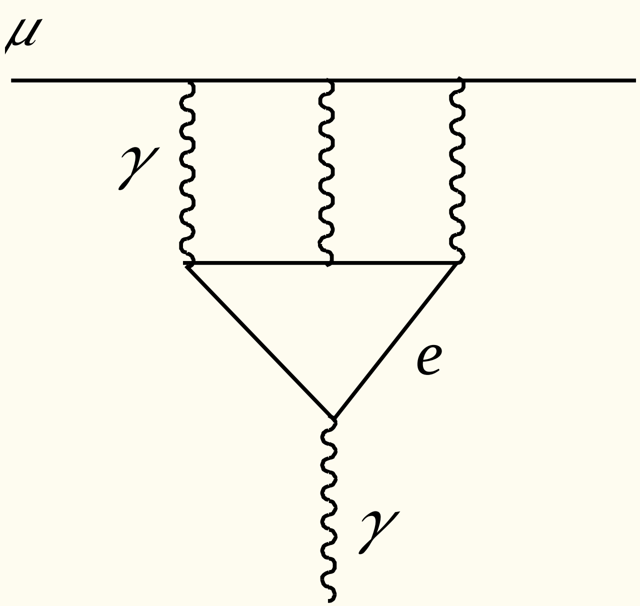
(based on e+e-)

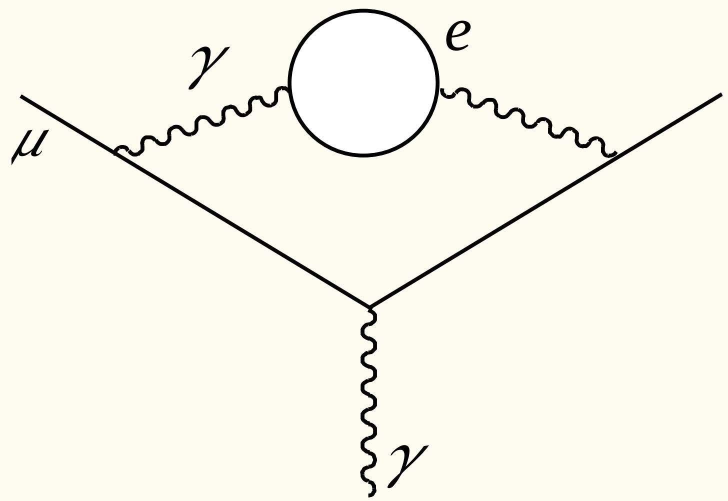
$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 123 \pm 89 \cdot 10^{-11}$$

$$\rightarrow 1.4\sigma$$

(tau)

# QED contributions: muon vs. electron




Enhancement factors:

$$\pi^{2n} \ln \frac{m_\mu}{m_e} \qquad\qquad \ln^n \frac{m_\mu}{m_e}$$

Leading five-loop effects must be included!

# QED contributions: problems at 4-loop order

Traditional approach (T. Kinoshita and M. Nio (Nara)):
numerical problems, digit deficiency

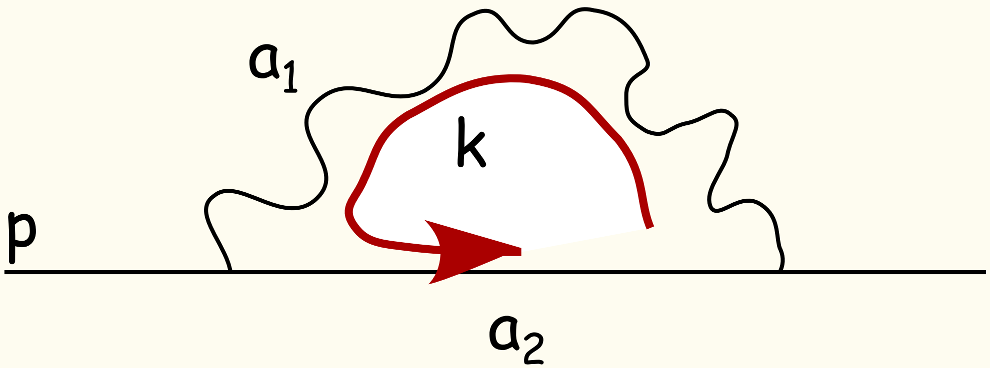
New approach (various groups, in progress):

Combine numerical and algebraic methods

Reduce all integrals to a smaller basis

Evaluate the primitive integrals numerically, with high accuracy.

