Theory of Lepton $g-2$: Improvement of QED Terms

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1. Introduction

- The precision of muon g-2 measurement has reached 0.5 ppm:

$$\mathbf{a}_\mu(\mathbf{exp}) = \mathbf{11\ 659\ 208\ (6) \times 10^{-10}}$$

Bennett *et al.*, arXiv:hep-ex/0401008

- Few years ago, apparent disagreement with theory generated a lot of excitement about possible new physics.

- Disagreement is still there but less conspicuous.

- In any case prediction of the Standard Model must be known with high enough precision to identify new physics.

- SM prediction consists of pure QED, electroweak, and hadronic parts.

- Since hadronic and electroweak terms of a_μ are covered extensively by other speakers, I will focus on the pure QED contribution which Dr. Nio and I have been working on lately.

- **Actually, 999937 ppm of correction to g=2 (Dirac's prediction) comes from QED, leaving only 60 ppm to hadron and 1.3 ppm to electroweak effect.**

- **QED contribution further breaks down to**

α **term**	**994623 ppm**	**known exactly**
α^2 **term**	**5064 ppm**	**known exactly**
α^3 **term**	**246 ppm**	**known exactly**
α^4 **term**	**3.3 ppm**	**numerical evaluation**
α^5 **term**	**0.045 ppm**	**crude estimate**

- **Note: the α^4 term is larger than experimental uncertainty.**

- **The α^4 term had been assumed to be well-settled.**
  Unfortunately this turned out to be not true.

  We found an error in a part of old calculation.

Kinoshita, Nio, PRL 90, 021803 (2003)

- **Is there any more error in QED ?**
- **To answer this question we decided to examine QED part once again.**
- **The results obtained thus far are as follows:**

  (a) All α^4 terms contributing to $\mathbf{a}_\mu - \mathbf{a}_\mathbf{e}$ have been evaluated by at least two independent formulations.

  No further error was found.

  (b) Some $\alpha^\mathbf{5}$ terms in a_μ and a_e have been calculated.

  (c) Numerical precision of a_μ and a_e has been improved substantially.

  (d) New (tentative) value of $\alpha(\mathbf{a}_\mathbf{e})$ is obtained.

- $\mathbf{a_\mu(QED)}$ **can be written in the general form:**

$$\mathbf{a_\mu(QED) = A_1 + A_2(m_\mu/m_e) + A_2(m_\mu/m_\tau) + A_3(m_\mu/m_e, m_\mu/m_\tau)}$$

$$\mathbf{A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \ldots,\ i = 1, 2, 3.}$$

- $\mathbf{A_1}$ **is common to** $\mathbf{a_e}$ **and** $\mathbf{a_\mu}$**.**

- $\mathbf{A_2^{(4)}}$, $\mathbf{A_2^{(6)}}$, $\mathbf{A_3^{(6)}}$ **evaluated by numerical int., analytic int., asymptotic expansion in** $\mathbf{m_\mu/m_e}$**, or power series expansion in** $\mathbf{m_\mu/m_\tau}$**. Thus known "exactly": Errors from those of** α, $\mathbf{m_\mu}$ **and** $\mathbf{m_\tau}$ **only.**

$$\mathbf{A_2^{(4)}(m_\mu/m_e) = 1.094\ 258\ 282\ 7(104)}$$

$$\mathbf{A_2^{(4)}(m_\mu/m_\tau) = 7.8059\ (25) \times 10^{-5}}$$

$$\mathbf{A_2^{(6)}(m_\mu/m_e) = 22.868\ 379\ 36(22)}$$

$$\mathbf{A_2^{(6)}(m_\mu/m_\tau) = 36.054(21) \times 10^{-5}}$$

$$\mathbf{A_3^{(6)}(m_\mu/m_e, m_\mu/m_\tau) = 0.52763\ (17) \times 10^{-3}}$$

Kinoshita NCB 51, 140(1967)

Laporta NCB 106, 675(1993)

Laporta, Remiddi, PLB 301, 440(1993)

Czarnecki, Skrzypek, PLB 449, 354(1999)

- **Higher-order terms - previous results:**
- $\mathbf{A_2^{(8)}}$ **term**

Evaluated mostly by numerical integration.

$$\mathbf{A_2^{(8)}(m_\mu/m_e) = 127.50(41)}$$

Kinoshita et al., PRD 41, 593(1990)
Laporta PLB 312, 495(1993)
Broadhurst et al. PLB298, 445(1993)

- $\mathbf{A_3^{(8)}}$ **term**

$$\mathbf{A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.079(3)}$$

Kinoshita et al., PRD 41, 593 (1990)

- $\mathbf{A_2^{(10)}}$ **term**

Crude estimate based on RG and a lot of guesswork:

$$\mathbf{A_2^{(10)}(m_\mu/m_e) = 930(170)}$$

Kinoshita et al., PRD 41, 593 (1990)
Karshenboim, Yad. Phys. 56, 252 (1993)

2. New evaluation of $\mathbf{A_2^{(8)}(m_\mu/m_e)}$

469 Feynman diagrams: Classified in four gauge-invariant sets.

- **Group I. Second-order muon vertex with lepton vp-loops.
  49 Feynman diagrams.
  Classified into *g-i* subsets I(a), I(b), I(c), I(d).**

- **Group II. Fourth-order muon vertex with lepton vp-loops.
  90 Feynman diagrams.**

- **Group III. Sixth-order muon vertex with electron vp-loop of
  2nd order. 150 Feynman diagrams.**

- **Group IV. Muon vertex containing *light-light scattering* subdiagram
  with further radiative corrections. 180 diagrams.
  Consists of *g-i* subsets IV(a), IV(b), IV(c), IV(d).**

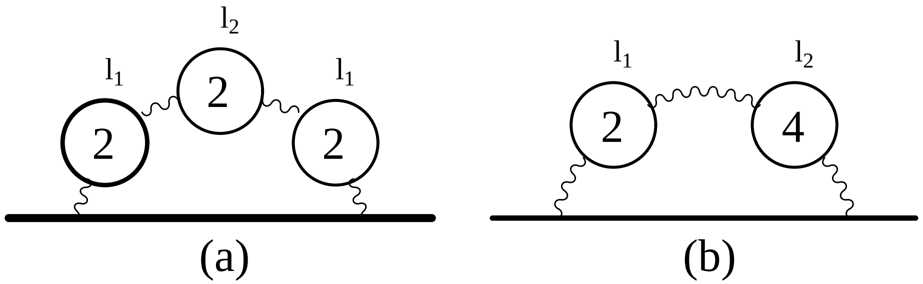

Figure 1: **Typical diagrams of Group I(a) and Group I(b). I(a) has 1 diagram. I(b) has 6 diagrams. In this and subsequent figures fermions propagate in external magnetic field.**

