LEPTON FLAVOR VIOLATION IN THE MINIMAL SO(10) GUT MODEL AND IN THE STANDARD MODEL WITH ADDITIONAL HEAVY DIRAC NEUTRINOS

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1. Motivation

a. Neutrino masses and mixings – experimental status

SuperK, K2K, SNO and Kamland experiments (ν_{2004}) :

$$\begin{aligned}\Delta m^2_{23} &= 2.1\times 10^{-3} eV^2, \Delta m^2_{12} \;=\; 8.3\times 10^{-5} eV^2,\\ \sin^2\theta_{23} &= 0.5, \sin^2\theta_{12} \;=\; 0.28, \sin^2\theta_{13} \;\leq 0.061\end{aligned}$$

$*$ first evidence of physics beyond SM.
$*$ bimaximal structure of MNS, experiments are expected to determine θ_{13} and CP violating phase in the lepton sector.
$*$ clear sign that the lepton flavor is not conserved; LFV is expected to be found in the charge leptons sector (neutrinoless LFV decays)

b. Improvement of the experimental upper bounds of the LFV violation decay rates.

2. Aims

∗ Presentation of the results of semileptonic LFV tau-lepton processes derived in MSSM with see-saw mechanism evaluated, and comparison with previous results derived in SM with heavy Dirac neutrinos.

∗ The structure of the neutrinoless LFV amplitudes.

∗ The best decays for finding LFV

3. MODELS

a. Particle content :

SM, $SU(3)_c \times SU(2)_L \times U(1)_Y$:

$$g,\, A,\, Z,\, W^\pm;\; H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+G^0) \end{pmatrix};\; \begin{pmatrix} u_L \\ V_L^{CKM} d_L \end{pmatrix},\, u_R,\, d_R;\; \begin{pmatrix} \nu_L \\ l_L \end{pmatrix},\, l_R$$

SM with additional heavy Dirac neutrinos (model A), [Valle et al. PRD34 (1986) 1642, PLB187 (1987) 303]

$$g,\, A,\, Z,\, W^\pm\boldsymbol{;}\; H\boldsymbol{;}\; \begin{pmatrix} u_L \\ V_L^{CKM} d_L \end{pmatrix},\, u_R,\, d_R\boldsymbol{;}\; \begin{pmatrix} K_L^{\dagger\nu}\nu_L + K_L^{\dagger N}\boldsymbol{N_L} \equiv K_L^\dagger n_L \\ l_L \end{pmatrix}\boldsymbol{,}\; \boldsymbol{N_R}\boldsymbol{,}\; l_R$$

MSSM with see-saw mechanism (model B) :

$$g,\, A,\, Z,\, W^\pm\boldsymbol{;}\; H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+G^0) \end{pmatrix}\boldsymbol{;}\; \begin{pmatrix} u_L \\ V_L^{CKM} d_L \end{pmatrix},\, u_R,\, d_R\boldsymbol{;}\; \begin{pmatrix} \nu_L \\ l_L \end{pmatrix},\, l_R,\, \boldsymbol{\nu_L}$$

$$\boldsymbol{\tilde{g}};\quad (\boldsymbol{\tilde{\chi}_1^0}, \boldsymbol{\tilde{\chi}_2^0}, \boldsymbol{\tilde{\chi}_3^0}, \boldsymbol{\tilde{\chi}_4^0})^\top = (\boldsymbol{\tilde{B}_L}, \boldsymbol{\tilde{W}_L^0}, \boldsymbol{\tilde{H}_{1L}^0}, \boldsymbol{\tilde{H}_{2L}^0})^\top\; O_N^\top,$$

$$(\boldsymbol{\tilde{\chi}_{1L}^-}, \boldsymbol{\tilde{\chi}_{2L}^-})^{\;\top} O_L^\top,\quad (\boldsymbol{\tilde{\chi}_{1L}^+}, \boldsymbol{\tilde{\chi}_{2L}^+})^{\;\top} O_R^\dagger$$

$$\begin{pmatrix} \boldsymbol{\tilde{u}_X} = U_{Xi}^u \boldsymbol{\tilde{u}_{Li}} + U_{Xi+3}^u \boldsymbol{\tilde{u}_{Ri}} \\ \boldsymbol{\tilde{d}_X} = U_{Xi}^d \boldsymbol{\tilde{d}_{Li}} + U_{Xi+3}^d \boldsymbol{\tilde{d}_{Ri}} \end{pmatrix};\; \begin{pmatrix} \boldsymbol{\tilde{\nu}_X} = U_{Xi}^\nu \boldsymbol{\tilde{\nu}_{Li}} + U_{Xi+3}^\nu \boldsymbol{\tilde{N}_{Ri}} \\ \boldsymbol{\tilde{l}_X} = U_{Xi}^l \boldsymbol{\tilde{l}_{Li}} + U_{Xi+3}^l \boldsymbol{\tilde{l}_{Ri}} \end{pmatrix}$$

b. Sources of LFV

Model A – LFV vertices

∘ LFV interactions:

$\bar{l}nW^-$, $\overline{n}lW^+$, $\overline{n}nZ$, $\bar{l}nG^-$, $\overline{n}lG^+$, $\overline{n}nG^0$, $\overline{n}nH$

∘ description of LFV: $\boldsymbol{K_L}$ matrix

$$\mathcal{L}^l_{CC} \; = \; \frac{g}{2\sqrt{2}} \, (\bar{l}\gamma^\mu(1-\gamma_5)\boldsymbol{K_L}^\dagger n \; W^- + h.c.)$$

∘ leading-order contributions to neutrinoless LFV processes : 1-loop diagrams (except W^+W^-)

∘ GIM mechanism: $K_L^\dagger K_L \; = \; 1$

∘ Notation: $\Omega_L \; = \; K_L K_L^\dagger$

Model B

∘ LFV interactions:

$\overline{\nu}\tilde{\chi}^{+}\tilde{e}$, $\overline{\tilde{\chi}^{+}}\nu\tilde{e}^{\dagger}$, $\overline{e}\tilde{\chi}^{-}\tilde{\nu}$, $\overline{\tilde{\chi}^{-}}e\tilde{\nu}^{\dagger}$, $\overline{\nu}\tilde{\chi}^{0}\tilde{\nu}$, $\overline{\tilde{\chi}^{0}}\nu\tilde{\nu}^{\dagger}$, $\overline{e}\tilde{\chi}^{0}\tilde{e}$, $\overline{\tilde{\chi}^{0}}e\tilde{e}^{\dagger}$,

$\tilde{e}^{\dagger}\tilde{e}Z$, $\tilde{\nu}^{\dagger}\tilde{\nu}Z$

∘ description of LFV:

$$
\begin{aligned}
&\bar{\nu}_i \boldsymbol{C^{R(\nu)}_{iAX}} P_R \widetilde{\chi}^{+}_{A}\tilde{e}_X + \bar{e}_i\left[\boldsymbol{C^{L(e)}_{iAX}} P_L + \boldsymbol{C^{R(e)}_{iAX}} P_R\right]\widetilde{\chi}^{-}_{A}\tilde{\nu}_X \\
&+\bar{\nu}_i \boldsymbol{N^{R(\nu)}_{iAX}} P_R \widetilde{\chi}^{0}_{A}\tilde{\nu}_X + \bar{e}_i\left[\boldsymbol{N^{L(e)}_{iAX}} P_L + \boldsymbol{N^{R(e)}_{iAX}} P_R\right]\widetilde{\chi}^{0}_{A}\tilde{e}_X + h.c., \\
&-g\overline{\tilde{\chi}^{-}_{A}}\not{Z}[(\boldsymbol{E^{L(\chi^{-})}_{AB}})P_L + \boldsymbol{E^{R(\chi^{-})}_{AB}})P_R)]\tilde{\chi}^{-}_{B}, \\
&-g\overline{\tilde{\chi}^{0}_{A}}\not{Z}[(\boldsymbol{E^{L(\chi^{0})}_{AB}})P_L + \boldsymbol{E^{R(\chi^{0})}_{AB}})P_R)]\tilde{\chi}^{0}_{B}, \\
&-g\overline{\tilde{f}^{*}_{X}} i \overleftrightarrow{\partial}_{\mu} \tilde{f}_Y Z_\mu[\boldsymbol{D^{\tilde{f}}_{XY}}]
\end{aligned}
$$

∘ leading-order contributions to neutrinoless LFV processes : 1-loop diagrams

∘ GIM mechanism ($H = L, R$):

$$
\begin{aligned}
C^{H(e)}_{jAX}C^{H(e)*}_{iAX} &= C^{H(\nu)}_{jAX}C^{H(\nu)*}_{iAX} = \delta_{ij}, \\
C^{H(e)}_{jBX}E^{H(\chi^{-})}_{BA}C^{H(e)*}_{iAX} &= C^{H(e)}_{jBX}E^{H(\chi^{-})}_{BA}C^{H(e)*}_{iAX} = \delta_{ij} \\
C^{H(e)}_{jAY}D^{\tilde{f}}_{XY}C^{H(e)*}_{iAX} &= C^{H(\nu)}_{jAY}D^{\tilde{f}}_{XY}C^{H(\nu)*}_{iAX} = \delta_{ij}
\end{aligned}
$$

4. Methods and processes for finding deviation from SM induced by LFV

Model A

$*$ Tree-level processes diagonal in lepton flavor

– neutrino doublet contains light and heavy neutrino components

– deviations from the SM couplings (nonuniversality of CC couplings, deviation from unitarity of CKM matrix, change of the invisible width of Z boson, etc.)