# Example: integration by parts




$$J\left(a_1,a_2\right)=\int d^Dk\,\frac{1}{\left(k^2\right)^{a_1}\left(k^2+2kp\right)^{a_2}}$$

$$0=p_\mu\int d^Dk\,\frac{\partial}{\partial k_\mu}\frac{1}{\left(k^2\right)^{a_1}\left(k^2+2kp\right)^{a_2}}$$

# New approach to the QED part

Obstacles:

Very large number of integrals

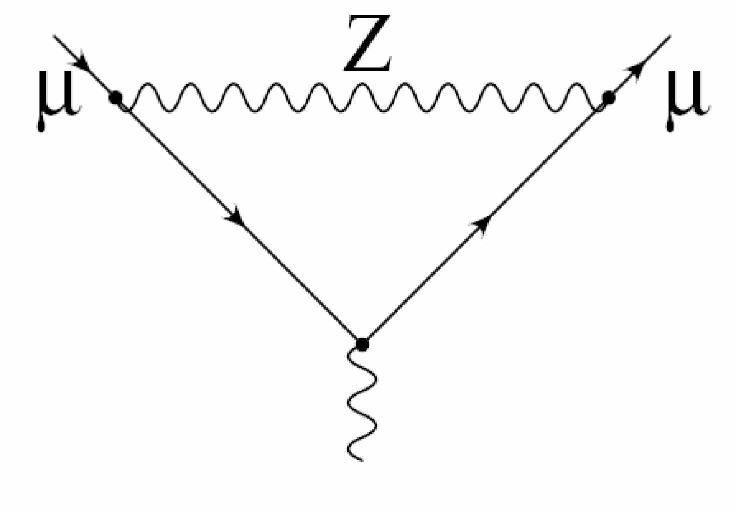
Reduction to primitive integrals

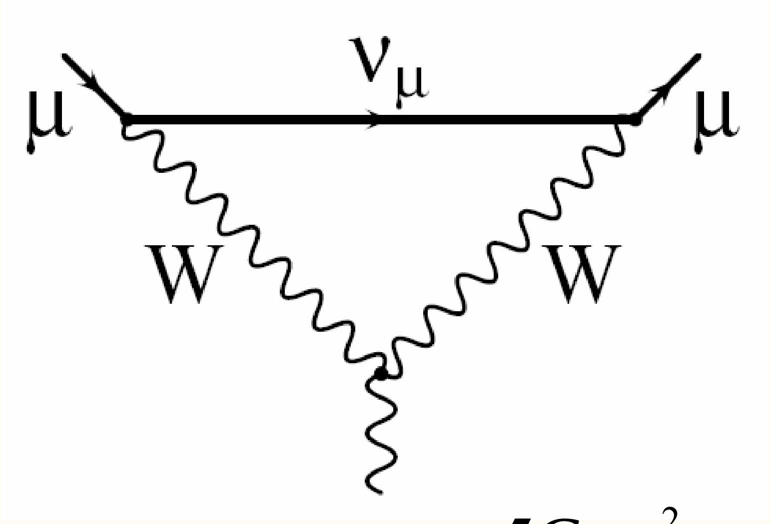
Evaluation of master integrals

Recent progress: algorithmic reduction (Laporta, 2001)