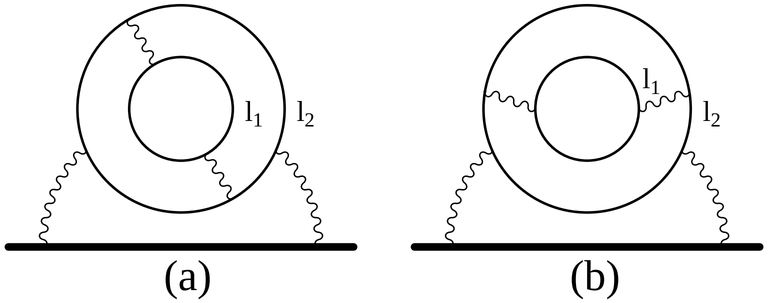

Figure 2: **Typical diagrams of Group I(c), which consists of 9 diagrams.**

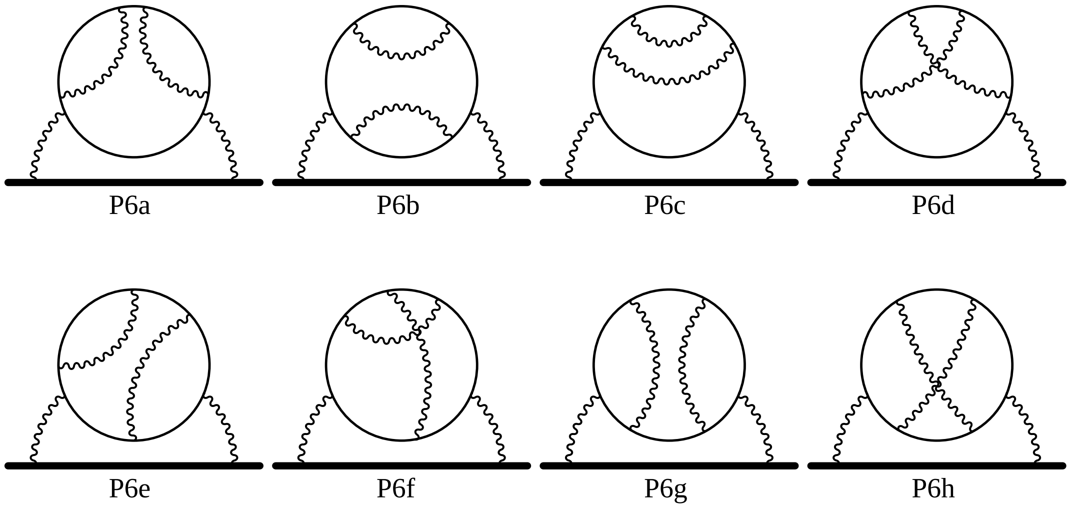

Figure 3: **Diagrams of Group I(d). It has 15 diagrams.**

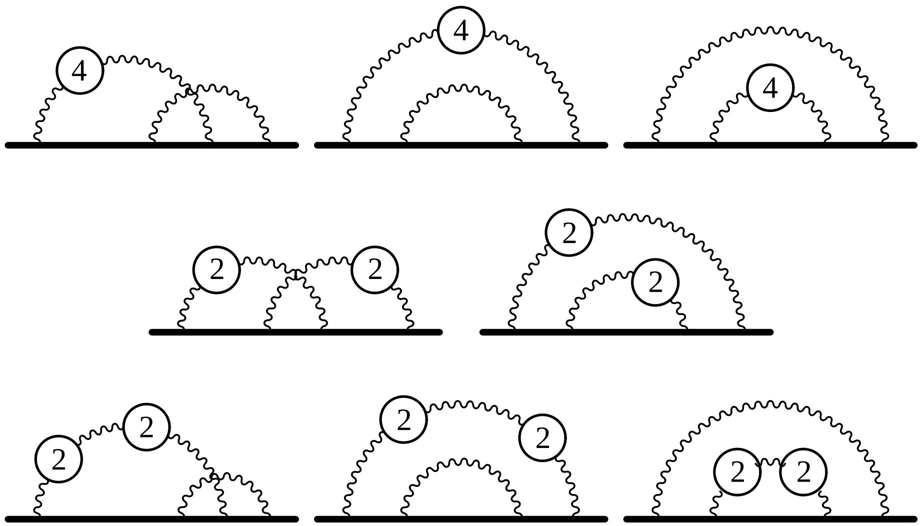

Figure 4: **Typical diagrams of Group II. 90 diagrams.**

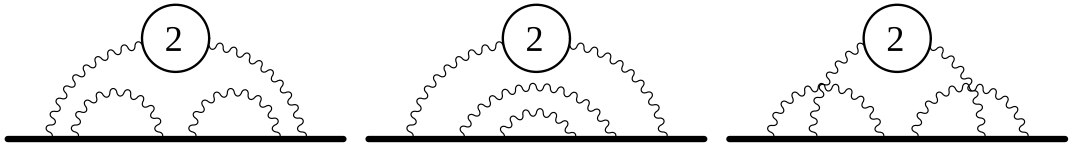

Figure 5: **Some of 150 diagrams of Group III.**

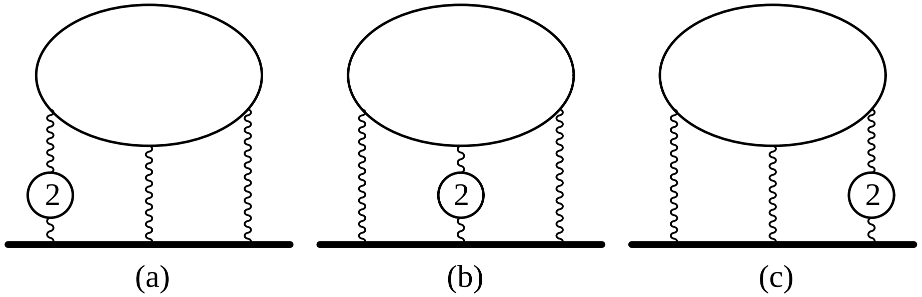

Figure 6: **Some of 54 diagrams of Group IV(a).**

- **Groups I and II evaluated by more than one methods, numerical, analytic or other:**

$$\mathbf{a}_{\mathbf{I}}^{(\mathbf{8})} = \mathbf{16.720\ 359\ (20)}$$

$$\mathbf{a}_{\mathbf{II}}^{(\mathbf{8})} = -\mathbf{16.674\ 591\ (68)}$$

- **Groups III and IV(a) evaluated only numerically. But their codes are fully tested in the sense that they are obtained from sixth-order codes which give results identical with analytic results:**

