The Radiative Return and Form Factors at Large Q^2

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III Nucleon Form Factors at B-Factories

IV Pion and Kaon Form Factors at large Q^2 and $\tau \longrightarrow \nu K^- K^0$

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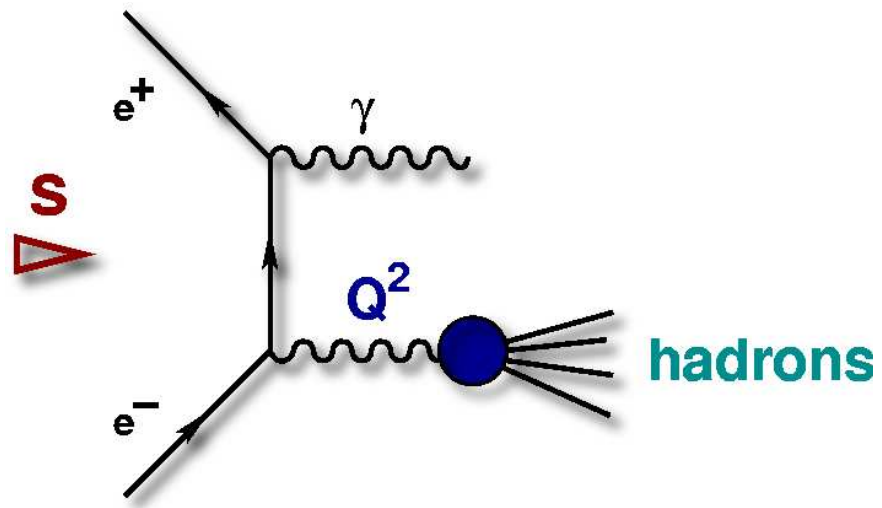
I BASIC IDEA

photon radiated off the initial e^+e^- (ISR) reduces the effective energy of the collision

$$d\sigma(e^+e^- \to \text{hadrons} + \gamma) = H(Q^2, \theta_\gamma)\, d\sigma(e^+e^- \to \text{hadrons})$$



- measurement of $R(s)$ over the full range of energies, from threshold up to $\sqrt{s}$
- large luminosities of factories compensate α/π from photon radiation
- radiative corrections essential (NLO)
- advantage over energy scan (BES, CMD2, SND): systematics (e.g. normalization) only once

High precision measurement of the hadronic cross-section at DAΦNE, CLEO-C, B-factories

Rough estimates for rates:

$\pi^+ \pi^- \gamma : E_\gamma > 100 MeV$

$\sqrt{s}$ $[GeV]$	$\int \mathfrak{L}$ $[fb^{-1}]$	#events, $\theta_{min} = 7^\circ$
1.02	1.35	16 $\cdot 10^6$
10.6	100	3.5 $\cdot 10^6$

multi-hadron-events (R $\equiv 2$) $\sqrt{s} = 10.6\, GeV$

Q^2-interval $[GeV]$	#events, $\theta_{min} = 7^\circ$
$[\, 1.5\,,\ 2.0\,]$	9.9 $\cdot 10^5$
$[\, 2.0\,,\ 2.5\,]$	7.9 $\cdot 10^5$
$[\, 2.5\,,\ 3.0\,]$	6.6 $\cdot 10^5$
$[\, 3.0\,,\ 3.5\,]$	5.8 $\cdot 10^5$

Lowest order

$$\frac{d\sigma}{dQ^2}\left(e^+e^- \to \gamma + \text{had}(Q^2)\right) = \sigma\left(e^+e^- \to \text{had}(Q^2)\right)$$

$$\times \frac{\alpha}{\pi s}\left\{ \begin{array}{l} \frac{s^2+Q^4}{s(s-Q^2)}\left(\log(s/m_e^2) - 1\right), \text{ no angular cut} \\ \frac{s^2+Q^4}{s(s-Q^2)} \log\left(\frac{1+\cos\theta_{min}}{1-\cos\theta_{min}}\right) - \frac{s-Q^2}{s}\cos\theta_{min} \end{array} \right\}$$

$\Rightarrow$ differential luminosity: $\frac{dL}{dQ^2}\left(Q^2, s\right) = \frac{\alpha}{\pi s}\left\{\cdots\right\} L(\text{at } s)$

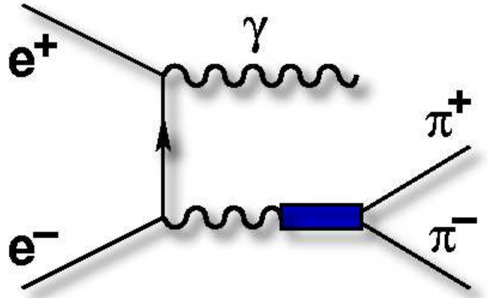
e.g. $\theta_{min} = 30^\circ$; $\sqrt{s} = 10.58$ GeV ; $Q = 1$ GeV ; $\Delta Q = 0.1$ GeV

$$\frac{dL}{dQ^2}\left(Q^2, s\right)\Delta Q^2 = 7.6 \cdot 10^{-6}\, L(\text{at } s)$$

100 fb^{-1} at 10.58 GeV $\Rightarrow$ 0.76 pb^{-1} per scan point at 1 GeV

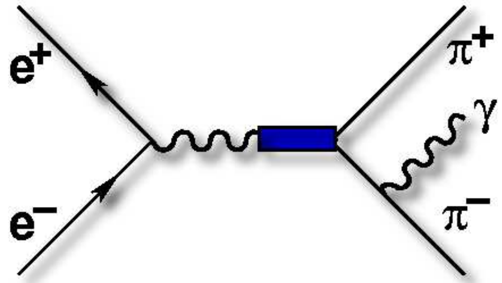
Basic Ingredients for Pion Formfactor

▶ ISR



pion form factor
⇨ to be tested

▶ FSR



radiation from point-like pions (probably overestimated)

▶ **additional radiation:** collinear (EVA MC)
or NLO calculation (PHOKHARA MC)

II MONTE CARLO GENERATORS

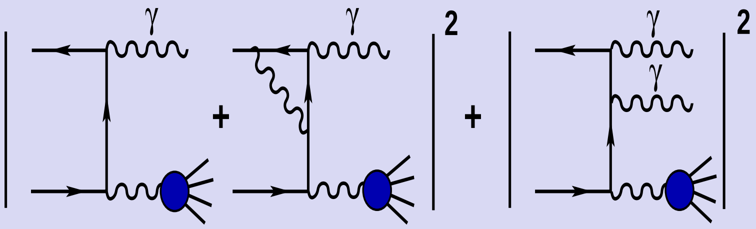
P
H
OTONS FROM
KARLSRUHE
H
ADRONICALLY
R
ADIATED

References etc. $\longrightarrow$ http://cern.ch/german.rodrigo/phokhara

PHOKHARA 2.0:

$\pi^+\pi^-$, $\mu^+\mu^-$, 4π

- **ISR at NLO:** virtual corrections to one photon events and two photon emission at tree level



- FSR at LO: $\pi^+\pi^-$, $\mu^+\mu^-$
- tagged or untagged photons
- modular structure