# Electroweak effects: pure and hadronic

Small part of the total $g$-2: 154(3)×10$^{-11}$

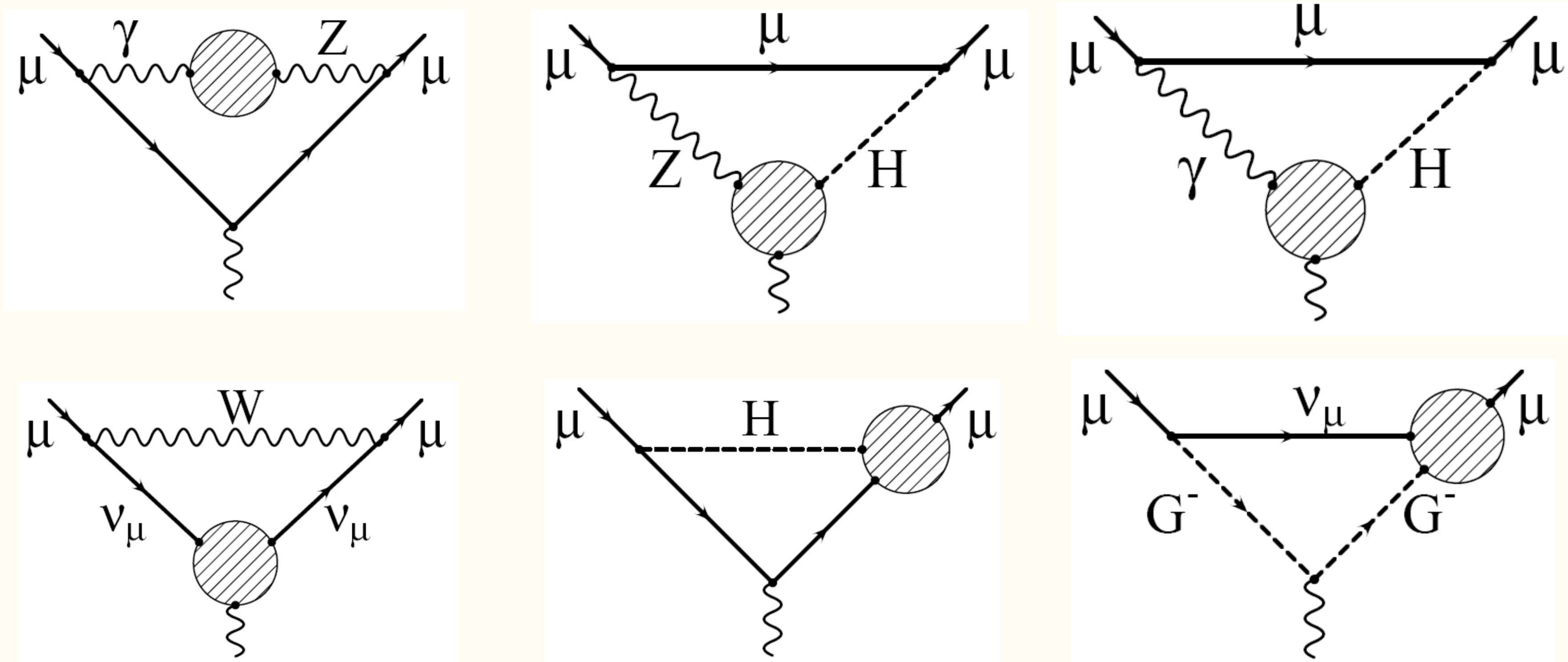



$$\left( -1 \quad +2\right) \cdot \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}$$

$$\simeq 195 \cdot 10^{-11}$$

# Higher-order electroweak effects




Most important: photonic corrections → large logs

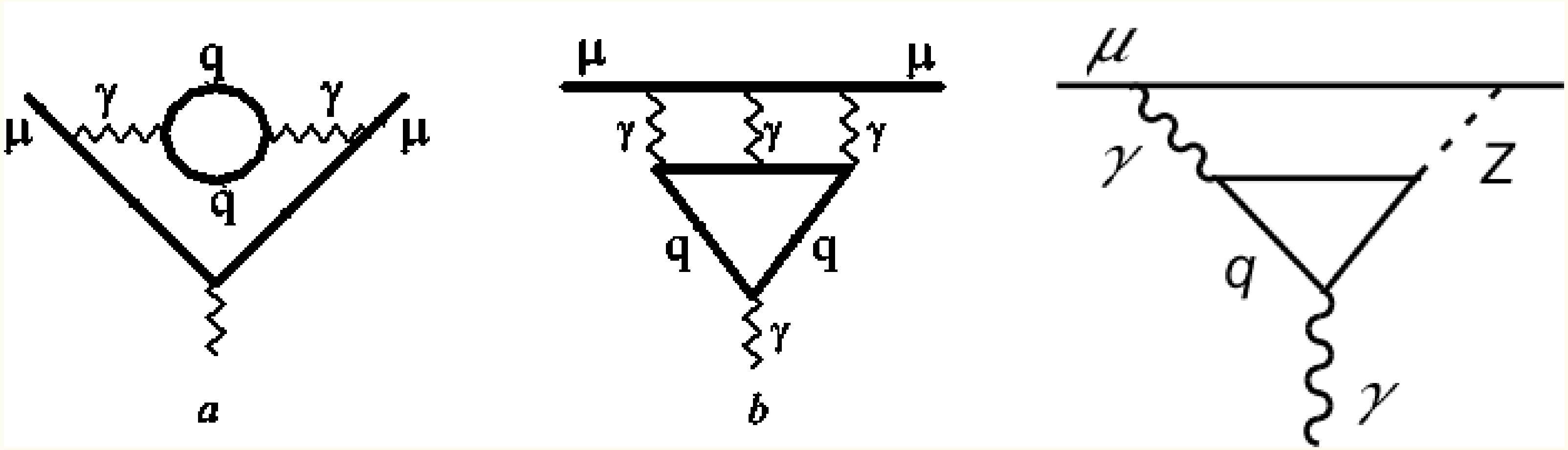
$$\frac{\alpha}{\pi} G_\mu m_\mu^2 \ln \frac{M_W^2}{m_\mu^2} \sim -23\% \text{ of one-loop}$$

Kukhto et al.
AC, Krause, Marciano
Heinemaier, Stockinger, Weiglein

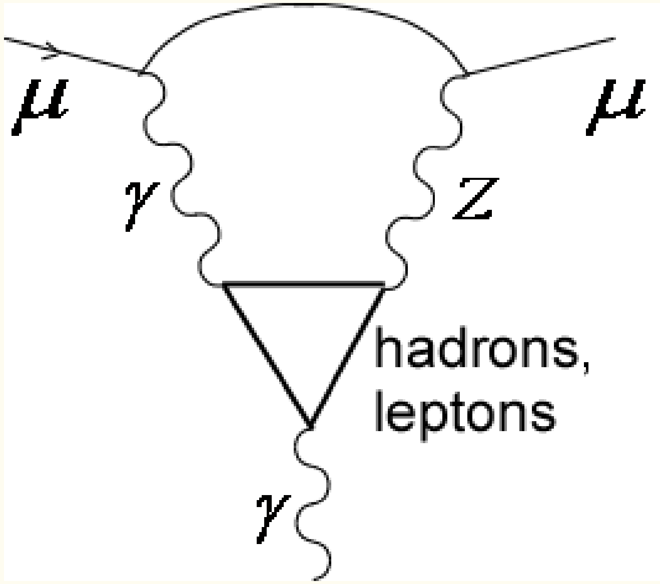
# Muon *g-2*: hadronic loops

Hadronic effects dominate theoretical uncertainty:

- Vacuum polarization
- Light-by-light scattering
- Electroweak triangle diagrams (numerically small)



# Electroweak-hadronic effects




Large logs $\ln(m_\mu/M_Z)$ appear in individual fermion contributions;

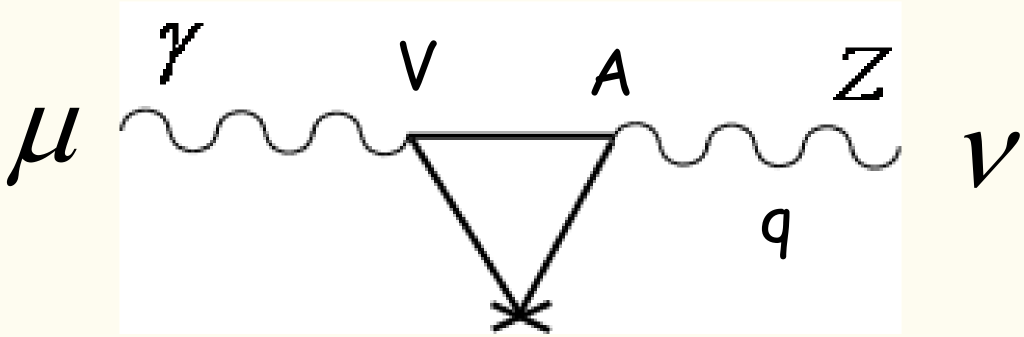
But cancel in the sum for each generation – like anomalies.

This cancellation between leptons and hardons was contoversial.

AC, Marciano, Vainshtein
vs.
Knecht, Peris, Perrottet, de Rafael

Useful illustration: similar techniques used in light-by-light

# Structure of the triangle




external V

$$\sim w_T\left(q^2\right)\left(-q^2\tilde{F}_{\mu\nu}+q_\mu q^\sigma\tilde{F}_{\sigma\nu}-q_\nu q^\sigma\tilde{F}_{\sigma\mu}\right) \quad + \quad w_L\left(q^2\right)\left(q_\nu q^\sigma\tilde{F}_{\sigma\mu}\right)$$

Perturbative result:

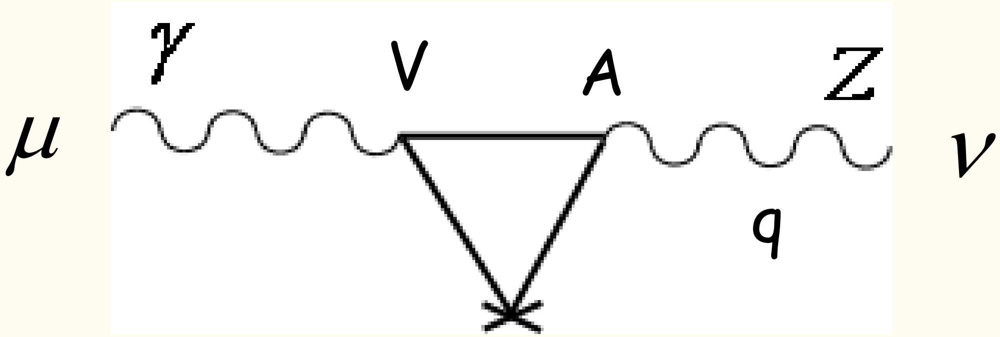
$$w_L = 2w_T \sim \frac{1}{q^2}$$

Anomaly:

$$q_\nu T^{\mu\nu} \sim q^2 w_L \neq 0$$

$$q_\mu T^{\mu\nu} = 0$$

# Vainshtein's non-renormalization theorem for $w_T$




$$T_{\mu\nu} \sim w_T\left(q^2\right)\left(-q^2\tilde{F}_{\mu\nu} + q_\mu q^\sigma \tilde{F}_{\sigma\nu} - q_\nu q^\sigma \tilde{F}_{\sigma\mu}\right) \;+\; w_L\left(q^2\right)\left(q_\nu q^\sigma \tilde{F}_{\sigma\mu}\right)$$

In the chiral limit, $w_T$ has no perturbative corrections

Idea of the proof:

$$\mathrm{Im}\, T_{\mu\nu} \sim q_\mu q^\sigma \tilde{F}_{\sigma\nu} + q_\nu q^\sigma \tilde{F}_{\sigma\mu}$$ (symmetric)

$$2w_T\left(q^2\right) = w_L\left(q^2\right)$$

Guidance from one-loop calculations!