$$\mathbf{a}_{\mathbf{III}}^{(\mathbf{8})} = \mathbf{10.793\ 43\ (414)}$$

$$\mathbf{M}_{\mathbf{6LL,P2}}^{(\mathbf{e,e})} = \mathbf{116.759\ 18\ (30)}$$

$$\mathbf{M}_{\mathbf{6LL,P2}}^{(\mathbf{e},\mu)} = \mathbf{2.697\ 44\ (15)}$$

$$\mathbf{M}_{\mathbf{6LL,P2}}^{(\mu,\mathbf{e})} = \mathbf{4.328\ 89\ (30)}$$

- **Remaining diagrams of Groups IV**

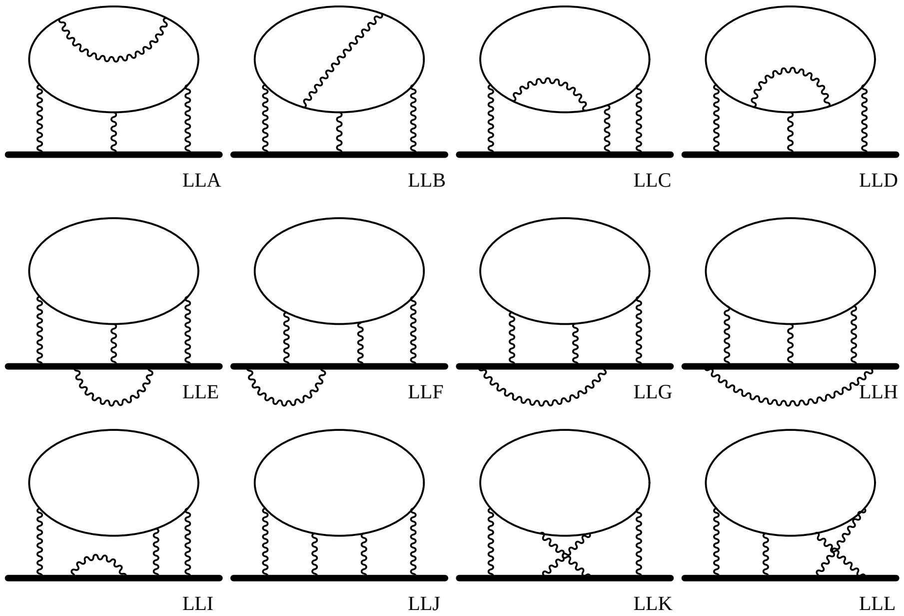

Figure 7: **Typical diagrams of Group IV(b), IV(c), IV(d).**

Group IV(b): LLA, LLB, LLC, LLD. 60 diagrams
Group IV(c): LLE, LLF, LLG, LLH, LLI. 48 diagrams
Group IV(d): LLJ, LLK, LLL. 18 diagrams

- IV(b), IV(c), and IV(d) have been evaluated in two independent ways.
- An error was discovered and corrected for IV(d).

Kinoshita, Nio, PRL 90, 021803 (2003)

- We found that IV(b) and IV(c) have no algebraic error.

Kinoshita, Nio, arXiv:hep-ph/0402206

- We are confident that all diagrams of Group IV and hence all terms contributing to $A_2^{(8)}(m_\mu/m_e)$ have analytic or numerical error.
- Combining results of two versions we obtain

$$a^{(8)}_{IV(b)} = -0.417\ 72\ (385)$$

$$a^{(8)}_{IV(c)} = \quad 2.908\ 10\ (454)$$

$$a^{(8)}_{IV(d)} = -4.432\ 43\ (\ 58)$$

- **New total value is**

$$\mathbf{A_2^{(8)}(m_\mu/m_e) = 132.682\ 3\ (72)}$$

The old value was

$$\mathbf{A_2^{(8)}(m_\mu/m_e) = 127.50(41)}$$

- **While size of $\mathbf{A_2^{(8)}(m_\mu/m_e)}$ is determined by $\mathbf{a_{IV(a)}}$, its error is dominated by $\mathbf{a_{IV(b)}}$, $\mathbf{a_{IV(c)}}$.**

- **$\mathbf{A_3^{(8)}}$ has also been re-evaluated:**

$$\mathbf{A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.0376(1)}$$

Difference with old value 0.079(3) is mostly due to diagrams not included in old estimate.

3. Evaluation of $\mathbf{A_2^{(10)}(m_\mu/m_e)}$: Progress report

- $A_2^{(10)}(m_\mu/m_e)$ **was estimated to be 930 (170), which contributes only 0.054 ppm to** a_μ**, well within current experimental uncertainty.**
- **But it will become important source of error in next generation of** a_μ **measurement.**
- **Better estimate of** $A_2^{(10)}(m_\mu/m_e)$ **will be needed eventually.**
- **Total number of diagrams contributing to** $A_2^{(10}(m_\mu/m_e)$ **is**

9080 !

- **It turned out, however, that it is not too difficult to improve** $A_2^{(10)}(m_\mu/m_e)$ **substantially.**
- **The first step is to classify them into gauge-invariant sets.**
- **There are 32 gauge-invariant sets within 6 supersets.**

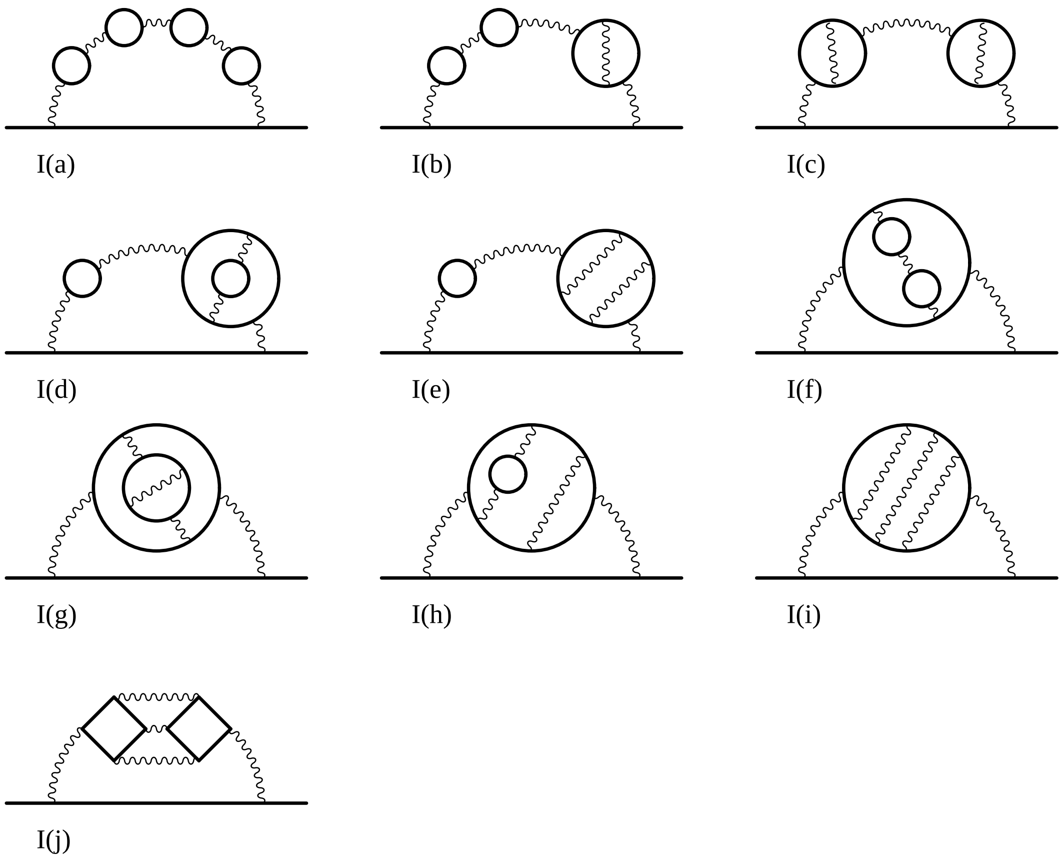

Figure 8: **Some diagrams of Set I.**

Set I is built from a second-order vertex. 498 diagrams contribute to $A_2^{(10)}(m_\mu/m_e)$**.**