– best and cleanest limits on the LFV parameters [Tommasini et al. PLB327(94)319] $\left((s_L^{\nu_l})^2 = \Sigma_i (K_L)^\dagger_{li}(K_L)_{il}\right)$

$$\begin{aligned} (s_L^{\nu_e})^2 &< 0.0071 \\ (s_L^{\nu_\mu})^2 &< 0.0014 \\ (s_L^{\nu_\tau})^2 &< 0.03(0.01) \end{aligned}$$

$*$ Processes nondiagonal in lepton flavor (LFV/LNV)

– clear signature for LFV

– no new limits on LFV parameters – limits only on some specific combinations of LFV parameters

$*$ Loop processes diagonal in lepton flavor

– limits only on some specific combinations of LFV parameters

Model B

$*$ LFV parameters are given in terms of off-diagonal components of the soft SUSY breaking parameters $\left(m^2_{\tilde{\ell}}\right)_{ij}$ and A_e^{ij}

$*$ loop effects
– only loop effect induce the deviation from the SM couplings (no SM neutrino mixing with particles beyond SM)
– flavor diagonal processes – μ magnetic moment – best limits on the LFV
– LFV processes – only $\mu \to e\gamma$ gives some constraints on the LFV parameters

$*$ dark matter constraint

5. Effective vertices of the LFV amplitudes written in terms of SM fields

General structure

In the following we present the terms of effective Lagrangians of the LFV vertices that give the leading terms in expansion in outer momenta and masses.

Effective Lagrangian for photon LFV vertex

$$\mathcal{L}^{\text{eff}}_{l_i l_j \gamma} = -e\,\bar{l}_j[(-\partial^2\gamma_\mu + \not{\partial}\partial_\mu)A^\mu(a_L P_L + a_R P_R) + \sigma_{\mu\nu}\partial^\nu A^\mu(b_L P_L + b_R P_R)]l_i$$

The structure of the vertex is a consequence of the gauge invariance of the photon LFV amplitude wich assures vanishing of the zeroth order in expansion in outer momenta and masses. First term contributes only in off-mass-shell processes. All four form factors appear in all LFV models.

Effective Lagrangian for Z-boson LFV vertex

$$\mathcal{L}^{\text{eff}}_{l_i l_j Z} = g\bar{l}_j[\gamma_\mu Z^\mu(F_Z^L P_L + F_Z^R P_R)]l_i$$

The Z-boson penguin amplitude contains zeroth order term in expansion in outer momenta. For that reason the Z-boson and photon amplitudes look so different. Both form factors can be found in all known models inducing LFV.

Effective Lagrangian for Higgs-boson LFV vertex

$$\mathcal{L}^{\mathrm{eff}}_{l_i l_j H} = g\bar{l}_j[F_H^L P_L + F_H^R P_R] l_i H$$

Effective Lagrangian for box $l_i \to l_j \overline{q}_a q_b$ LFV vertex

$$\begin{aligned}\mathcal{L}^{\mathrm{box}}_{\mathrm{eff}} = \sum_{\bar{q}_a q_b = \bar{u}u, \bar{d}d, \bar{s}s, \bar{d}s, \bar{s}d} \Big[& b^L_{1ab}(\bar{\ell}_j \gamma^\mu P_L \ell_i)(\bar{q}_a \gamma_\mu P_L q_b) + b^R_{1ab}(\bar{\ell}_j \gamma^\mu P_R \ell_i)(\bar{q}_a \gamma_\mu P_R q_b) \\ & + b^L_{2ab}(\bar{\ell}_j \gamma^\mu P_L \ell_i)(\bar{q}_a \gamma_\mu P_R q_b) + b^R_{2ab}(\bar{\ell}_j \gamma^\mu P_R \ell_i)(\bar{q}_a \gamma_\mu P_L q_b) \\ & + b^L_{3ab}(\bar{\ell}_j P_L \ell_i)(\bar{q}_a P_L q_b) + b^R_{3ab}(\bar{\ell}_j P_R \ell_i)(\bar{q}_a P_R q_b) \\ & + \bar{b}^L_{3ab}(\bar{\ell}_j P_L \ell_i)(\bar{q}_a P_R q_b) + \bar{b}^R_{3ab}(\bar{\ell}_j P_R \ell_i)(\bar{q}_a P_L q_b) \\ & + b^L_{4ab}(\bar{\ell}_j \sigma_{\mu\nu} P_L \ell_i)(\bar{q}_a \sigma^{\mu\nu} P_R q_b) + b^R_{4ab}(\bar{\ell}_j \sigma_{\mu\nu} P_R \ell_i)(\bar{q}_a \sigma^{\mu\nu} P_L q_b) \Big].\end{aligned}$$

All the above form factors may appear in the general box LFV vertex. For instance, all of them appear in the model B. In the model A only the first eight four form factors do contribute.