# $w_{T,L}$ in QCD (chiral limit)

Perturbative: $$w_L = 2w_T = \frac{2}{Q^2} \qquad Q^2 \equiv -q^2$$

Non-perturbative:

| | Large $Q^2$ | Small $Q^2$ |
| --- | --- | --- |
| $w_L$ | $\frac{2}{Q^2}$ | $\frac{2}{Q^2}$ (pion pole) |
| $w_T$ | $\frac{1}{Q^2} - \frac{(0.7\,\text{GeV})^4}{Q^6} + O\left(\frac{1}{Q^8}\right)$ | $\frac{1}{m_{a_1}^2 - m_\rho^2}\left(\frac{m_{a_1}^2 - m_\pi^2}{Q^2 + m_\rho^2} - \frac{m_\rho^2 - m_\pi^2}{Q^2 + m_{a_1}^2}\right)$ (model for $w_T$) |

# Contributions to g-2

$$\Delta a_\mu \sim \left(\frac{\alpha}{\pi}\right)^2 \frac{m_\mu^2}{M_Z^2} \int_{m_\mu^2}^{\infty} dQ^2 \left( w_L + \frac{M_Z^2}{M_Z^2+Q^2} w_T \right)$$

Asymptotics:

$$w_{T,L} \xrightarrow{Q^2\to\infty} \frac{1}{Q^2} \sum_f I_{3f} N_f Q_f^2$$

$$\int^{\infty} dQ^2 \;\; w_L \;:\; \text{diverges}$$

theory inconsistent
unless anomalies cancel

$$\int^{\infty} dQ^2 \; \frac{M_Z^2}{M_Z^2+Q^2} w_T \sim \ln M_Z^2$$

# "Pure" hadronic contributions

## Recent progress

### Updated studies of g-2 using $e^+e^-$ data

Davier, Eidelman, Höcker, Zhang
Hagiwara, Martin, Nomura, Teubner

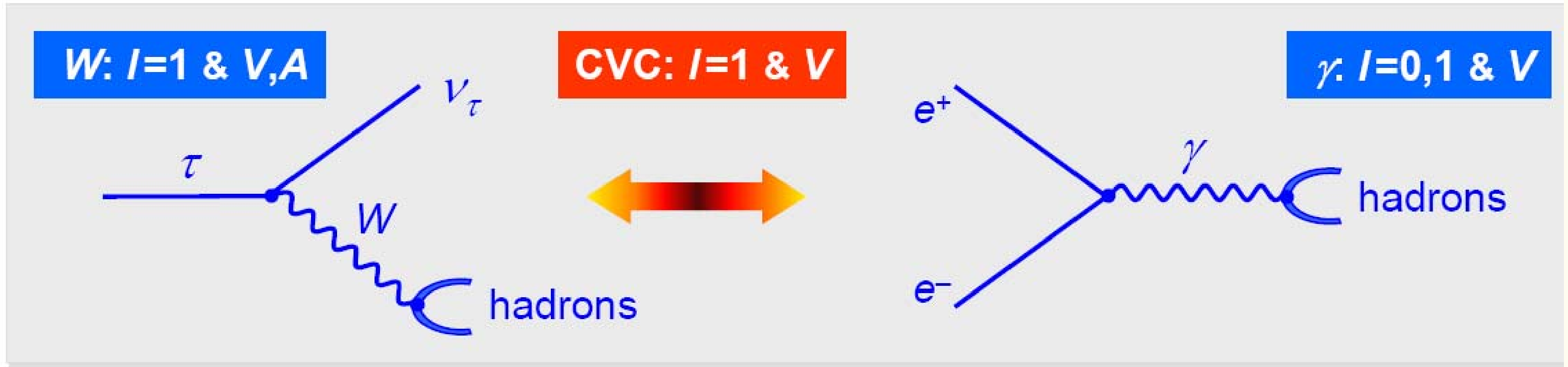
### Novosibirsk results tested by Daphne