❶ LL at a fixed order + subleading terms **(1 %)**

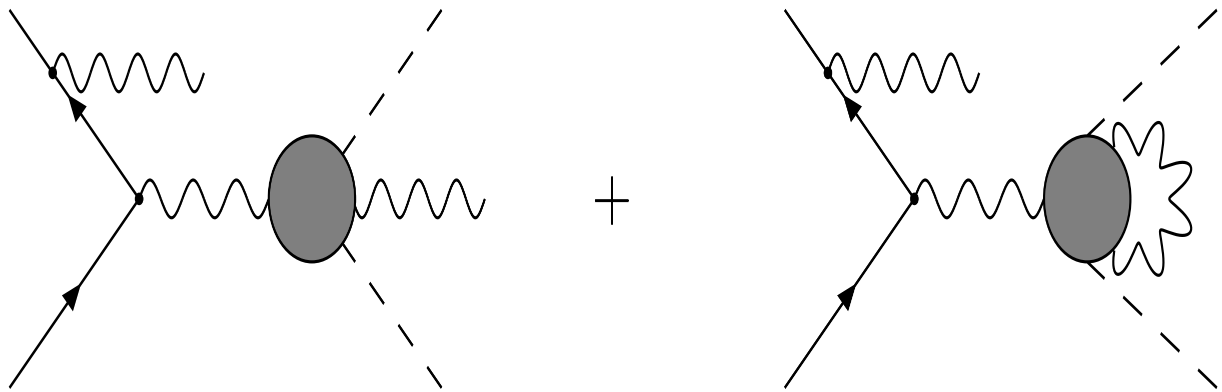
❷ Full angular dependence

❸ Momentum conservation

❹ Tagged or untagged photon

PHOKHARA 3.0

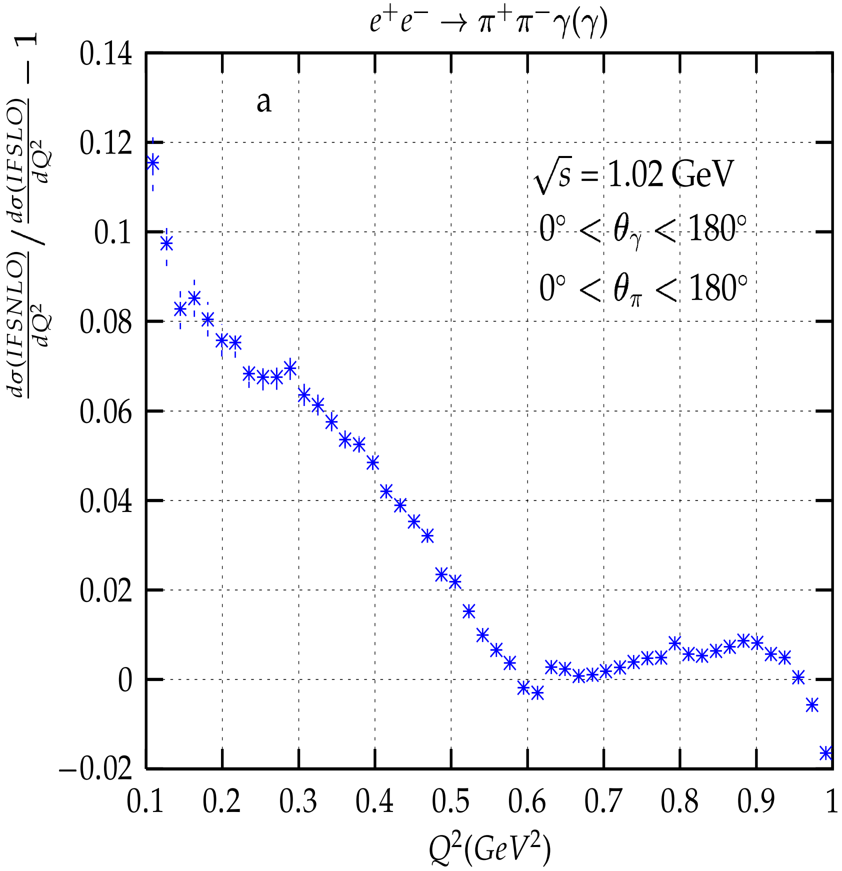
- specifically developed for $\pi^+\pi^-$ (plus photons)
- allows for simultaneous emission of photons
  from initial and final state,
  including virtual corrections (interference neglected).

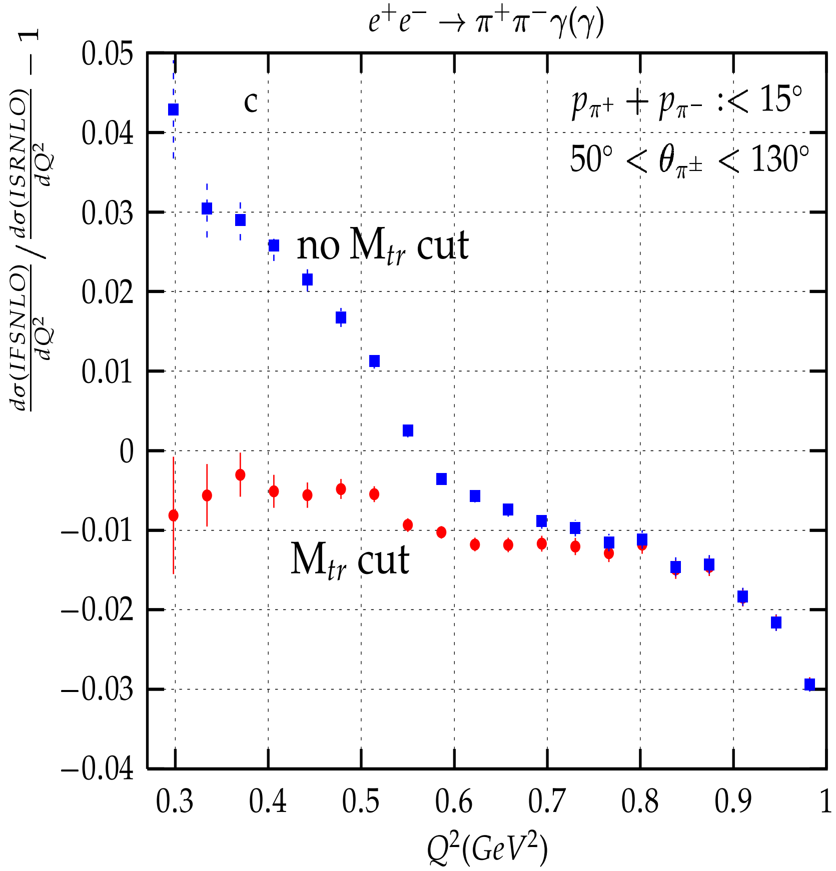


$\Rightarrow$ dominated by "two step process": $e^+e^- \to \gamma\ \rho\ (\to \gamma\ \pi\pi)$

$\Rightarrow$ importance of $\pi\pi\gamma$ as input for a_μ

Large effect for $Q^2 < m_\rho^2$ eliminated by suitable cuts on $\pi^+\pi^-$ configuration (suppress 2γ events)





$\Rightarrow$ Talk by D. Leone

or measure photon

Experimental Perspectives

BABAR, BELLE

higher Q^2 available

$\Rightarrow$ measurement of R(Q^2) from threshold up to at least 5 GeV.

Examples:

- $\pi\pi$
- $4\pi^{\pm}$
- K K $\pi\pi$
- K K K K

PHOKHARA 4.0

- $\mu^+\mu^-\gamma$ **with FSR at NLO**
- **vacuum polarisation can be switched on**
- **nucleon pair production included**

To be done:

- **three mesons: 3π ($\rightarrow \rho\pi$), $KK\pi$**
- **$KK\pi\pi$, $4K$**
- **narrow resonances**

parameters of J/ψ , ψ' :

observable: $\Gamma_e \frac{\Gamma_f}{\Gamma_{tot}}$; $f = \mu^+\mu^-$, $\pi^+\pi^-$, 3π , 4π , $4K$, ...

compare : $\frac{\sigma_f}{\sigma_{\mu^+\mu^-}}(off\ resonance) \stackrel{?}{=} \frac{\sigma_f}{\sigma_{\mu^+\mu^-}}(on\ resonance)$

$f = \mu^+\mu^-$, $\pi^+\pi^-$, 4π , ... virtual photon only (I=1)

$f = 3\pi$, $K\bar{K}$, $K\bar{K}\pi$, ... 3 gluon intermediate state (I=0)

III NUCLEON FORM FACTORS

(with Czyż, Nowak, Rodrigo, hep-ph/0403062)