The effective Lagrangian for the leptonic LFV box amplitude $l_i \to l_j l_k l_l$ is similar in structure to the leptonic LFV box amplitude.

6. Semileptonic neutrinoless LFV processes

$*$ $l \to l'\gamma$ LFV processes are given also because one of them constraints the parameter space.

a. Classification of neutrinoless leptonic and semileptonic amplitudes

process	A-model diagrams	B-model diagrams
$l \to l'\gamma$	γ	γ
$\tau \to lP^0$ (cqf)	γ, Z and *l-q-box*	γ, Z and *l-q-box*
$\tau \to lP^0$ (ncqf)	*l-q-box*	*l-q-box*
$\tau \to lV^0$ (cqf)	γ, Z and *l-q-box*	γ, Z and *l-q-box*
$\tau \to lV^0$ (ncqf)	*l-q-box*	*l-q-box*
$\tau^- \to l'^- P_1 P_2$ (cqf)	all except *l-box*	γ, Z, H and *l-q-box*
$\tau^- \to l'^- P_1 P_2$ (ncqf)	*l-q-box* and W^+W^-	*l-q-box*
$\tau^- \to l'^- P_1 P_2$ (cqf,H)	H and W^+W^-	H

γ, Z, H, *l-q-box*, *l-box*, W^+W^- denote photon-penguin, Z-penguin, lepton-quark-box, lepton-box (one-loop amplitudes), and W^+W^- amplitudes (tree-level amplitude)

.

b. Basic blocks

∗ In every model all amplitudes can be expressed in terms of several combinations of loop functions and coupling constants – ”basic blocks” of the amplitudes.
∗ In the limit of large masses some of the basic blocks dominate – all decay rates of the processes with amplitudes having the same basic block behave in the same way – the ratios of these branching ratios are well defined numbers or simple expressions.

Model A :

* **Basic blocks** ($\lambda_{N_i} = m_{N_i}^2/M_W^2$)

$$
\begin{aligned}
\boldsymbol{\mathcal{A}_{ll'}} &= \sum_{N_i} (K_L)_{N_i l} (K_L^*)_{N_i l'} = \boldsymbol{s_L^{\nu_l} s_L^{\nu_{l'}} \tilde{x}_{ll'}} \quad \sim \quad \mathbf{1} \\
\boldsymbol{\mathcal{B}_{ll'}} &= \sum_{N_i} (K_L)_{N_i l} (K_L^*)_{N_i l'} \ln \lambda_{N_i} \quad \sim \quad \boldsymbol{s_L^{\nu_l} s_L^{\nu_{l'}} \tilde{x}_{ll'} \ln \lambda_N} \\
\boldsymbol{\mathcal{C}_{ll'}} &= \sum_{N_i N_j} (K_L)_{N_i l} (\Omega_L)_{N_i N_j} (K_L^*)_{N_j l'} \\
&\quad \times \frac{\lambda_{N_i} \lambda_{N_j}}{\lambda_{N_i} - \lambda_{N_j}} \ln \frac{\lambda_{N_i}}{\lambda_{N_j}} \quad \sim \quad \boldsymbol{s_L^{\nu_l} \sum_i (s_L^{\nu_{l_i}})^2 s_L^{\nu_{l'}} \tilde{y}_{ll'} \; \lambda_N} \\
\boldsymbol{\mathcal{D}_{ll' l_1 l_2}} &= \sum_{N_i N_j} (K_L)_{N_i l} (K_L)_{N_j l_2} \Big((K_L^*)_{N_i l'} (K_L)_{N_j l_1} + (K_L^*)_{N_i l_1} (K_L)_{N_j l'} \Big) \\
&\quad \times \frac{\lambda_{N_i} \lambda_{N_j}}{\lambda_{N_i} - \lambda_{N_j}} \ln \frac{\lambda_{N_i}}{\lambda_{N_j}} \quad \sim \quad \boldsymbol{s_L^{\nu_l} s_L^{\nu_{l'}} s_L^{\nu_{l_1}} s_L^{\nu_{l_2}} \tilde{z}_{ll' l_1 l_2} \; \lambda_N}
\end{aligned}
$$

∗ explicit mass dependence

$$
\mathcal{A}_{ll'} \sim \mathbf{1}, \qquad \mathcal{B}_{ll'} \sim \boldsymbol{\ln m_N}, \qquad \mathcal{C}_{ll'} \sim m_N^2, \qquad \mathcal{D}_{ll' l_1 l_2} \boldsymbol{\sim} \boldsymbol{m_N^2}
$$