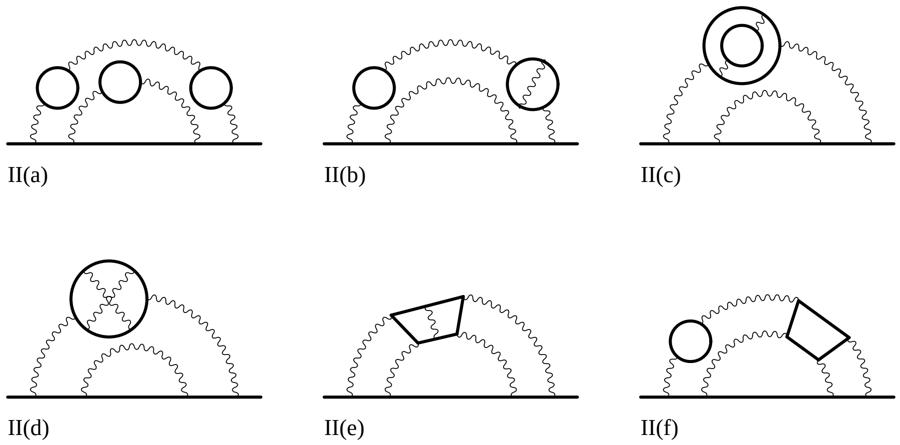

Figure 9: **Diagrams of Set II.**

Set II is built from fourth-order proper vertices. 1176 diagrams contribute to $A_2^{(10)}(m_\mu/m_e)$.

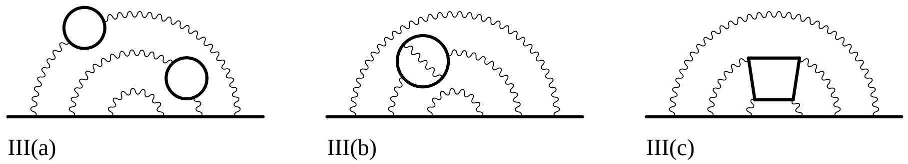

Figure 10: **Diagrams of Set III.**

Set III is built from sixth-order proper vertices. 1740 diagrams contribute to $A_2^{(10)}(m_\mu/m_e)$.

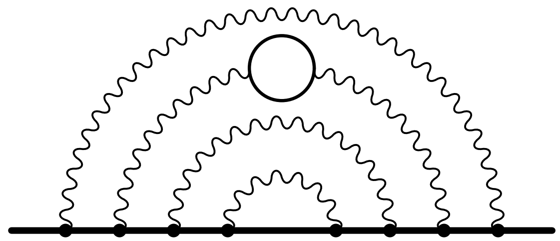

Figure 11: **Diagrams of Set IV.**

Set IV is built from eighth-order proper vertices. 2072 diagrams contribute to $A_2^{(10)}(m_\mu/m_e)$**.**

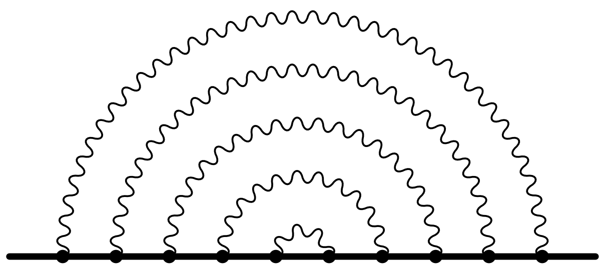

Figure 12: **Diagrams of Set V.**

Set V consists of 10th-order proper vertices with no closed lepton loop. No contribution to $A_2^{(10)}(m_\mu/m_e)$**.**

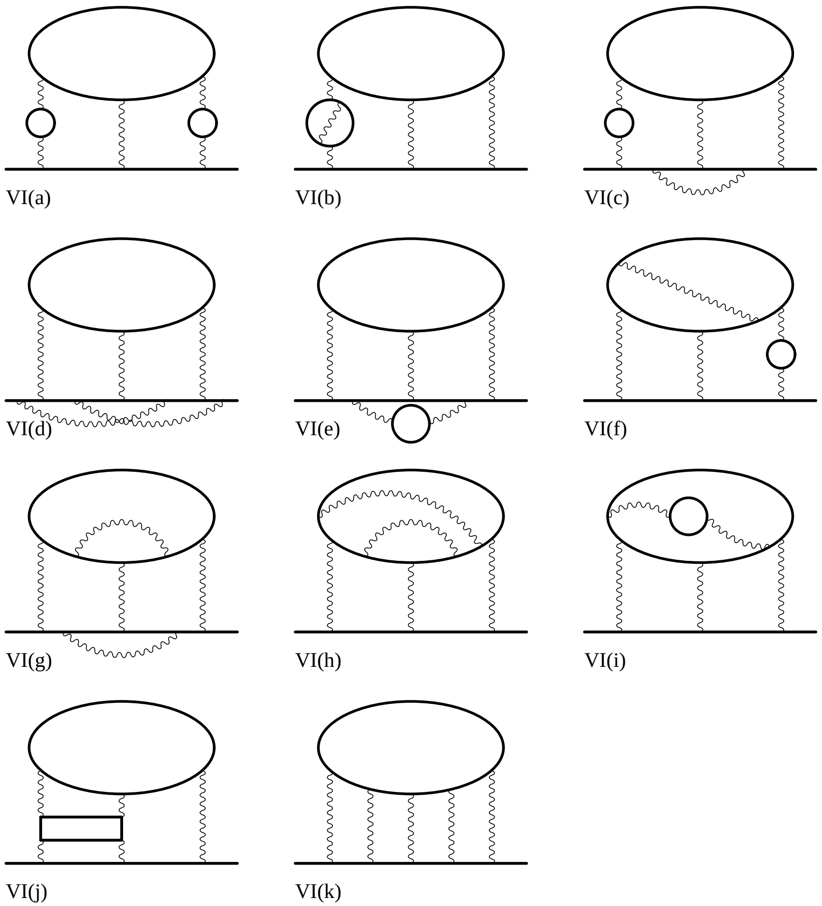

Figure 13: **Diagrams of Set VI.**

Set VI consists of diagrams containing various *l-l-scattering* **subdgrams. 3594 contribute to** $A_2^{(10)}(m_\mu/m_e)$**.**

- **Fortunately, not difficult to see which g-i sets give large contribution.**
- **They are sets having light-by-light scattering subdiagram and/or vacuum-polarization subdiagram.**
- **Both are sources of $\ln(m_\mu/m_e)$, as was noted and exploited previously.**

Kinoshita et al., PRD 41, 593 (1990)

Karshenboim, Yad. Phys. 56, 252 (1993)