$Q^2 \gtrsim 4m_N^2$ accessible at B-factories

$\Rightarrow$ study $e^+e^- \rightarrow \gamma N\bar{N}$ (with $N = p$ or n)

hadronic current:

$$J_\mu = -ie \cdot \bar{u}(q_2) \left(F_1^N(Q^2)\,\gamma_\mu - \frac{F_2^N(Q^2)}{4m_N}\,[\gamma_\mu, \not{Q}] \right) v(q_1)\,,$$

$$Q = q_1 + q_2\,, \quad q = (q_1 - q_2)/2$$

or

$$G_M = F_1 + F_2\,, \quad G_E = F_1 + \frac{Q^2}{4m^2}\,F_2$$

Result:

$$d\sigma = \frac{1}{2s} L_{\mu\nu} H^{\mu\nu} \, d\Phi_2(p_1 + p_2; Q, k) \, d\Phi_2(Q; q_1, q_2) \, \frac{dQ^2}{2\pi} \, ,$$

$$L_{\mu\nu} H^{\mu\nu} = \frac{(4\pi\alpha)^3}{Q^2} \Bigg\{ \left(|G_M^N|^2 - \frac{1}{\tau} |G_E^N|^2 \right) \times \frac{32 s}{\beta_N^2 (s - Q^2)} \left(\frac{1}{y_1} + \frac{1}{y_2} \right) \left(\frac{(p_1 \cdot q)^2 + (p_2 \cdot q)^2}{s^2} \right) + 2 \left(|G_M^N|^2 + \frac{1}{\tau} |G_E^N|^2 \right) \left[\left(\frac{1}{y_1} + \frac{1}{y_2} \right) \frac{(s^2 + Q^4)}{s(s - Q^2)} - 2 \right] \Bigg\} \, ,$$

where

$$y_{1,2} = \frac{s - Q^2}{2s} (1 \mp \cos\theta_\gamma) \, , \quad \tau = \frac{Q^2}{4 m_N^2} \, , \quad \beta_N^2 = 1 - \frac{4 m_N^2}{Q^2}$$

Separation of $|G_M|^2$ and $|G_E|^2$ through angular distribution:

$$L_{\mu\nu}H^{\mu\nu} = \frac{(4\pi\alpha)^3}{Q^2}\frac{(1+\cos^2\theta_\gamma)}{(1-\cos^2\theta_\gamma)} \times 4\left(|G_M^N|^2\,(1+\cos^2\hat{\theta}) + \frac{1}{\tau}|G_E^N|^2\,\sin^2\hat{\theta}\right)$$

$\hat{\theta}$ = angle of nucleon with respect to γ-direction in hadronic rest frame

(valid for $s/Q^2 \ll 1$, corrections and "optimal frame" $\longrightarrow$ hep-ph/0403062

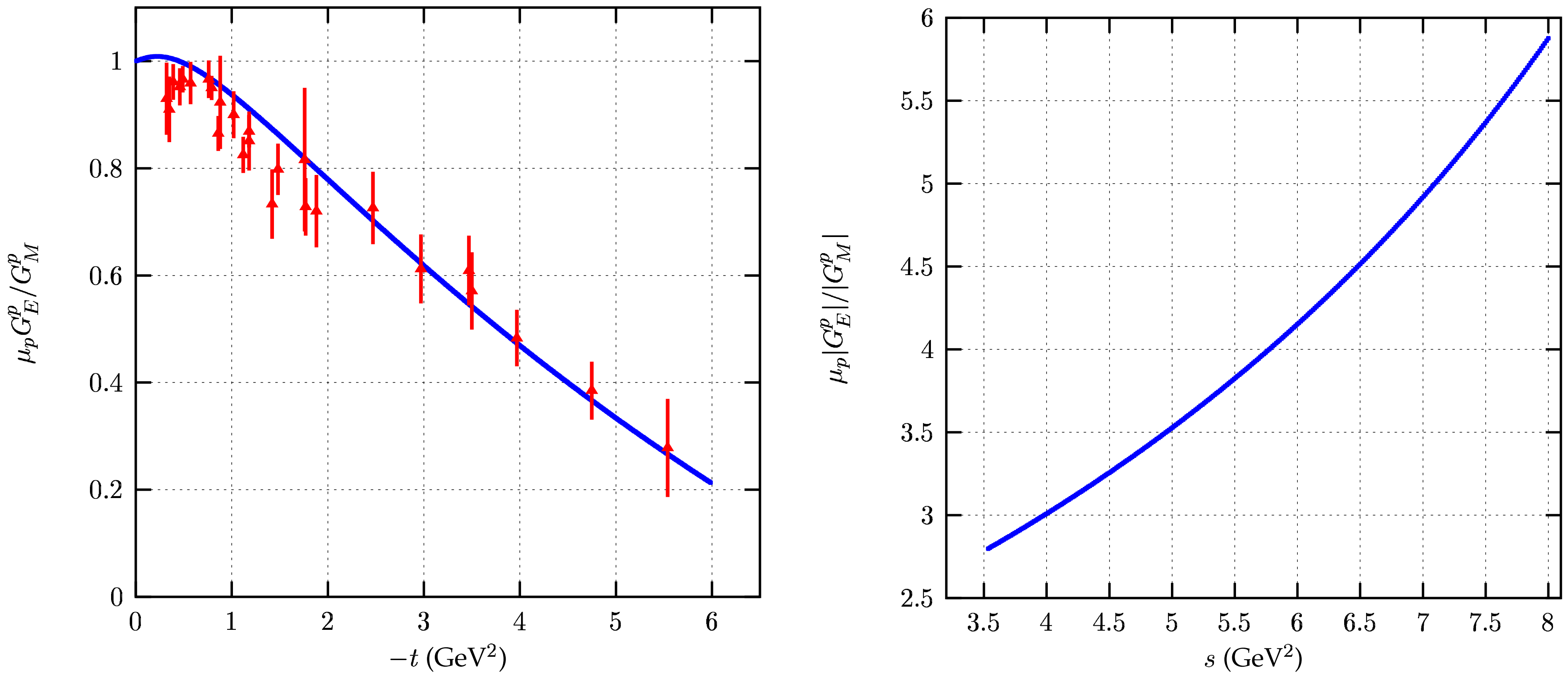
$\Rightarrow$ additional rotation by $\theta_D = \frac{1}{2}\arctan\left(\frac{2s_\gamma c_\gamma}{\gamma\,(\beta^2 + c_\gamma^2 - s_\gamma^2/\gamma^2)}\right) \approx \frac{1}{\gamma}\frac{s_\gamma c_\gamma}{1+c_\gamma^2}$

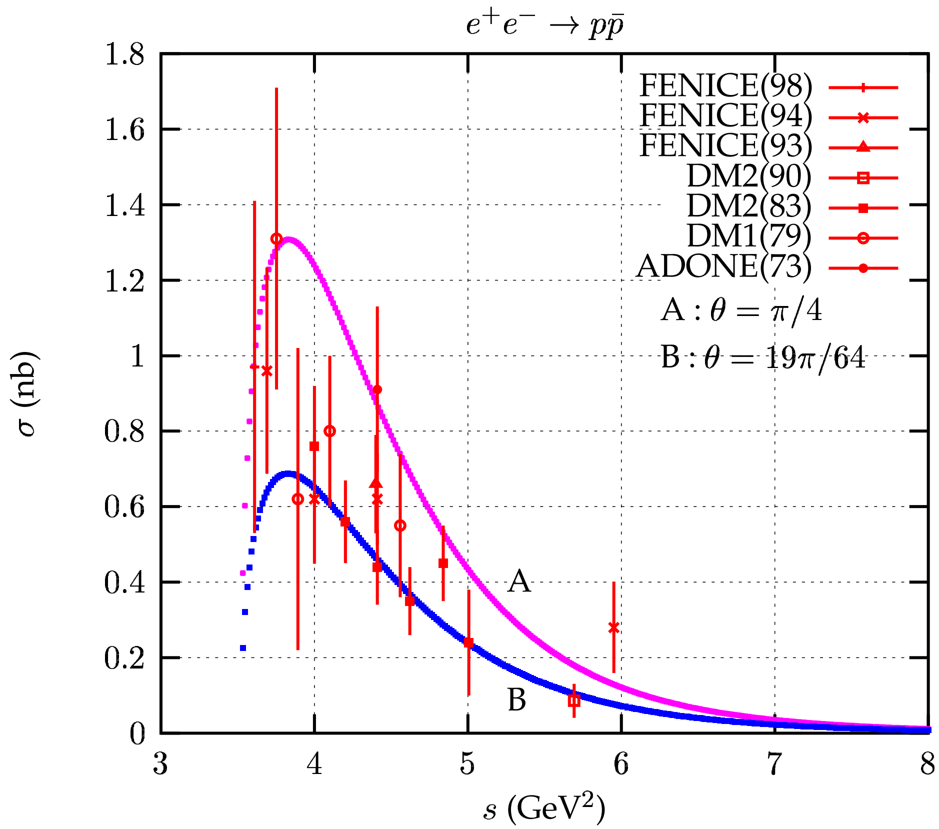
with $s_\gamma = \sin\theta_\gamma$, $\beta = (s - Q^2)/(s + Q^2)$, $\gamma = (s + Q^2)/2\sqrt{sQ^2}$)