∗ perturbative unitarity $s_L^{\nu_l} \sim \boldsymbol{1/m_N}$ for $m_N \to \infty$
∗ complete mass dependence

$$
\mathcal{A}_{ll'} \sim \boldsymbol{1/m_N^2}, \qquad \mathcal{B}_{ll'} \sim \boldsymbol{\ln m_N / m_N}, \qquad \mathcal{C}_{ll'} \sim \boldsymbol{1/m_N^2}, \qquad \mathcal{D}_{ll' l_1 l_2} \sim \boldsymbol{1/m_N^2}
$$

∗ no LFV in the $m_N \to \infty$ limit;
∗ There is a window of m_N values where LFV can be comparable with the present experimental limits. Within that window the Z-boson and box amplitude give the dominant contribution in the Feynman gauge.

*** Dominant basic blocks and decays**

$\mathcal{A}_{ll'}$: $\quad l \to l'\gamma;$

$\mathcal{A}_{ll'}$ and $\mathcal{B}_{ll'}$: $\quad \tau \to lV^0(ncqf),\ lP^0(ncqf),\ lP_1P_2(ncqf);$

$\mathcal{C}_{ll'}$: $\quad \tau \to lP^0(cqf),\ lV^0(cqf),\ lP_1P_2(cqf),\ lP_1P_2(cqfH);$

The most interesting decay within the group with the same dominant basic block — process with the largest BR.

2. **Model B**

$*$ LFV amplitudes have more basic blocks – can be divided into three groups
$\circ$ γ-penguin ($\mathcal{A}^{\gamma}_{ll'}$),
$\circ$ Z-boson penguin ($\mathcal{A}^{Z}_{ll'}$) and
$\circ$ box amplitude ($\mathcal{A}^{box}_{ll'}$)
group.
$*$ within each group terms have the same functional dependence on m_{SUSY} (the characteristic mass of the SUSY particles) and on couplings,

$$\mathcal{A}^{\gamma}_{ll'} \sim 1/m^2_{SUSY} \qquad \mathcal{A}^{Z}_{ll'} \sim \ln m_{SUSY} \qquad \mathcal{A}^{box}_{ll'} \sim 1/m^2_{SUSY}$$

and

$$\mathcal{A}^{\gamma}_{ll'} \sim C^2_{l;\tilde{\chi};\tilde{l}} \qquad \mathcal{A}^{Z}_{ll'} \sim C^2_{l;\tilde{\chi};\tilde{l}} O^2 \qquad \mathcal{A}^{box}_{ll'} \sim C^4_{l;\tilde{\chi};\tilde{l}}$$

Where $C^2_{l;\tilde{\chi};\tilde{l}}$ represents lepton-slepton-(chargino or neutralino) coupling and O is a building block of the (chargino or neutralino)–Z-boson or slepton–Z-boson coupling.
$*$ For $m_{SUSY} \to \infty$, $C^2_{l;\tilde{\chi};\tilde{l}}$ and $O \sim 1/m^{\alpha}_{SUSY}$, $\alpha \geq 1$; therefore all terms have either approximatelly the same behaviour in mass or photon amplitude dominates.
$*$ The LFV amplitude dissapears in the $m_{SUSY} \to \infty$ limit.

*** Dominant basic blocks and decays**

Within the groups with the same dominant basic block one can compare the amplitudes and decay rates, and find which process within the group is the most interesting for finding LFV.

Comparison of experiment and theory, SL decays

Notation: $\boldsymbol{x}_{ll'} = \frac{\mathcal{A}_{ll'}}{\mathcal{A}_{ll'}^{max}}, \quad \boldsymbol{y}_{ll'} = \frac{\mathcal{C}_{ll'}}{\mathcal{C}_{ll'}^{max}},$