- **Largest contribution comes from Set VI(a) [252 diagrams].**
- **Next largest comes from Set VI(b) [162 diagrams].**
- **We have now evaluated them precisely:**

$$\mathbf{A_2[VI(a)] = 629.1407\ (118)} \quad \textbf{(it was 570 (140))}$$

$$\mathbf{A_2[VI(b)] = 181.1285\ (\ 51)} \quad \textbf{(it was 176 (35))}$$

Other possibly large sets computed (not yet fully double-checked) are:

$$\begin{aligned}
\mathbf{A_2[I(a)]} &= \mathbf{22.553\ 17\ (25)}^* \\
\mathbf{A_2[I(b)]} &= \mathbf{30.667\ 54\ (33)}^* \\
\mathbf{A_2[I(c)]} &= \mathbf{5.141\ 38\ (15)}^* \\
\mathbf{A_2[I(d)]} &= \mathbf{8.892\ 07\ (102)} \\
\mathbf{A_2[I(e)]} &= \mathbf{-1.219\ 20\ (71)} \\
\mathbf{A_2[I(f)]} &= \mathbf{3.685\ 10\ (13)} \\
\mathbf{A_2[II(a)]} &= \mathbf{-70.4717\ (38)}^* \\
\mathbf{A_2[II(b)]} &= \mathbf{-34.7717\ (26)}^* \\
\mathbf{A_2[II(f)]} &= \mathbf{-77.5224\ (414)} \\
\mathbf{A_2[VI(c)]} &= \mathbf{-36.5763\ (1141)} \\
\mathbf{A_2[VI(e)]} &= \mathbf{-4.3215\ (1341)} \\
\mathbf{A_2[VI(f)]} &= \mathbf{-38.1396\ (1591)} \\
\mathbf{A_2[VI(i)]} &= \mathbf{-27.3373\ (1147)}
\end{aligned}$$

Parts of data with * contain analytic results.

Laporta, PLB 328, 522 (1994)

- **Largest remaining contribution is likely to come from VI(k) [120 diagrams] whose leading log term was studied previously :**

$$\mathbf{A_2[VI(k)] \simeq C_n \pi^4 \log(m_\mu/m_e) + \ldots .}$$

Elkhovskii, Yad. Phys. 49, 1059 (1989)

$$\mathbf{C_n = 0.438 \ldots .}$$

Milstein,Elkhovskii, Phys. Lett. 233B, 11 (1989)

- **It was estimated from this that**

$$\mathbf{A_2[VI(k)] = 185\ (85).}$$

Karshenboim, Yad. Phys. 56, 252 (1993)

- **The huge π^4 factor is due to the fact that $\log(m_e)$ comes from the integration domain where all exchanged photons have $|k| << m_e$.**

- **In this domain, electron moves non-relativistically in the Coulomb potential of the muon, each Coulomb photon contributing a factor $i\pi$ when its momentum is integrated out.**

- **Elkhovskii's argument leaves several questions unanswered:**

(1) Naive power counting shows that electron loop may have $1/m_e^2$ rather than $\log(m_e)$ behavior for $m_e \to 0$. It is not clear how $1/m_e^2$ singularity is weakened by integration over photon momenta.

(2) Leading log estimate may be too crude since $m_\mu/m_e \simeq 206$ is far from asymptotic. It may have to be reduced substantially, as was the case with light-by-light-scattering diagram of sixth-order.

- **To answer these questions it is best to evaluate them explicitly.**

- **It turned out that this is not too difficult.**

- **Our first step is to reduce the number of integrals to 12 using**

$$\mathbf{\Lambda}^\nu(\mathbf{p},\mathbf{q}) \simeq -\mathbf{q}_\mu\left[\frac{\partial \mathbf{\Lambda}_\mu(\mathbf{p},\mathbf{q})}{\partial \mathbf{q}_\nu}\right]_{\mathbf{q}=\mathbf{0}} - \frac{\partial \mathbf{\Sigma}(\mathbf{p})}{\partial \mathbf{p}_\nu},$$

derived from the Ward-Takahashi identity, then reduce it further to 9 using time-reversal symmetry.

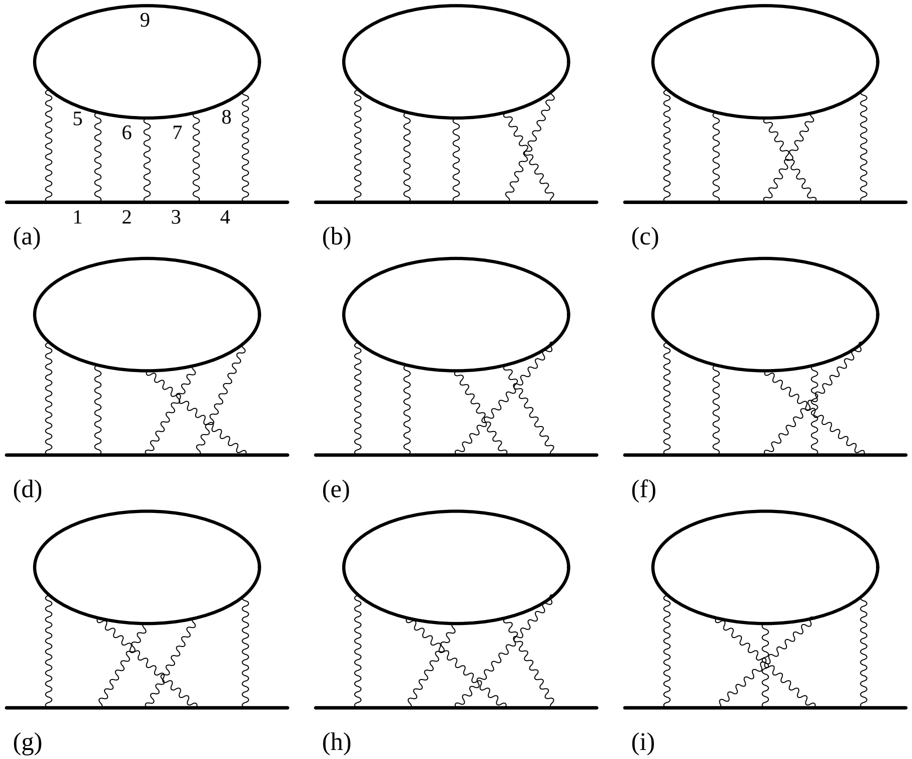

Figure 14: **Ward-Takahashi-summed diagrams of Set VI(k).**

- **Starting from RHS of W-T identity, FORM generates more than 90,000 terms occupying about 30,000 lines of FORTRAN code.**
- **Not too bad: Only 30 times larger than eighth-order integrals.**
- **Numerical integration (over 13-dim. Feynman parameter space) is carried out using VEGAS.**
- **Our result thus far (not yet fully double-checked) is**

$$\mathbf{A_2[VI(k)] = 86.69\ (0.09)}.$$

- **There is no indication of $1/m_e^2$ behavior. Further check needed.**
- **But clearly the previous value was an overestimate by ~ 100.**
- **Using Karshenboim's guess for VI(j)**

$$\mathbf{A_2[VI(j)] = 0 \pm 40}, \qquad \text{[This will be evaluated next.]}$$

we obtain a new estimate for the 10th-order term

$$\mathbf{A_2^{(10)}(m_\mu/m_e) = 677 \pm 40}.$$

- **Then the total QED contribution to a_μ becomes**

$$\mathbf{a_\mu(QED) = 116\ 584\ 717.88\ (0.02)(0.27)(0.85) \times 10^{-11}}$$

- **If one uses values of hadronic v-p and light-by-light scattering terms**

$$\mathbf{a}_\mu(\mathbf{had.vp}) = \mathbf{6963\ (62)\ (36)} \times \mathbf{10}^{-11}$$