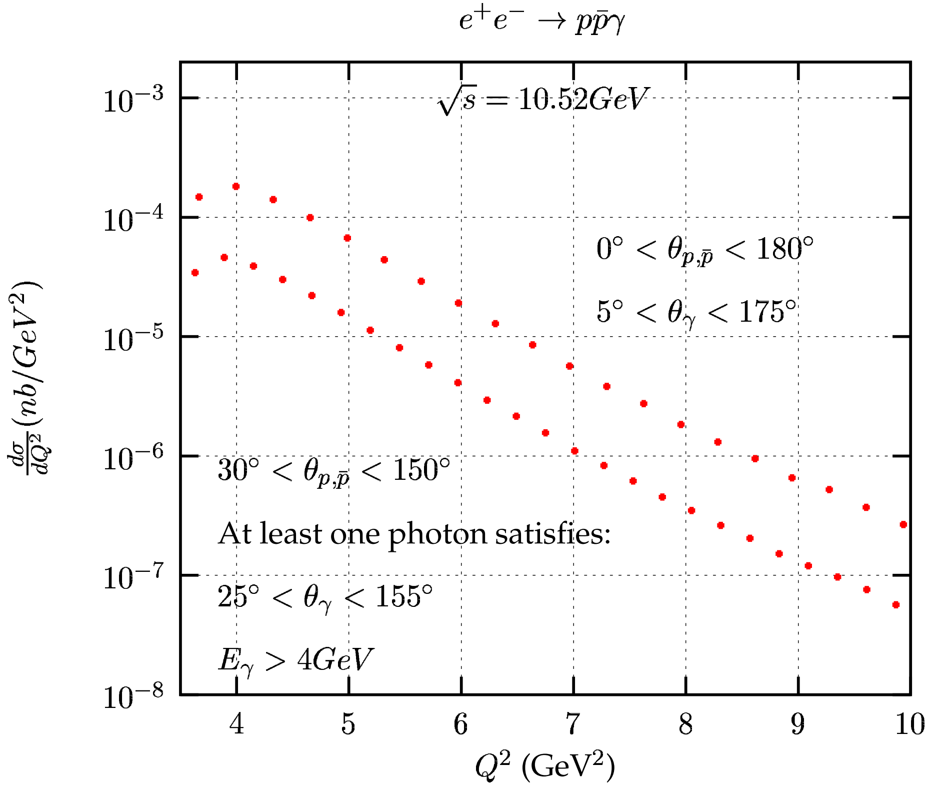
Similarity to $e^+e^- \longrightarrow N\bar{N}$:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta_N}{4Q^2}\left(|G_M^N|^2\,(1+\cos^2\theta) + \frac{1}{\tau}|G_E^N|^2\,\sin^2\theta\right)$$

Implementation on basis of model for form factor:





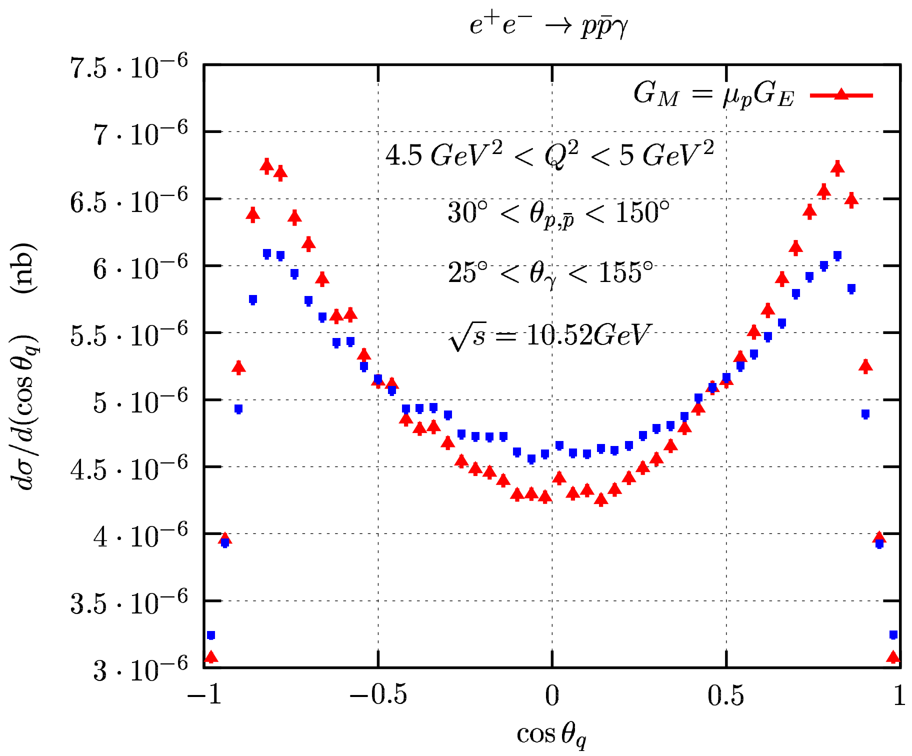


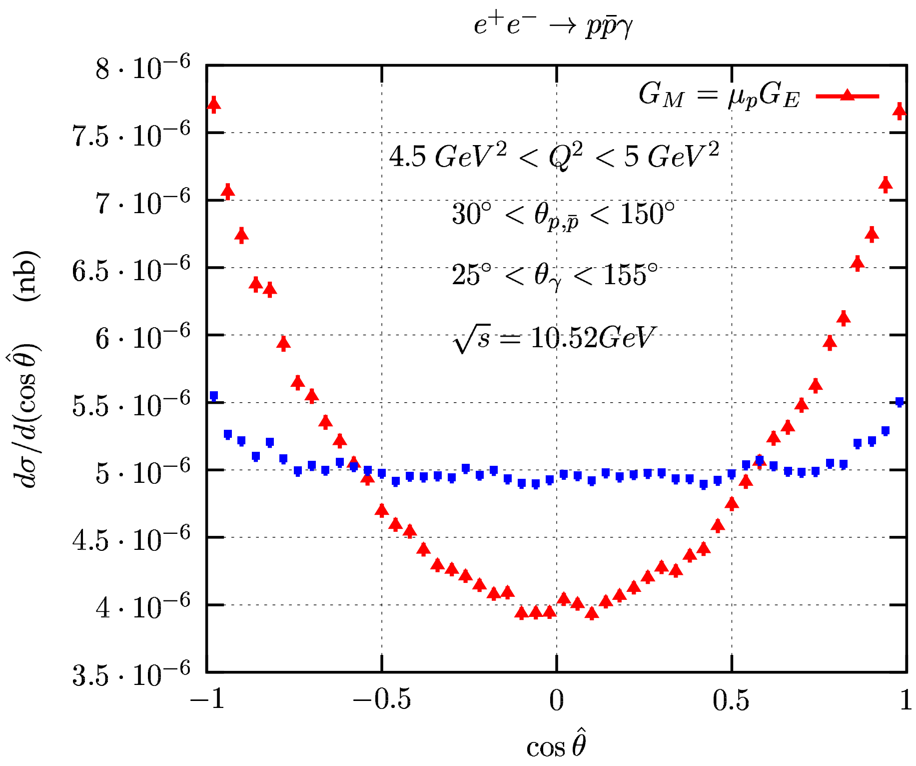
$$e^+e^- \to p\bar{p}$$

$$e^+e^- \to p\bar{p}\gamma$$

implementation in PHOKHARA

Angular distributions of nucleon





lab frame

hadronic rest frame

(two choices for G_M/G_E)