Process	B_{exp}^{UB}	$B_{th}^{UB}(A)$	$B_{th}^{UB}(B)$
$\tau \to e^- \pi^0$	$< 3.7 \times 10^{-6}$	$< 2.8 \times 10^{-6}$	$< 3 \times 10^{-8}$
$\tau \to \mu^- \pi^0$	$< 4.0 \times 10^{-6}$	$< 5.4 \times 10^{-7}$	$< 1.3 \times 10^{-7}$
$\tau \to e^- K_S^0$	$< 9.1 \times 10^{-7}$	$< 9.8 \times 10^{-16}$	
$\tau \to \pi^- K_S^0$	$< 9.5 \times 10^{-7}$	$< 1.9 \times 10^{-16}$	
$\tau \to e^- \eta$	$< 8.2 \times 10^{-6}$	$< 4.0 \times 10^{-7}$	
$\tau \to \mu^- \eta$	$< 9.6 \times 10^{-6}$	$< 4.8 \times 10^{-8}$	
$\tau \to e^- \rho^0$	$< 2.0 \times 10^{-6}$	$< 2.7 \times 10^{-6}$	
$\tau \to \mu^- \rho^0$	$< 6.3 \times 10^{-6}$	$< 5.3 \times 10^{-7}$	
$\tau \to e^- K^{*0}$	$< 5.1 \times 10^{-6}$	$< 2.4 \times 10^{-16}$	
$\tau \to \mu^- K^{*0}$	$< 7.5 \times 10^{-6}$	$< 4.7 \times 10^{-16}$	
$\tau \to e^- \overline{K}^{*0}$	$< 7.4 \times 10^{-6}$	$< 2.4 \times 10^{-15}$	
$\tau \to \mu^- \overline{K}^{*0}$	$< 7.5 \times 10^{-6}$	$< 4.7 \times 10^{-16}$	
$\tau \to e^- \pi^+ \pi^-$	$< 2.2 \times 10^{-6}$	$< 2.7 \times 10^{-6}$	
$\tau \to e^+ \pi^- \pi^-$	$< 1.9 \times 10^{-6}$		
$\tau \to \mu^- \pi^+ \pi^-$	$< 8.2 \times 10^{-6}$	$< 5.2 \times 10^{-7}$	
$\tau \to \mu^+ \pi^- \pi^-$	$< 3.4 \times 10^{-6}$		

$\tau \to e^-\pi^+K^-$	$< 6.4 \times 10^{-6}$	$< 3.3 \times 10^{-15}$
$\tau \to e^-\pi^-K^+$	$< 3.8 \times 10^{-6}$	$< 3.3 \times 10^{-15}$
$\tau \to e^+\pi^-K^-$	$< 2.1 \times 10^{-6}$	
$\tau \to e^-K_S^0K_S^0$	$< 2.2 \times 10^{-6}$	$< 7.3 \times 10^{-9}$
$\tau \to e^-K^+K^-$	$< 6.0 \times 10^{-6}$	$< 1.1 \times 10^{-6}$
$\tau \to e^+K^-K^-$	$< 3.8 \times 10^{-6}$	
$\tau \to \mu^-\pi^+K^-$	$< 7.5 \times 10^{-6}$	$< 6.4 \times 10^{-16}$
$\tau \to \mu^-\pi^-K^+$	$< 7.4 \times 10^{-6}$	$< 6.4 \times 10^{-16}$
$\tau \to \mu^+\pi^-K^-$	$< 7.0 \times 10^{-6}$	
$\tau \to \mu^-K_S^0K_S^0$	$< 3.4 \times 10^{-6}$	$< 6.3 \times 10^{-9}$
$\tau \to \mu^-K^+K^-$	$< 1.5 \times 10^{-5}$	$< 2.1 \times 10^{-7}$
$\tau \to \mu^+K^-K^-$	$< 6.0 \times 10^{-6}$	
$\tau \to e^-\pi^0\pi^0$	$< 6.5 \times 10^{-6}$	$< 4.8 \times 10^{-18}$
$\tau \to \mu^-\pi^0\pi^0$	$< 1.4 \times 10^{-5}$	$< 7.9 \times 10^{-19}$
$\tau \to e^-\pi^0\eta$	$< 2.4 \times 10^{-5}$	0
$\tau \to \mu^-\pi^0\eta$	$< 2.2 \times 10^{-5}$	0

$\mu \to e^-\gamma$	$< 1.2 \times 10^{-11}$	$\mathbf{8.08 \times 10^{-9} x_{\mu e}^2}$	$\mathbf{1.2 \times 10^{-11}}$
$\mu \to e^-e^+e^-$	$< 1.0 \times 10^{-12}$	$\mathbf{6.41 \times 10^{-7} y_{\mu e}^2}$	
$\mu \to e^-2\gamma$	$< 7.2 \times 10^{-11}$		
$\tau \to e^-\gamma$	$< 2.7 \times 10^{-6}$	$3.38 \times 10^{-8} x_{\tau e}^2$	1.0×10^{-9}
$\tau \to \mu^-\gamma$	$< 1.1 \times 10^{-6}$	$6.68 \times 10^{-9} x_{\tau\mu}^2$	4.5×10^{-9}
$\tau \to e^-e^+e^-$	$< 2.9 \times 10^{-6}$	$2.69 \times 10^{-6} y_{\tau e}^2$	
$\tau \to e^-\mu^+\mu^-$	$< 1.8 \times 10^{-6}$	$1.44 \times 10^{-6} y_{\tau e}^2$	
$\tau \to e^+\mu^-\mu^-$	$< 1.5 \times 10^{-6}$	$1.32 \times 10^{-9} y_{\tau\mu\mu e}^2$	
$\tau \to \mu^-e^+e^-$	$< 1.7 \times 10^{-6}$	$3.71 \times 10^{-7} y_{\tau\mu}^2$	
$\tau \to \mu^+e^-e^-$	$< 1.5 \times 10^{-6}$	$6.67 \times 10^{-9} y_{\tau ee\mu}^2$	
$\tau \to \mu^-\mu^+\mu^-$	$< 1.9 \times 10^{-6}$	$4.48 \times 10^{-7} y_{\tau\mu}^2$	