Davier,et al., EPJ C 31, 503 (2003)

$$\mathbf{a}_\mu(\mathbf{had.ll}) = \mathbf{136\ (25)} \times \mathbf{10}^{-11}$$

Melnikov, Vainshtein, arXiv:hep-ph/0312226

and other small correction, the SM value of a_μ will be

$$\mathbf{a}_\mu(\mathbf{SM}) = \mathbf{116\ 591\ 870.9\ (76.2)} \times \mathbf{10}^{-11}$$

$$\mathbf{a}_\mu(\mathbf{exp}) - \mathbf{a}_\mu(\mathbf{SM}) = \mathbf{209\ (97)} \times \mathbf{10}^{-11}$$

where "theory" error comes mostly from hadronic v-p terms.

4. Electron $g-2$

$$\mathbf{a_e(QED) = A_1 + A_2(m_e/m_\mu) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu, m_e/m_\tau)}$$

$$\mathbf{A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \ldots, \quad i = 1, 2, 3}$$

$\mathbf{A_1^{(2)} = 0.5}$	**1 diagram**	**(analytic)**
$\mathbf{A_1^{(4)} = -0.328\ 478\ 965\ \ldots}$	**7 diagrams**	**(analytic)**
$\mathbf{A_1^{(6)} = 1.181\ 241\ 456\ \ldots}$	**72 diagrams**	**(analytic)**
$\mathbf{A_1^{(8)} = -1.709\ 3\ (42)}$	**891 diagrams**	**(numerical)**
$\mathbf{A_1^{(10)} = 0\ (3.8)}$	**12672 diagrams**	**(crude guess)**

Laporta, Remiddi, PLB 379, 283 (1996)

Kinoshita, PRL 75, 4728 (1995)

- **$\mathbf{A_2, A_3}$ and weak and hadronic contributions to $\mathbf{a_e}$ are very small and known with sufficient accuracy.**

- $\mathbf{A_1^{(8)}}$ **is still tentative. Most difficult are 518 diagrams represented by 47 Ward-Takahashi-summed diagrams shown below.**

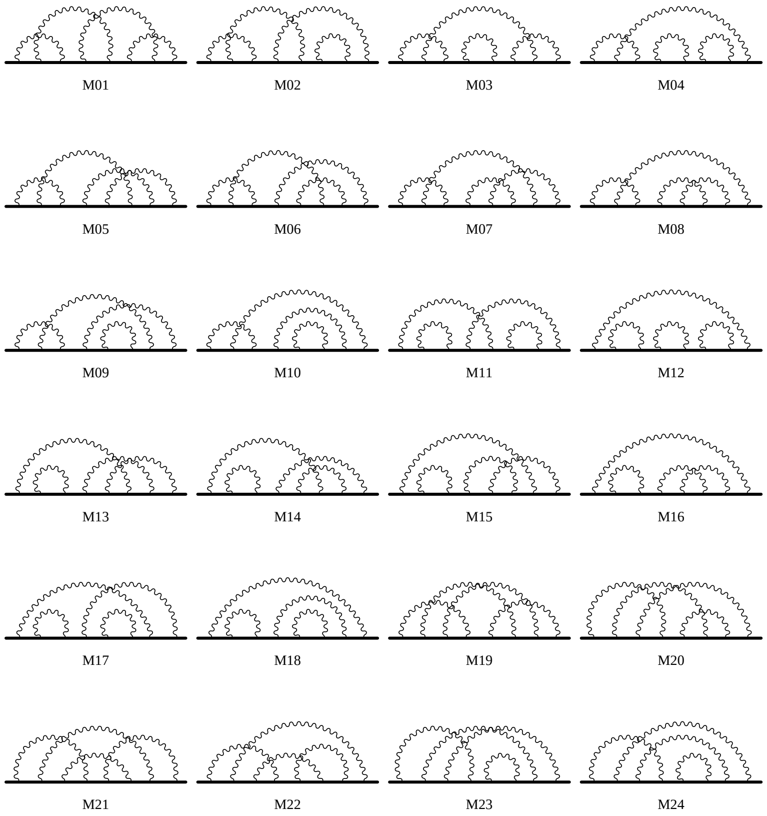

Figure 15: **Diagrams M01 - M24 of Group V.**

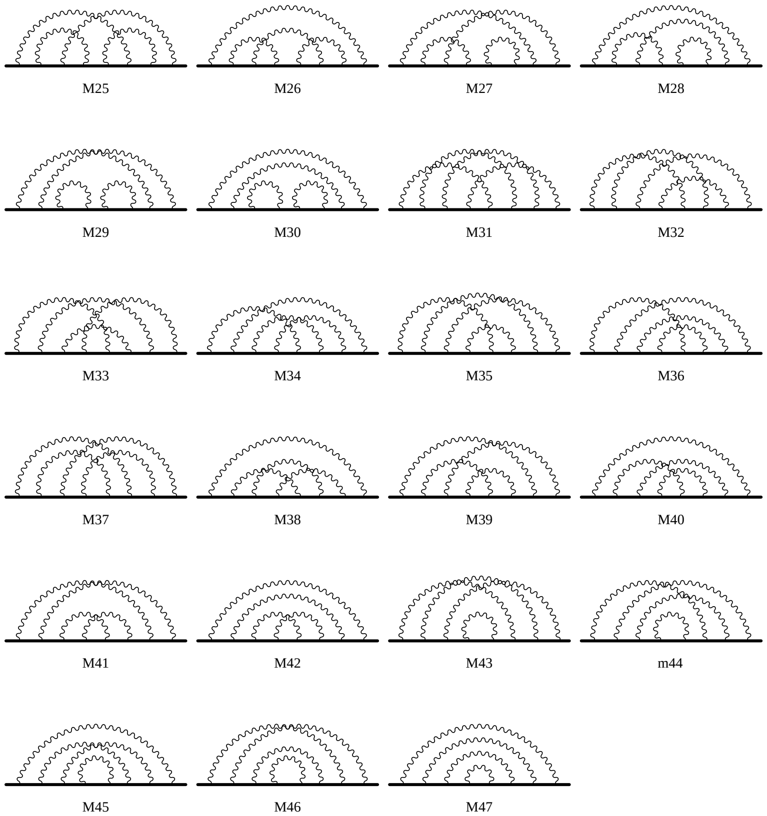

Figure 16: **Diagrams M25 - M47 of Group V.**

- **Algebraic error of $A_1^{(8)}$ from Group V have been eliminated completely by**

(a) Use of common template for all diagrams.