Comments

- **similar results for neutron pair production**
- **NLO corrections from ISR included** (corrections $\sim$ 1–2%)
- **no FSR**

thousands of events around 4–5 GeV^2
several events up to 7–8 GeV^2

IV MESON FORM FACTORS at LARGE Q^2

(with Bruch, Khodjamirian, hep-ph/0409080)

radiative return will explore large Q^2

convenient representation for F_π :

generalized VDM with ρ, ρ', ...

combined with Veneziano-type tower of resonances (Dominguez)

$$F_\pi(s) = \sum_{n=0}^{\infty} c_n \frac{m_n^2}{m_n^2 - s},$$

$$c_n = \frac{(-1)^n \Gamma(\beta - 1/2)}{\sqrt{\pi}(\frac{1}{2} + n)\Gamma(n+1)\Gamma(\beta - 1 - n)},$$

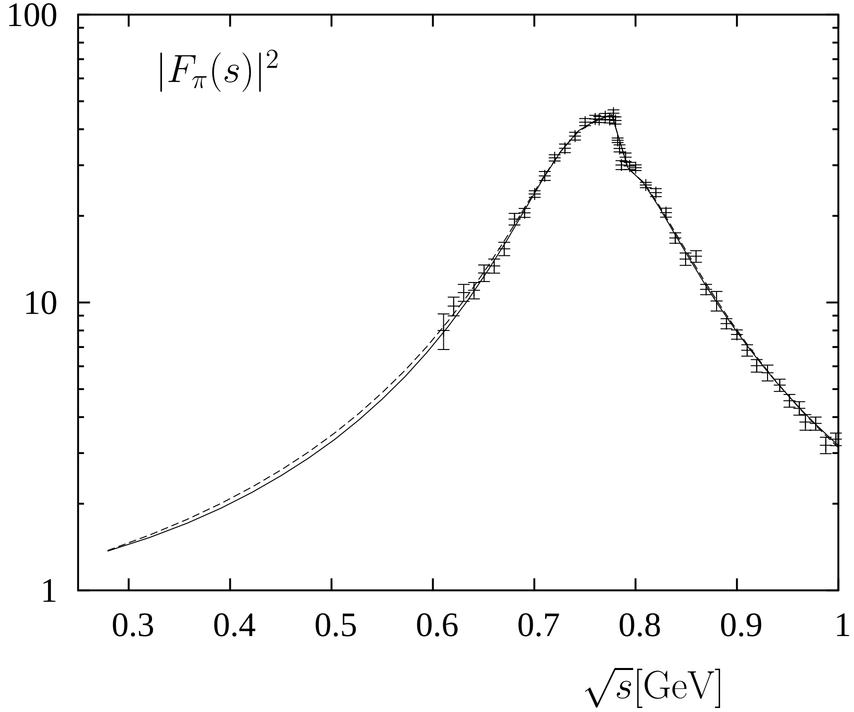
$$m_n^2 = m_\rho^2(1 + 2n),$$

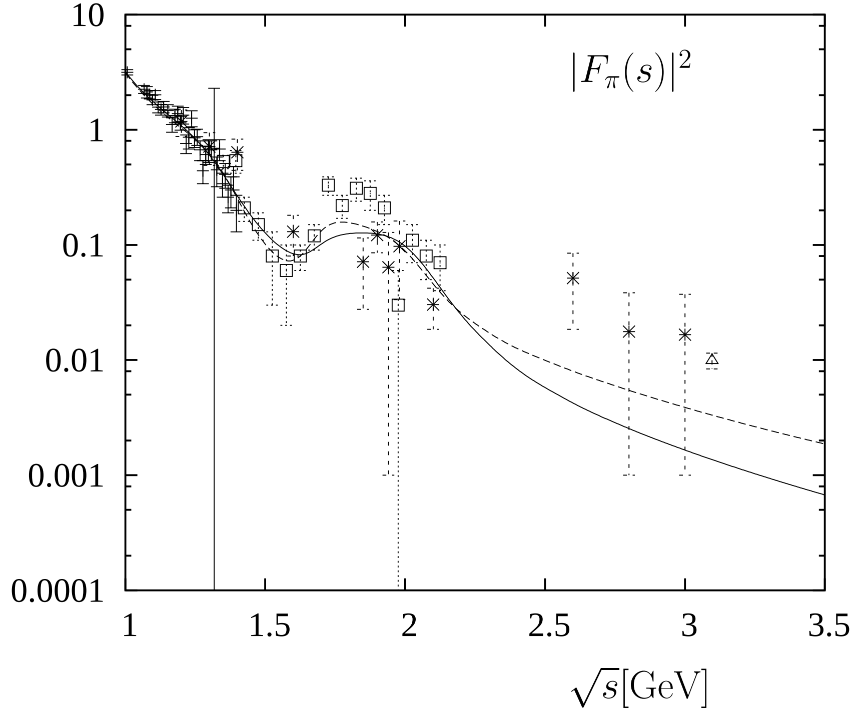
$$\beta = \text{free parameter}$$

Modifications:

- finite widths
- parameters of ρ, ρ', ρ'' fitted to data
- Breit-Wigner for ρ, ρ', ρ'' with Q^2-dependent widths
  $\Rightarrow$ reasonable agreement between model and fit

Parameter	Input	Fit(KS)	Fit(GS)	dual-QCD$_{N_c=\infty}$	PDG value
m_ρ	-	773.9 ± 0.6	776.3 ± 0.6	input	775.5 ± 0.5
Γ_ρ	-	144.9 ± 1.0	150.5 ± 1.0	input	150.3 ± 1.6
m_ω	783.0	-	-	-	782.59 ± 0.11
Γ_ω	8.4	-	-	-	8.49 ± 0.08
$m_{\rho'}$	-	1357 ± 18	1380 ± 18	1335	1465 ± 25
$\Gamma_{\rho'}$	-	437 ± 60	340 ± 53	266	400 ± 60
$m_{\rho''}$	1700	-	-	1724	1720 ± 20
$\Gamma_{\rho''}$	240	-	-	344	250 ± 100
$m_{\rho'''}$	-	-	-	2040	-
$\Gamma_{\rho'''}$	-	-	-	400	-
c_0	-	1.171 ± 0.007	1.098 ± 0.005	1.171	-
β	c_0	2.30 ± 0.01	2.16 ± 0.015	2.3(input)	-
c_ω	0.00184(KS) 0.00195(GS)	-	-	-	- -
c_1	-	-0.119 ± 0.011	-0.069 ± 0.009	-0.1171	-
c_2	-	0.0115 ± 0.0064	0.0216 ± 0.0064	-0.0246	
c_3	$\sum c_n=1$	-0.0438 ∓ 0.02	-0.0309 ∓ 0.02	-0.00995	-
$\sum_{n=4}^{\infty} c_n$	-0.01936	-	-	-0.01936	-
$\chi^2/d.o.f.$	-	155/101	153/101	-	-