6. MSSM model with see-saw based on minimal SO(10) model

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a. MSSM model with see-saw

(A. Masiero et al., PRL 57, 961 (1986), J. Hisano et al., PRD 53 (1996) 2442)

1. three (five) universal parameters defining soft SUSY breaking terms at the SUSY breaking scale:

m_0, $M_{1/2}$, A_0 (, B and μ)

(scalar mass, gaugino mass, coefficient of trilinear coupling, parameters of the Higgs potential)

2. superpotential

$$\begin{aligned} W_Y &= Y_u^{ij}(u_R^c)_i q_j H_u + Y_d^{ij}(d_R^c)_i q_j H_d \\ &+ Y_\nu^{ij}(\nu_R^c)_i \ell_j H_u + Y_e^{ij}(e_R^c)_i \ell_j H_d + \frac{1}{2} M_{R_{ij}} (\nu_R^c)_i (\nu_R^c)_j + \mu H_d H_u \ , \end{aligned}$$

3. Soft breaking terms – parameters:

$\left(m_{\tilde{q}}^2\right)_{ij}$, $\left(m_{\tilde{u}}^2\right)_{ij}$, $\left(m_{\tilde{d}}^2\right)_{ij}$, $\left(m_{\tilde{\ell}}^2\right)_{ij}$, $\left(m_{\tilde{\nu}}^2\right)_{ij}$, $\left(m_{\tilde{e}}^2\right)_{ij}$, $m_{H_u}^2$, $m_{H_d}^2$,

B_ν, $M_{R_{ij}}$, A_u^{ij}, A_d^{ij}, A_ν^{ij}, A_e^{ij}, M_1, M_2, M_3

4. Universality boundary conditions

$$
\begin{aligned}
&\left(m^2_{\tilde{q}}\right)_{ij} = \left(m^2_{\tilde{u}}\right)_{ij} = \left(m^2_{\tilde{d}}\right)_{ij} = m_0^2\delta_{ij} \; , \\
&\left(m^2_{\tilde{\ell}}\right)_{ij} = \left(m^2_{\tilde{\nu}}\right)_{ij} = \left(m^2_{\tilde{e}}\right)_{ij} = m_0^2\delta_{ij} \; , \\
&m^2_{H_u} = m^2_{H_d} = m_0^2 \; , \\
&A_u^{ij} = A_0 Y_u^{ij} \; , \;\; A_d^{ij} = A_0 Y_d^{ij} \; , \\
&A_\nu^{ij} = A_0 Y_\nu^{ij} \; , \;\; A_e^{ij} = A_0 Y_e^{ij} \; , \\
&M_1 = M_2 = M_3 = M_{1/2} \; ,
\end{aligned}
$$

5. RGE evolution of soft SUSY terms to EW scale

6. determination of the μ parameter from Higgs minimization condition

$$
|\mu|^2 \;\; = \;\; \frac{m^2_{H_d} - m^2_{H_u}\tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}M_Z^2.
$$

7. Comment: LFV is determined by off-diagonal components of $\left(m^2_{\tilde{\ell}}\right)_{ij}$ and A_e^{ij}. They are induced by the LFV interactions through the neutrino Dirac Yukawa couplings, e.g.

$$
\begin{aligned}
\mu\frac{d}{d\mu}\left(m^2_{\tilde{\ell}}\right)_{ij} \;\; &= \;\; \mu\frac{d}{d\mu}\left(m^2_{\tilde{\ell}}\right)_{ij}|_{\text{MSSM}} \\
&+ \;\; \frac{1}{16\pi^2}\left(m^2_{\tilde{\ell}}Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m^2_{\tilde{\ell}} + 2Y_\nu^\dagger m^2_{\tilde{\nu}} Y_\nu + 2m^2_{H_u}Y_\nu^\dagger Y_\nu + 2A_\nu^\dagger A_\nu\right)_{ij}
\end{aligned}
$$

8. The mass matrices are determined by Higgs potential F terms, D terms, and soft SUSY breaking terms. Diagonalization of the mass matrices leads to the physical fields we mentioned before. Corresponding "weak" basis Lagrangian can be expressed in terms of physical fields. With that the Feynman rules of the model are defined.