(b) Analytic and non-trivial check of renormalization terms with known lower-order integrals.

(c) For diagrams containing two 2nd-order electron s.-e. parts, the complete 8th-order integrand can be generated by an integration by part from a 6th-order integral which has been verified by comparison with analytic result.

- **Numerical work of $\mathbf{A_1^{(8)}}$ is nearly finished. The latest value is**

$$\mathbf{A_1^{(8)}} = -\mathbf{1.709\ 3}\ (\mathbf{42})$$

- **Errorbars 9 times smaller than the old one.**

Kinoshita, Nio, preliminary, 2004

- **Current status of $A_1^{(10)}$ is as follows:**

$$\mathbf{A_1^{(10)}}[\text{partial sum}] = -\mathbf{2.179}\ (\mathbf{6})$$

from 904 diagrams.

- **Of course the value from remaining 11768 diagrams is unknown.**

- **For years biggest obstacle in testing QED using a_e was *unavailability* of α with high enough precision.**

- **Several precise values of α^{-1} available at present are shown:**

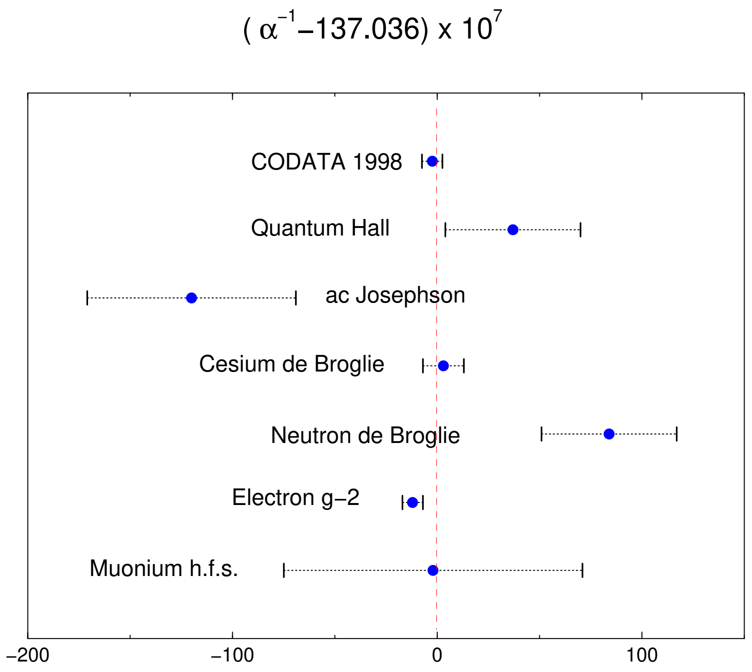

Figure 17: **Comparison of various α^{-1}**

- **MOST ARE NOT GOOD ENOUGH TO TEST QED !**

For instance, α obtained from quantum Hall effect gives

$$a_e(\mathbf{qH}) = \mathbf{1\ 159\ 652\ 146.8\ (0.2)(28.0)} \times \mathbf{10^{-12}}$$
$$\mathbf{(QED)}(\alpha(\mathbf{qH}))$$

- **Fortunately the situation has improved recently by improved interferometry measurement, combined with cesium D_1 line measurement:**

$$\alpha^{-1}(\mathbf{h}/M_{C_s}) = \mathbf{137.036\ 000\ 3\ (10)} \qquad \mathbf{(7.4\ ppb)}$$

A. Wicht et al. in Proc. of 6th Symp. on Freq. Standards and Metrology (World Sci., 2002), pp.193 -212

- **New theoretical value (assuming $\mathbf{A}_1^{(\mathbf{10})} = \mathbf{0.0(3.8)}$) is**

$$a_e(h/M_{Cs}) = \mathbf{1\ 159\ 652\ 176.41\ (0.12)(0.26)(8.48) \times 10^{-12}}$$
$$\mathbf{(8th)(10th)}(\alpha(h/M_{Cs}))$$

$$a_e(exp) - a_e(h/M_{Cs}) = \mathbf{11.8\ (9.5) \times 10^{-12}}$$

- **Calculated part of $A_1^{(10)}$ will reduce this by**

$$-\mathbf{2.179}(\alpha/\pi)^{\mathbf{5}} = -\mathbf{0.147} \times \mathbf{10^{-12}}\ .$$

This is within estimated uncertainty of α^5 term.

5. New value of α based on electron g-2

- **Alternative (and more sensible) test of QED is to calculate α from theory and measurement of $\mathbf{a_e}$.**
- **This leads to (still tentative) value of $\alpha(\mathbf{a_e})$:**

$$\alpha^{-1}(a_e) = \mathbf{137.035\ 998\ 90\ (1.5)\ (3.1)\ (50)} \quad \mathbf{(3.7\ ppb)}$$
$$(\alpha^4)\ (\alpha^5)\ (\mathbf{expt})$$

- **Uncertainty comes predominantly from the measurement of a_e.**
- **Uncertainty in α^4 term is smaller than that of α^5 term.**

Until reliable (even if crude) estimate of α^5 term is obtained, further reduction of uncertainty of $\mathbf{A_1^{(8)}}$ cannot improve theory significantly.

- **This is why α^5 term is being looked at.**
- **$A_1^{(10)}$ has contribution from 12672 Feynman diagrams, in which Set 5 (6354 diagrams) is the most difficult to evaluate although it is solvable in principle.**

- **Gabrielse's new experiment is in advanced stage.**

- **As is seen from his figures, it is very likely that the experimental uncertainty is reduced substantially in the near future.**

- **If the uncertainty is reduced by an order of magnitude, the uncertainty in $\alpha(a_e)$ will be brought down to the level of 0.3 ppb.**