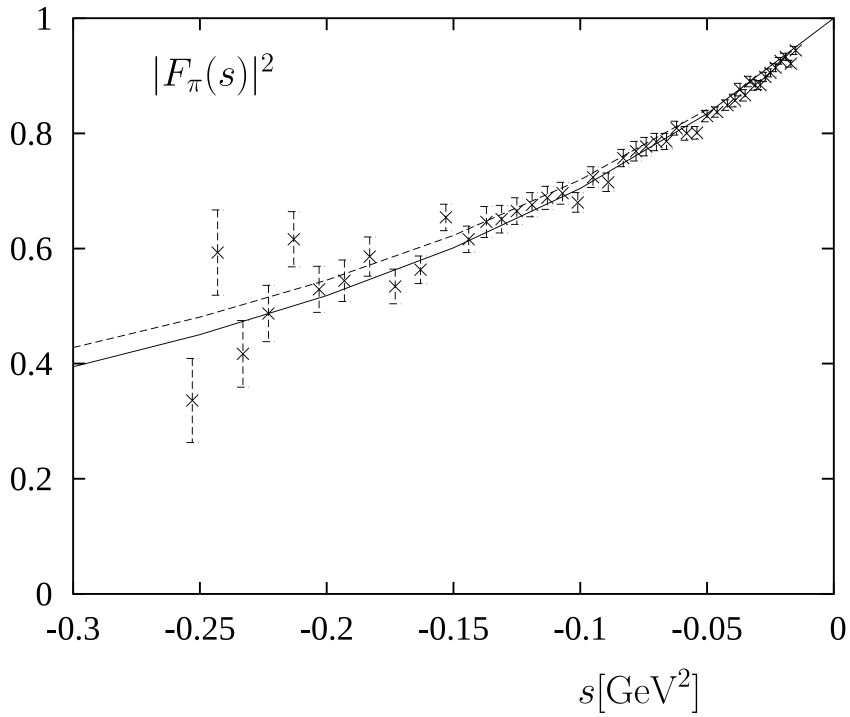


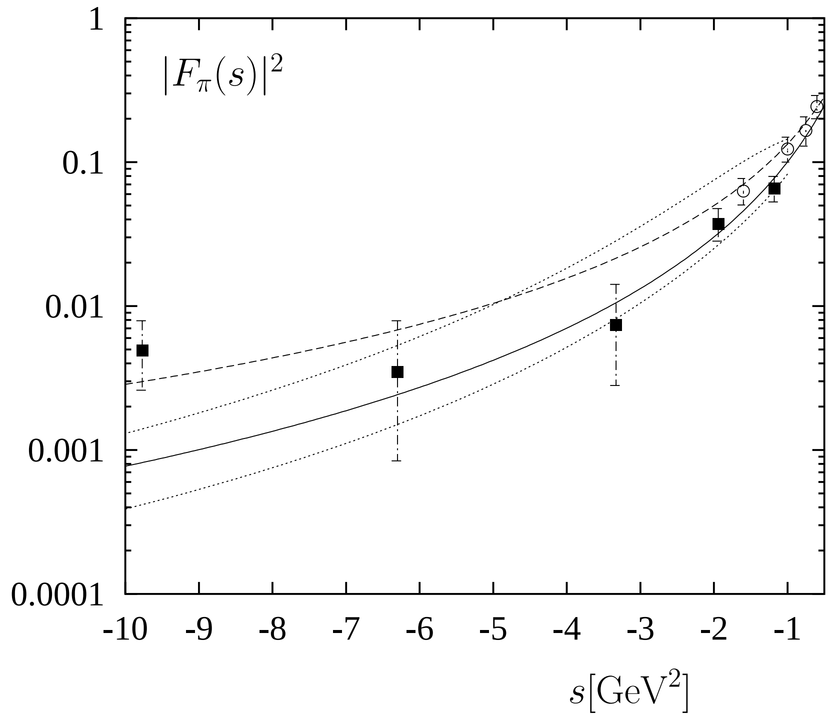


data point at 3.1 GeV ($J/\Psi \longrightarrow \pi\pi$) cannot be accomodated

spacelike region:

good agreement with data and with sum rules





$e^+e^- \to K^+K^-\,,\ K^0\bar{K}^0$

isospin symmetry:

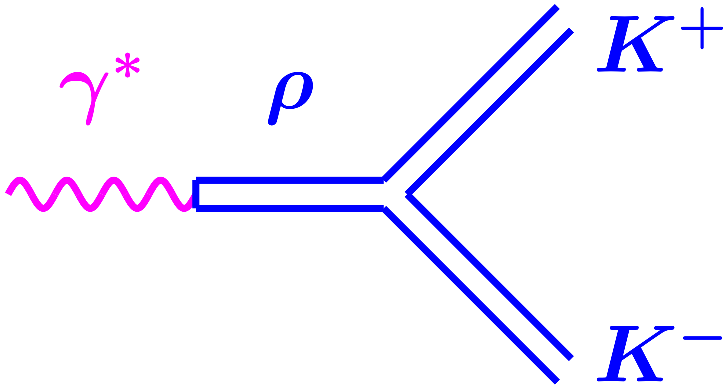
$$F_{K^+} = +F^{(I=1)} + F^{(I=0)}$$
$$F_{K^0} = -F^{(I=1)} + F^{(I=0)}$$

resonances:

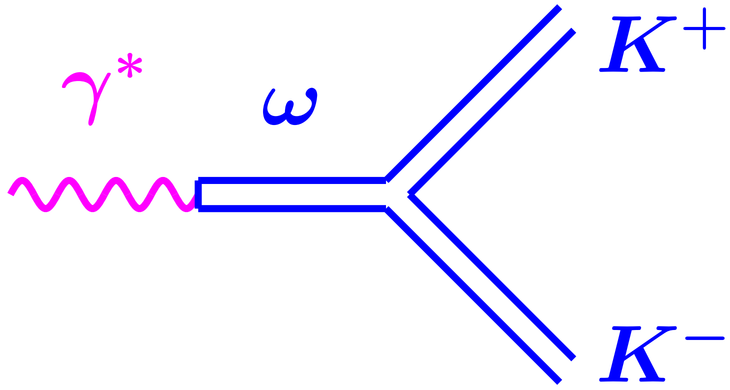
$$\begin{aligned} F_{K^+}(s) = &+\frac{1}{2}\Big(c^K_\rho BW_\rho(s) + c^K_{\rho'} BW_{\rho'}(s) + c^K_{\rho''} BW_{\rho''}(s)\Big) \\ &+ \frac{1}{6}\Big(c^K_\omega BW_\omega(s) + c^K_{\omega'} BW_{\omega'}(s) + c^K_{\omega''} BW_{\omega''}(s)) \\ &+ \frac{1}{3}\Big(c_\phi BW_\phi(s) + c_{\phi'} BW_{\phi'}(s)\Big)\,, \end{aligned}$$

$$\begin{aligned} F_{K^0}(s) = &-\frac{1}{2}\Big(c^K_\rho BW_\rho(s) + c^K_{\rho'} BW_{\rho'}(s) + c^K_{\rho''} BW_{\rho''}(s)\Big) \\ &+ \frac{1}{6}\Big(c^K_\omega BW_\omega(s) + c^K_{\omega'} BW_{\omega'}(s) + c^K_{\omega''} BW_{\omega''}(s)\Big) \\ &+ \frac{1}{3}\Big(\eta_\phi c_\phi BW_\phi(s) + c_{\phi'} BW_{\phi'}(s)\Big) \end{aligned}$$

quark model:



$\frac{1}{\sqrt{2}} f_\rho \quad g_{\rho KK}$



$\frac{1}{3\sqrt{2}} f_\omega \quad g_{\omega KK}$

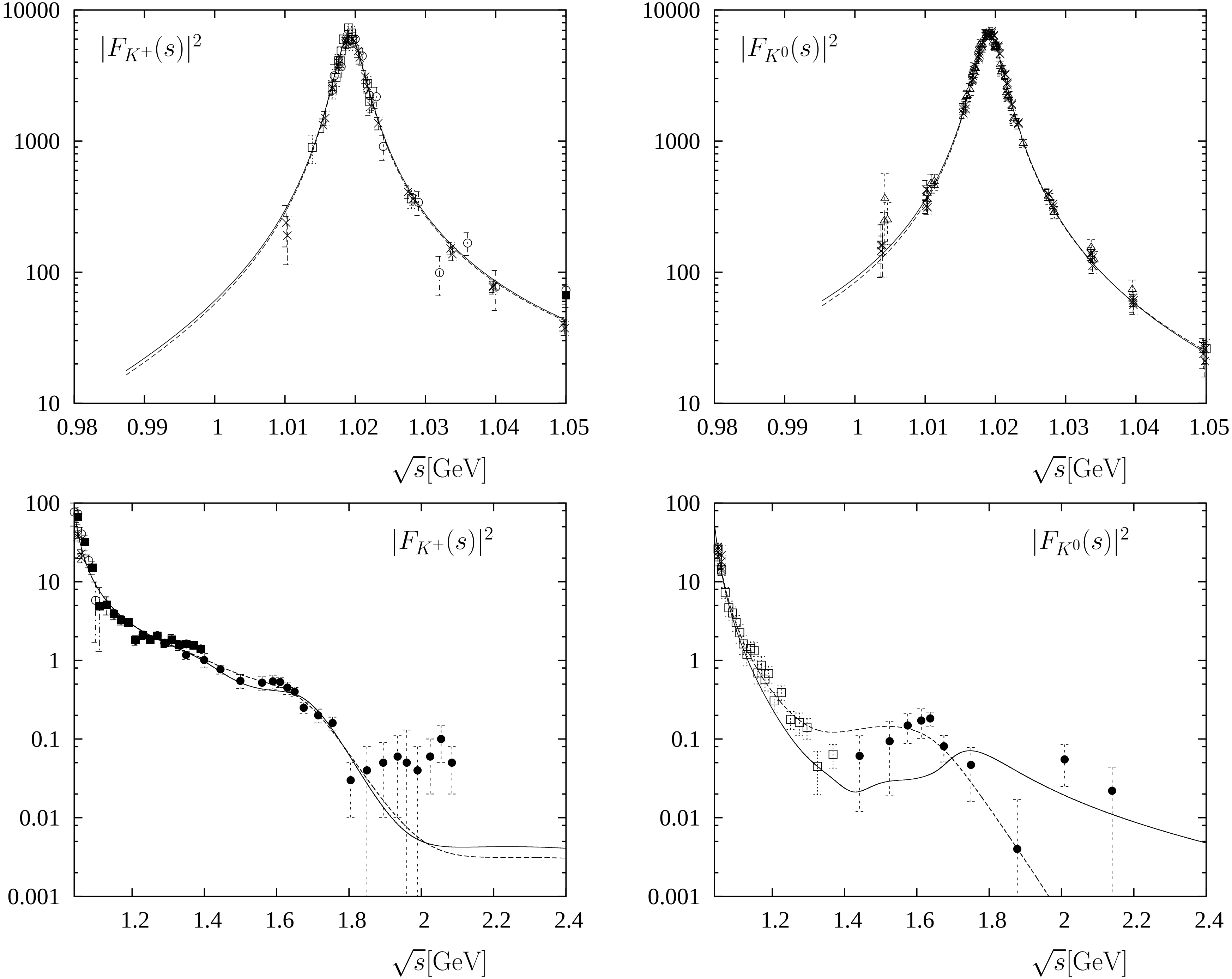
constraint: $f_\rho = f_\omega \, , \quad g_{\rho KK} = g_{\omega KK}$

$\Rightarrow c_\rho = c_\omega$

fit performed with **(solid curves)**
or without **(dashed curves)** this constraint

Results:

Parameter	Input	Fit(1)	Fit(2)	PDG value
m_ϕ	-	1019.372 ± 0.02	1019.355 ± 0.02	1019.456 ± 0.02
Γ_ϕ	-	4.36 ± 0.05	4.29 ± 0.05	4.26 ± 0.05
$m_{\phi'}$	1680	-	-	1680 ± 20
$\Gamma_{\phi'}$	150	-	-	150 ± 50
m_ρ	775	-	-	775.8 ± 0.5
Γ_ρ	150	-	-	150.3 ± 1.6
$m_{\rho'}$	1465	-	-	1465 ± 25
$\Gamma_{\rho'}$	400	-	-	400 ± 60
$m_{\rho''}$	1720	-	-	1720 ± 20
$\Gamma_{\rho''}$	250	-	-	250 ± 100
m_ω	783.0	-	-	782.59 ± 0.11
Γ_ω	8.4	-	-	8.49 ± 0.08
$m_{\omega'}$	1425	-	-	1400-1450
$\Gamma_{\omega'}$	215	-	-	180-250
$m_{\omega''}$	1670	-	-	1670 ± 30
$\Gamma_{\omega''}$	315	-	-	315 ± 35
c_ϕ	-	1.018 ± 0.006	0.999 ± 0.007	-
$c_{\phi'}$	$1 - c_\phi^K$	-0.018 ∓ 0.006	0.001 ∓ 0.007	-
c_ρ^K	-	1.195 ± 0.009	1.139 ± 0.010	-
$c_{\rho'}^K$	-	-0.112 ± 0.010	-0.124 ± 0.012	-
$c_{\rho''}^K$	$1 - c_\rho^K - c_{\rho'}^K$	-0.083 ∓ 0.019	-0.015 ∓ 0.022	-
$c_\omega^K(1)$	c_ρ^K	1.195 ± 0.009	-	-
$c_\omega^K(2)$	-	-	1.467 ± 0.035	-
$c_{\omega'}^K(1)$	$c_{\rho'}^K$	-0.112 ± 0.010	-	-
$c_{\omega'}^K(2)$	-	-	-0.018 ± 0.024	-
$c_{\omega''}^K$	$1 - c_\omega^K - c_{\omega'}^K$	-0.083 ∓ 0.019	-0.449 ∓ 0.059	-
$\chi^2/d.o.f.$	-	328/242	281/240	-