b. $SO(10)$ **basis of MSSM – determination of Yukawa couplings**

T. Fukuyama and N. Okada, JHEP **0211**, 011 (2002)
K. Matsuda, Y. Koide, T. F. and H. Nishiura, PR**D 65**, 033008 (2002)
H.S. Goh, R.N. Mohapatra, Siew-Phang Ng, PL**B 570**, 215 (2003),
PR**D 68**, 115008 (2003)

1. Higgs superfields : **10**, **126**

2. Fermion mass matrices :

$$\begin{aligned} M_u &= c_{10} M_{10} + c_{126} M_{126}, \\ M_d &= M_{10} + M_{126}, \\ M_D &= c_{10} M_{10} - 3c_{126} M_{126}, \\ M_e &= M_{10} - 3M_{126}, \\ M_R &= c_R M_{126}, \end{aligned}$$

3. GUT relation (follows from the mass matrices) – valid only at the GUT scale

$$M_e = c_d \left(M_d + \kappa M_u \right) \ ,$$

$$\begin{aligned} c_d &= -\frac{3c_{10} + c_{126}}{c_{10} - c_{126}}, \\ \kappa &= -\frac{4}{3c_{10} + c_{126}}. \end{aligned}$$

4. Choice of basis:
real and diagonal M_u, $M_u = D_u$
$M_d = V^*_{\rm CKM}\, D_d\, V^\dagger_{\rm CKM}$

5. Equations for κ and $|c_d|$

$$
\begin{aligned}
\left(\frac{\mathrm{tr}[\widetilde{M_e}^\dagger \widetilde{M_e}]}{m_e^2+m_\mu^2+m_\tau^2}\right)^2 &= \frac{\mathrm{tr}[(\widetilde{M_e}^\dagger \widetilde{M_e})^2]}{m_e^4+m_\mu^4+m_\tau^4}, \\
\left(\frac{\mathrm{tr}[\widetilde{M_e}^\dagger \widetilde{M_e}]}{m_e^2+m_\mu^2+m_\tau^2}\right)^3 &= \frac{\det[\widetilde{M_e}^\dagger \widetilde{M_e}]}{m_e^2\, m_\mu^2\, m_\tau^2},
\end{aligned}
$$

$\widetilde{M_e} \equiv V_{\mathrm{CKM}}^*\, D_d\, V_{\mathrm{CKM}}^\dagger + \kappa D_u$

6. Solution for κ and $|c_d|$ in terms of six quark masses, three angles and one CP-phase in the CKM matrix and three charged lepton masses – more precisely from these quantities at the GUT scale for given $\tan\beta$, obtained by RGE evolution starting at the EW scale. Phase of c_d is undetermined.

7. Solution for M_{10} and M_{126} in terms of κ and $|c_d|$

$$
\begin{aligned}
M_{10} &= \frac{3+|c_d|e^{i\sigma}}{4} V_{\mathrm{CKM}}^*\, D_d\, V_{\mathrm{CKM}}^\dagger + \frac{|c_d|e^{i\sigma}\kappa}{4} D_u, \\
M_{126} &= \frac{1-|c_d|e^{i\sigma}}{4} V_{\mathrm{CKM}}^*\, D_d\, V_{\mathrm{CKM}}^\dagger - \frac{|c_d|e^{i\sigma}\kappa}{4} D_u,
\end{aligned}
$$

(relation valid only at the GUT scale)

8. The solution of GUT relation exists. E.g. for $\tan\beta \sim 45$.

$$
\kappa = -0.00999071 + 0.000496015\, i\ ,
$$
$$
|c_d| = 8.13989\ .
$$

9. With that solution the fermion mass matrices and Yukawa couplings are determined as functions phase of $|c_d|$, σ.

10. Solution of the RGE for soft SUSY breaking parameters and the Yukawa couplings in the MSSM to determine the masses and mixings for the SUSY particles. With that MSSM parameters are determined.

7. Conclusions from the work in progress

1. The MSSM SL LFV and $l \to l'\gamma$ processes have been studied and evaluated in MSSM with see-saw based on minimal $SO(10)$ model, and compared with the corresponding expressions in the SM with additonal Dirac neutrinos.

2. The structure of the basic blocks has been found to be the same on general level, but the MSSM (model B) basic blocks tend faster to zero as characteristic mass of physics beyond the SM tends to infinity.

3. The MSSM parameters have dark matter upper bound constraint and have the lower bound constraint from μ magnetic moment and $\mu \to e\gamma$, while the model A parameters have the upper bound on parameter space only from lepton flavor diagonal processes.

4. The preliminary result for SL LFV and $l \to l'\gamma$ decays show that all LFV decay rates are smaller than PDG upper limits except for $\mu \to e\gamma$ which has been used to constraint the MSSM parameter space. The same is for model A.