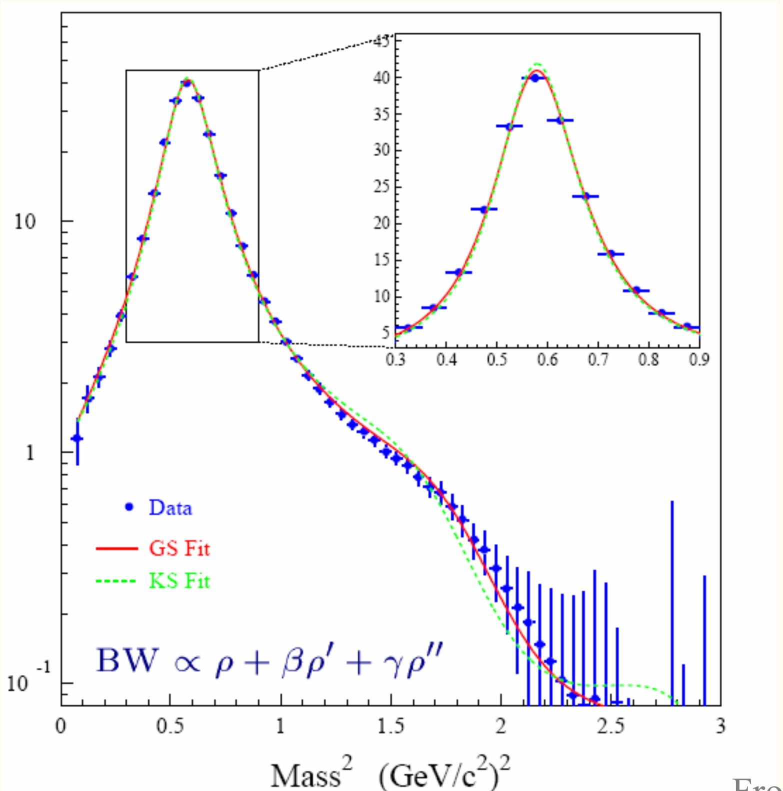
See talks by Kühn, Leone, Shwartz

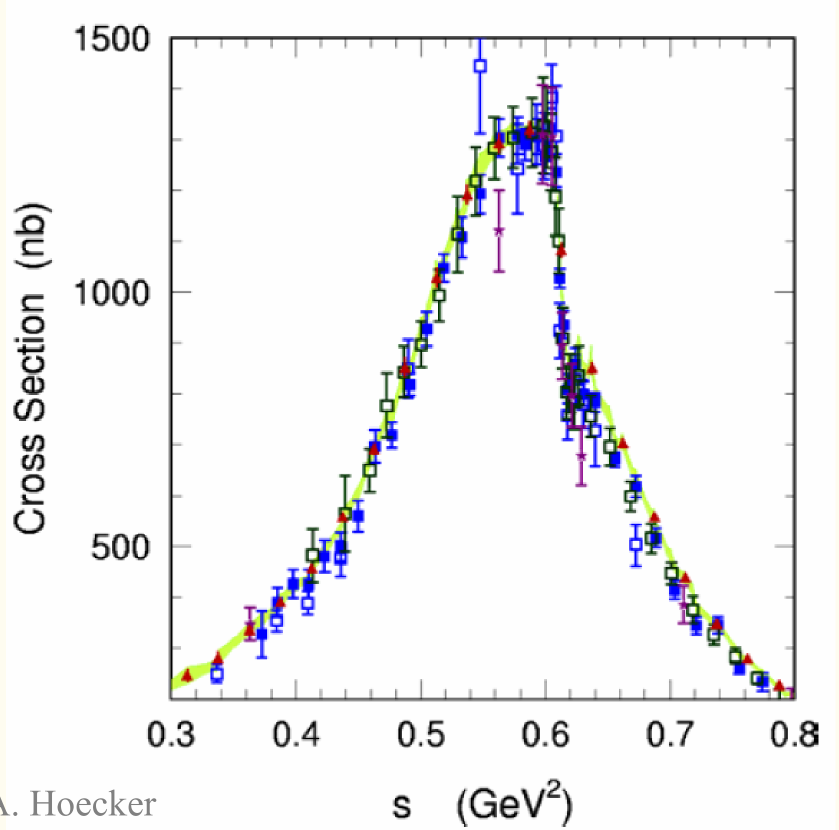
### Shift of the light-by-light prediction

Melnikov and Vainshtein

# Vacuum polarization: $\tau$ decays vs. $e^+e^-$



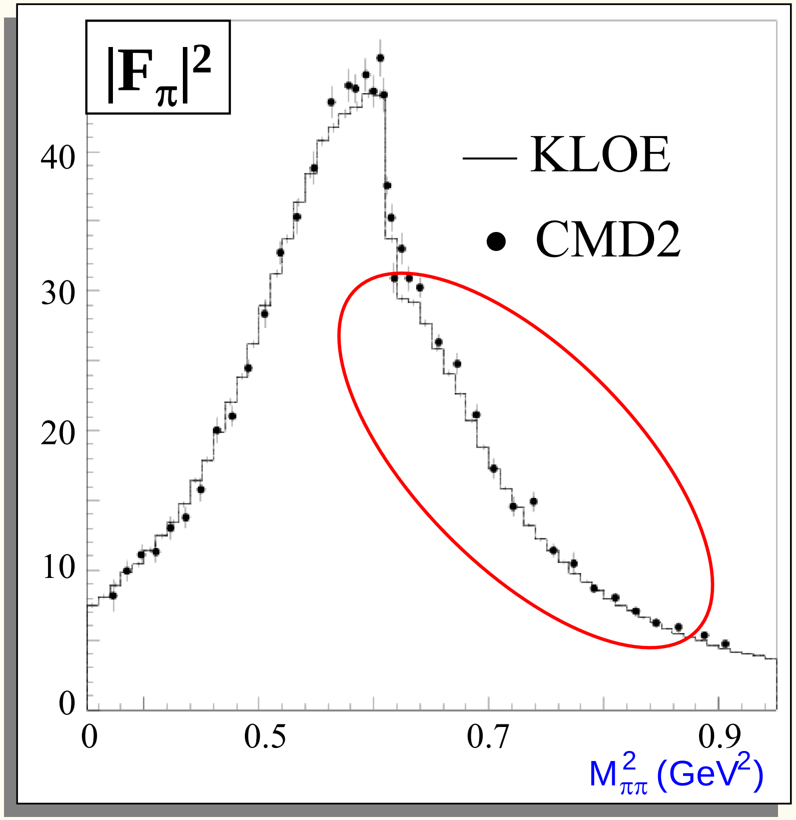




From M. Davier, A. Hoecker

# Vacuum polarization: $e^+e^-$

- $e^+e^-$ data have greatly improved
- New results from KLOE confirm Novosibirsk CMD2




From A. Denig

# Hadronic contributions: outstanding problems

How to reconcile $e^+e^-$ and $\tau$ data?