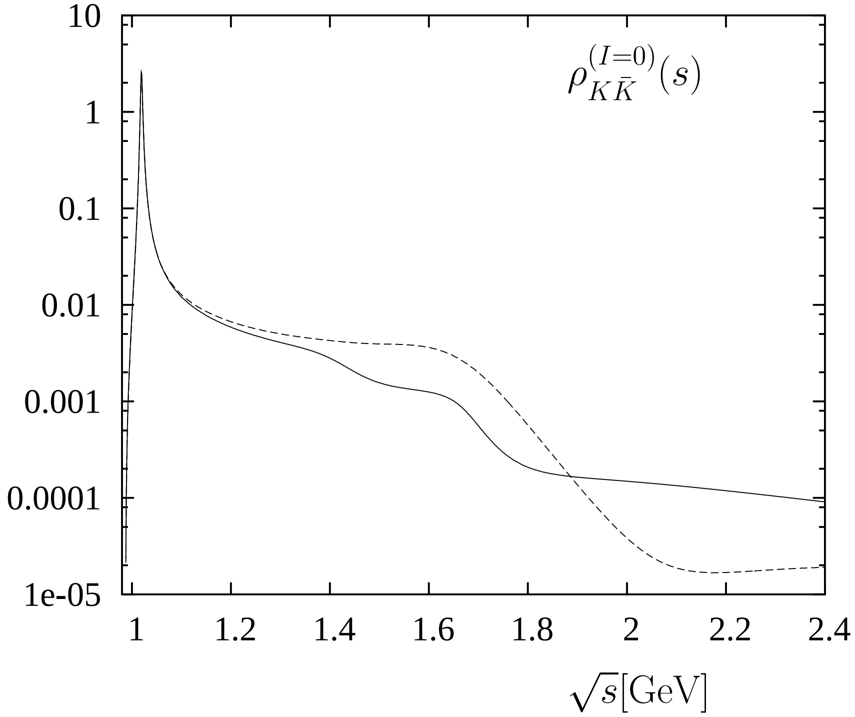


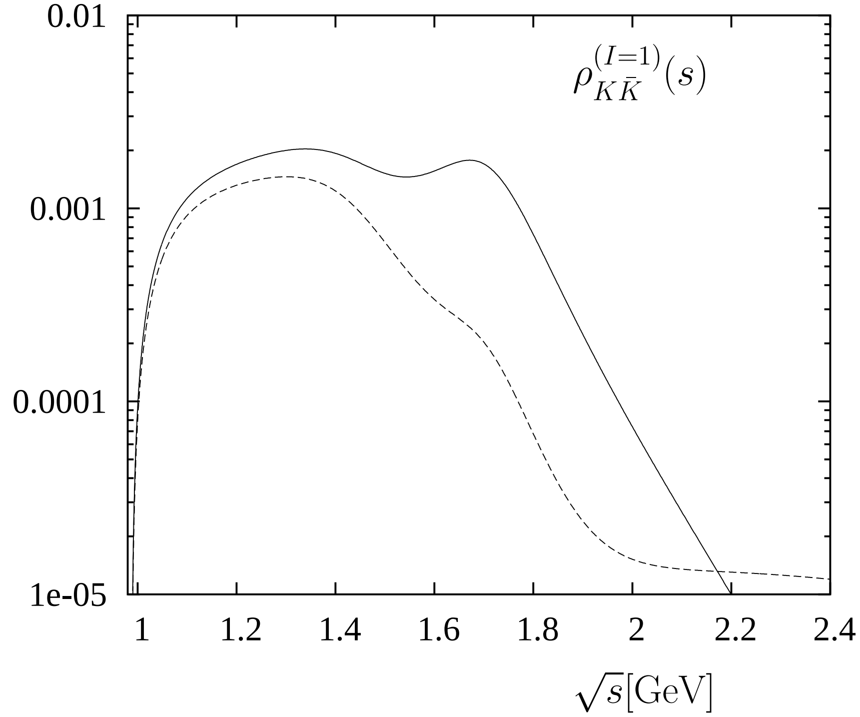
|F_{K+}(s)|²
|F_{K0}(s)|²
10000
1000
100
10
0.98
0.99
1
1.01
1.02
1.03
1.04
1.05
√s[GeV]
100
10
1
0.1
0.01
0.001
1.2
1.4
1.6
1.8
2
2.2
2.4

Spectral function separated for $I = 0$ and $I = 1$

(useful for electroweak analysis!)

$$\rho_{K\bar{K}}^{(I=0,1)}(s) = \frac{1}{12\pi} \left| \frac{F_{K^+}(s) \pm F_{K^0}(s)}{2} \right|^2 \left(\frac{2p_K(s)}{\sqrt{s}} \right)^3$$





significant model dependence above 1.5 GeV (poor data for $|F_{K^0}|^2$!)

$\tau \to K^- K^0 \nu$

Predictions based on isospin symmetry and $I = 1$ part of form factor:

$$\left(\frac{1}{BR(\tau \to \mu^- \bar{\nu}_\mu \nu_\tau)}\right) \frac{dBR(\tau \to K^- K^0 \nu_\tau)}{d\sqrt{Q^2}} = \frac{|V_{ud}|^2}{2m_\tau^2} \left(1 + \frac{2Q^2}{m_\tau^2}\right) \left(1 - \frac{Q^2}{m_\tau^2}\right)^2 \left(1 - \frac{4m_K^2}{Q^2}\right)^{3/2} \times \sqrt{Q^2}\, |F_{K^- K^0}(Q^2)|^2$$

and $F_{K^- K^0} = -F_{K^+} + F_{K^0}$

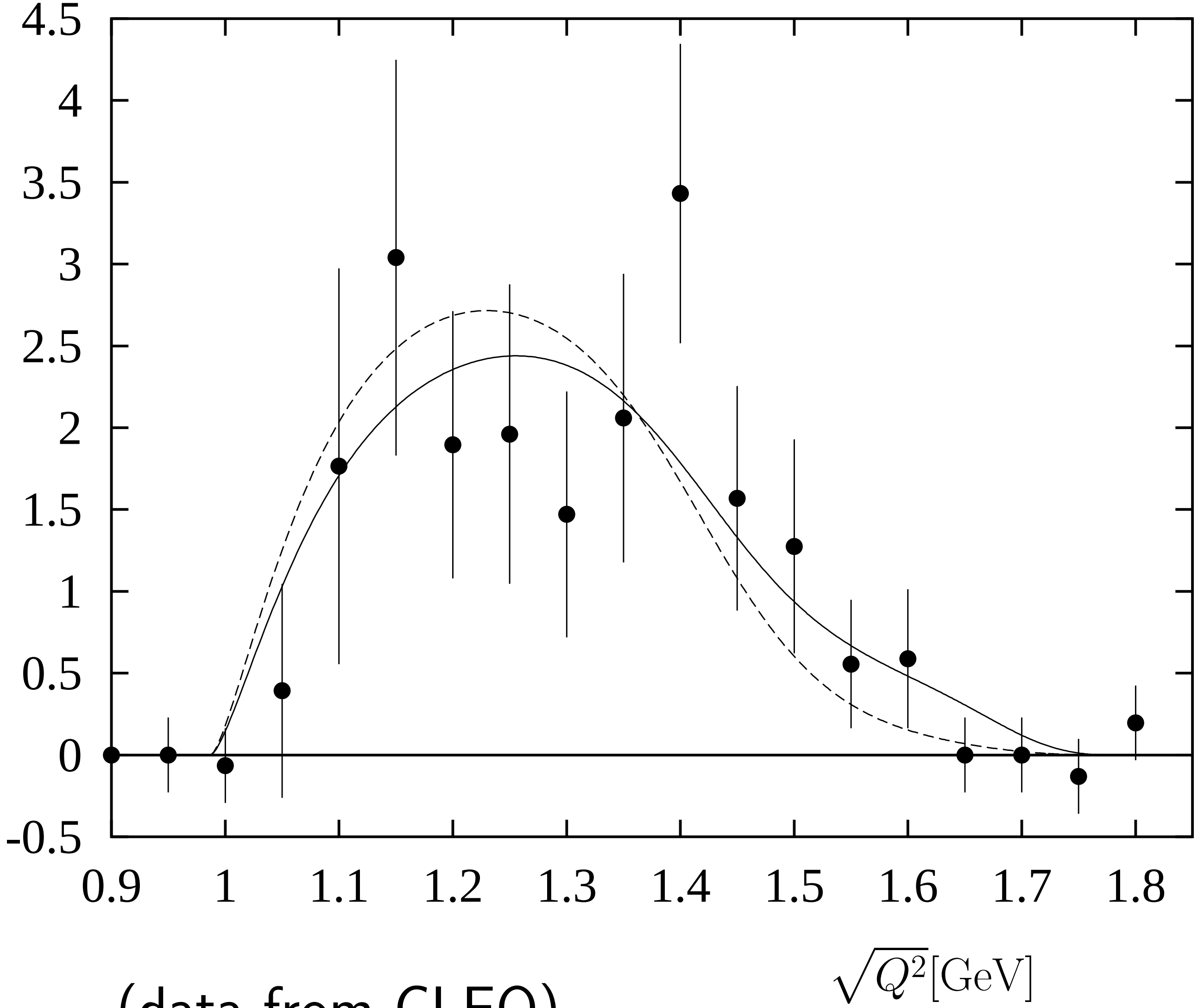
$\Rightarrow BR(\tau \to K^- K^0 \nu_\tau) = 0.19 \pm 0.01\% \quad (0.13 \pm 0.01\%)$

to be compared with

$$BR(\tau \to K^- K^0 \nu_\tau) = 0.154 \pm 0.016\%.$$

Q^2 distribution:

will provide further constraints!



(data from CLEO)

V Conclusions

- **continuous development of PHOKHARA**
  $\Rightarrow$ radiative corrections
  $\Rightarrow$ more channels
  $\Rightarrow$ cooperation between theory and experiment crucial

- **nucleon form factors:**
  G_E and G_M can be measured for a wide range of Q^2

- **pion form factor:** structures at large Q^2

  kaon form factors: K^+K^- & $K^0\bar{K}^0 \Rightarrow K^-K^0$
  $\Rightarrow$ **prediction for** $\tau \to \nu K^- K^0$