- **This will set a new standard for all measurements of α.**

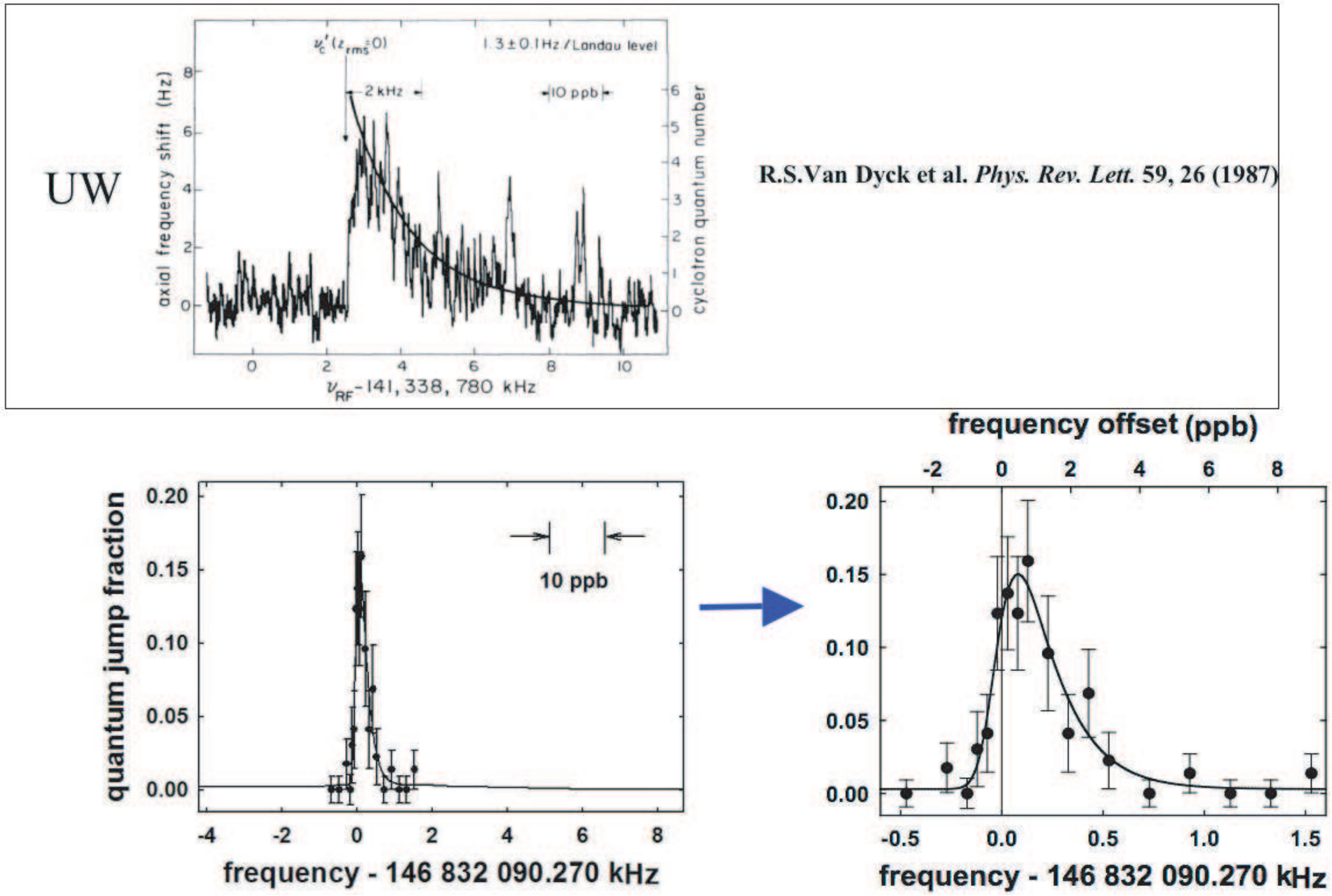

Figure 18: **Cyclotron Resonance Line (Gabrielse).**

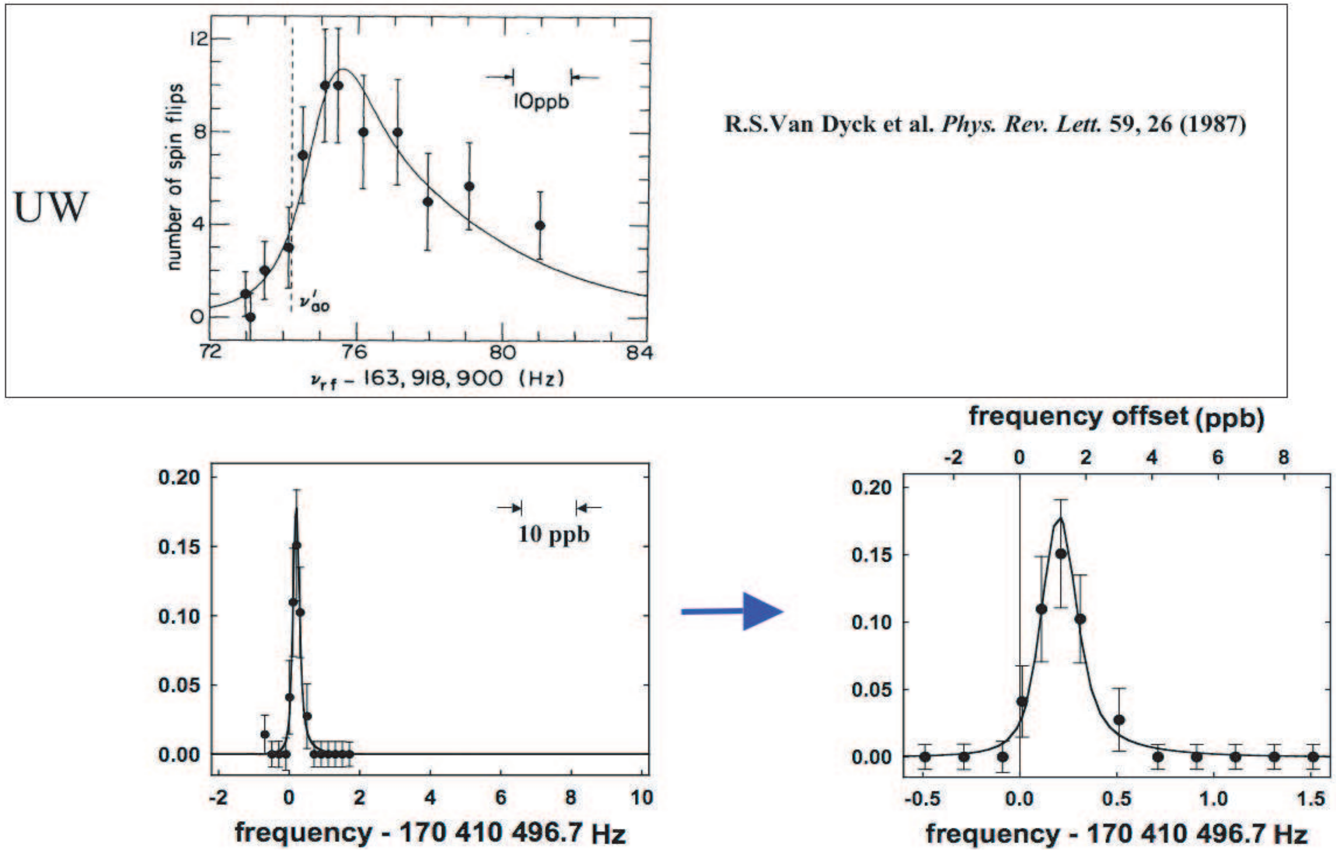

Figure 19: **Anomaly Resonance Line (Gabrielse).**

- **Primary motivation for pushing $\alpha(a_e)$ to higher and higher precision is to see where QED might break down.**
- **For this purpose we need another α of comparable precision, which is not yet available.**
- **Importance of measuring α's by diverse means cannot be overemphasized, since they enables us to test *all* theories and measurements on equal footing.**
- **Ultimately it provides check of internal consistency of quantum mechanics itself.**
- **At the moment ther is no serious disagreement among α's.**
- **But disagreement may develop as precision of α improves.**
- **What if such a disagreement persists ?**
- **Might this be a hint of new physics ?**