Can we improve the light-by-light prediction?

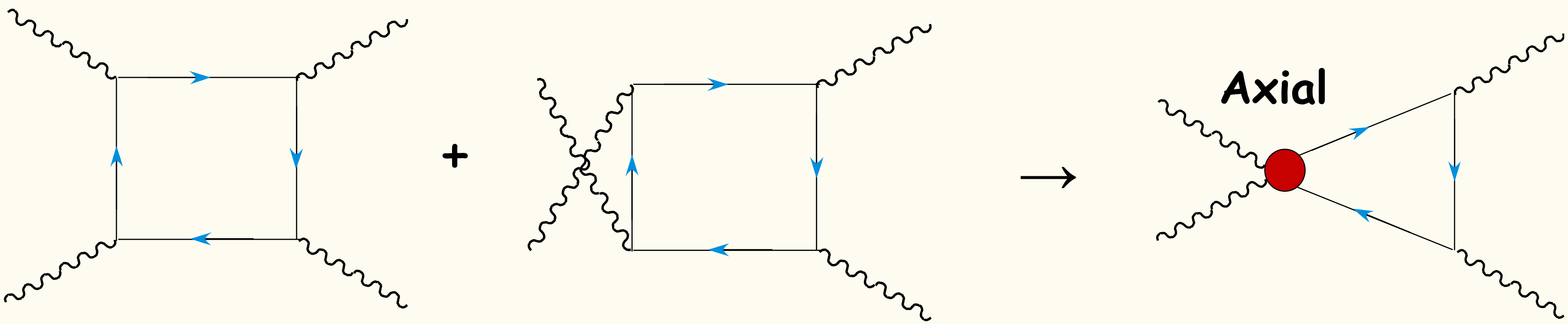
# Light-by-light scattering

Recent evaluations:

| | |
|---|---|
| Knecht, Nyffeler | 80(40) |
| Hayakawa, Kinoshita | 90(15) |
| Bijnens, Pallante, Prades | 83(32) |
| Melnikov, Vainshtein | 136(25) |

# Effects enhanced by $N_c$

Quark box: pQCD asymptotics




The same structure as in the EW-hadronic loops.

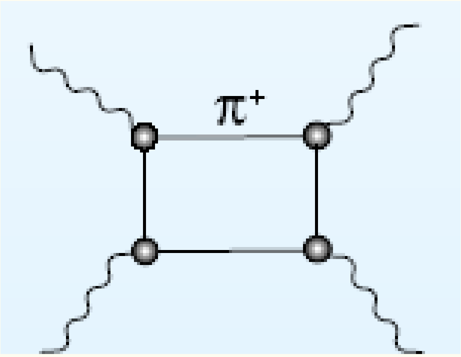
Dominant contribution: $\pi^0$ pole
Crucial observation: $1/Q^2$ asymptotics → no formfactor in $\pi^*\gamma^*\gamma$ if one of the photons soft.

$$\Delta a_\mu^{\mathrm{PS}} \simeq (76.5 + 2 \cdot 18) \cdot 10^{-11}$$

$$\Delta a_\mu^{\mathrm{PV}} \simeq 22 \cdot 10^{-11}$$

# What about terms subleading in $N_c$?

Example: pion box.
It is chirally enhanced, $m_\mu^2 / m_\pi^2$



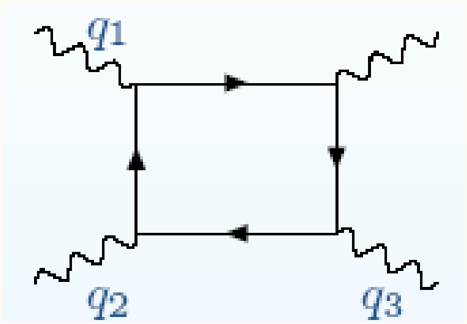

Numerical effect: small.
Previous estimates:

| | | |
|---|---|---|
| $-4.5(8.5)\times10^{-11}$ | HLS | Hayakawa, Kinoshita, Sanda |
| $-19(5)\times10^{-11}$ | VMD | Bijnens, Pallante, Prades |

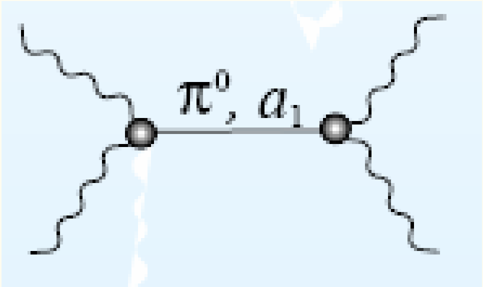
Melnikov & Vainshtein: $0\pm10\times10^{-11}$
(cancellations with higher orders in the chiral expansion)

# Summary on the light-by-light scattering

From Melnikov and Vainshtein




Matching of hadronic model with perturbative QCD, at asymptotic momentum transfer.
Large contribution of high virtualities.




Dominant in $N_c \to \infty$: pion pole

Still room for improvement: subleading terms

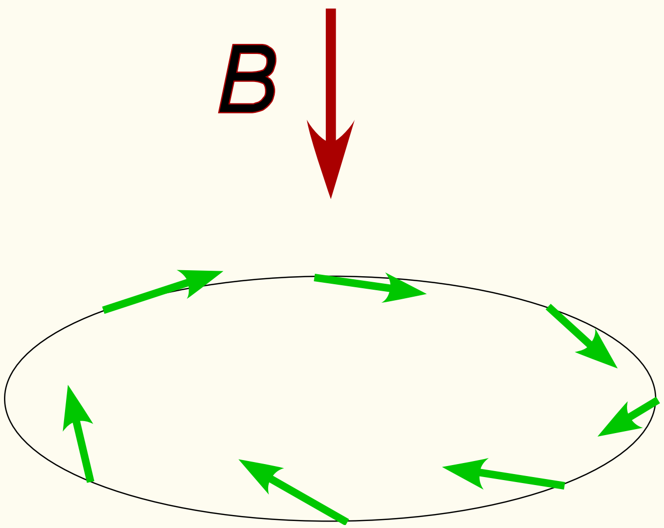
# Pomeranchuk and Sakharov on $g\text{-}2 = \frac{\alpha}{\pi}$

**If this is true, it's exceptionally important;**
**if it isn't true, that, too, is exceptionally important.**
(Pomeranchuk after Sakharov's talk, 1949)

**I felt like the messenger of the gods.** (Sakharov)

# How do we determine g-2?

Measure $\omega_a = \frac{g-2}{2}\frac{e}{m_\mu}B$




$B$ from NMR: $\omega_p = \frac{2\mu_p B}{\hbar}$

$\frac{e}{m_\mu}$ from $\mu_\mu \equiv g\frac{e\hbar}{4m_\mu}$

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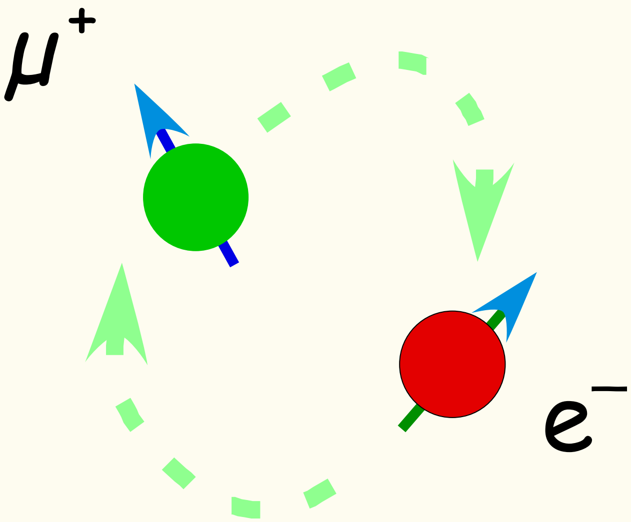
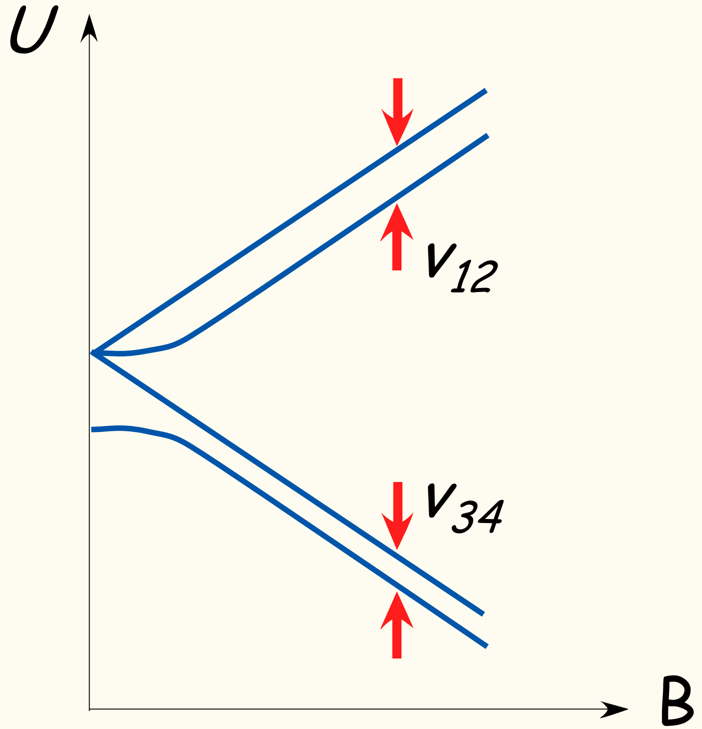
Master formula: $\frac{g-2}{2} = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p}$

Measured by E821

From muonium

# Muonium spectrum determines $\mu_\mu/\mu_p$




$$\nu_{34} - \nu_{12} \sim \mu_\mu B \quad \Rightarrow \quad \mu_\mu / \mu_p$$

Measured to relative $1.2 \cdot 10^{-7}$ (like $15 \cdot 10^{-11}$ in $a_\mu$)

Will need improvement for the "next g-2"

Mu: also $m_\mu/m_e$ and tests of QED